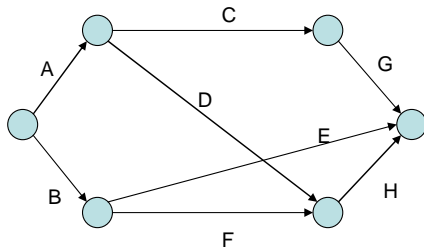


Mechanism Design

Week 9

Selfish Routing



- 8 people play as agents $A - H$; the others act as mediators.
- Agents' utility functions: $u_i = \text{payment} - \text{cost}$ if your edge is chosen; 0 otherwise.
- Mediators: find me a path from source to sink, at the lowest cost you can.
- Agents: agree to be paid whatever you like; claim whatever you like; however, you can't show your paper to anyone.

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- What is the difference between direct and indirect implementation?

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- What should we conclude from it?

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 - General form of quasilinear utility: $u_i(o, \theta) = u_i(x, \theta) - f_i(p_i)$
for monotone increasing f_i
 - We'll come back to f_i at the end if there's time.
 - For now: assume that f_i is the identity function $f_i(x) = x$
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$$\theta_i = \theta'_i \Rightarrow u_i(o, \theta) = u_i(o, \theta')$$

Quasilinear Mechanisms with Conditional Utility Independence

- Given conditional utility independence, we can write i 's utility function as $u_i(o, \theta_i)$
 - it does not depend on the other agents' types
- An agent's **valuation** for choice $x \in X$: $v_i(x) = u_i(x, \theta_i)$
 - the maximum amount i would be willing to pay to get x
 - in fact, i would be indifferent between keeping the money and getting x
- Alternate definition of **direct mechanism**:
 - ask agents i to declare $v_i(x)$ for each $x \in X$
- Define \hat{v}_i as the valuation that agent i declares to such a direct mechanism
 - may be different from his true valuation v_i
- Also define the tuples \hat{v}, \hat{v}_{-i}

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- What is revenue maximization? Why might we prefer minimization?
- How might we define fairness?
- What does it mean to minimize the price of anarchy?

Fun game

- Look at your piece of paper: it contains an integer x
- Go around the room offering everyone the following gamble:
 - they pay you x
 - you flip a coin:
 - heads: they win and get paid $2x$
 - tails: they lose and get nothing.
 - Players can accept the gamble or decline.
 - Answer honestly (imagining the amounts of money are real)
 - play the gamble to see what would have happened.
 - Keep track of:
 - Your own “bank balance” from others’ gambles you accepted.
 - The number of people who accepted your offer.

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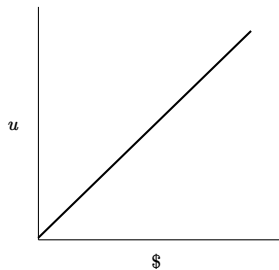
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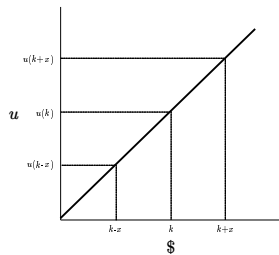
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 - Possibly different amounts, depending on how risky it is

Risk Neutrality

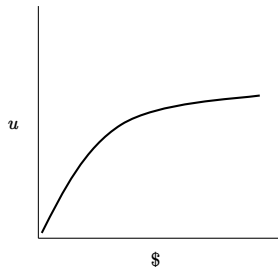


(a) Risk neutrality

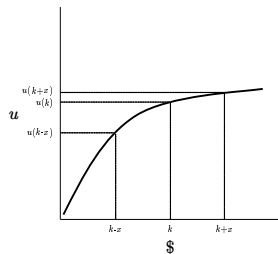


(b) Risk neutrality: fair lottery

Risk Aversion

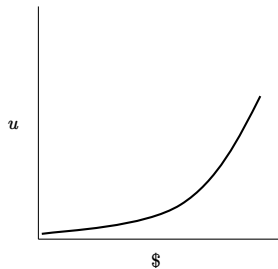


(c) Risk aversion

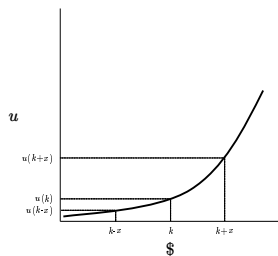


(d) Risk aversion: fair lottery

Risk Seeking



(e) Risk seeking



(f) Risk seeking: fair lottery