Mechanism Design for Multicast Cost Sharing in Wireless Networks

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1 Introduction

A large amount of Internet applications, involve multiple clients getting service from a single server. Unicast as the traditional routing scheme is inefficient in terms of resource allocation dealing with such applications. Unicast scheme necessitates that the source forwards the same data multiple times for each user, placing a large load on the server and wasting the resources. Multicast routing as an alternative scheme tries to reduce this problem by building a tree from source as the root to the receivers; data is sent once through each link; intermediate routers send replicas of the data to each of their outgoing edges determined by the tree. Since many users are simultaneously trying to benefit a single resource, multicast requires a mechanism to determine which user can receive the data and what is the cost for her. Multicast cost sharing has been recently studied in many theoretical contexts for mechanism design.

While this problem has been widely studied for wired networks, there has not been much effort for the wireless case. Wireless networks are gradually becoming the prominent communication method between people. The broadcast nature of these networks, makes them different from wired networks. The cost of the communication in these networks is not simply the sum of the links costs. This complicates the mechanism design for multicast cost sharing in these networks. In this report we mainely study two papers ([2, 7]) on wireless multicast cost sharing, and discuss some open research issues in this area. Definitely, the study of the available research on widely-studied wired case, will help a lot. In the next section we briefly introduce some related works in wired case ([6, 8, 3, 4, 5, 1]). In section 3, we study the general case where the wireless graph is modeled as a complete graph (N, c), where N is the set of network nodes (radio stations), and $c : N^2 \rightarrow \Re$ is the transmission cost. The mechanism assigns power to each station. If the power assigned to node $i \in N$ is $P(i)$, all the nodes $j \in N$ that $c(i, j) \leq P(i)$ can receive the transmission. In section 4 we study the more practical case where $c(i, j) \propto distance(i, j)^\alpha$, for $1 \leq \alpha \leq 6$ (called the attenuation factor). In section 5, we discuss [2] and [7], and bring up some open research issues on wireless multicast cost sharing. The report is concluded in section 6.

2 Multicast Cost Sharing

Consider a set of network nodes (agents)[1] N, a set of receivers $R(v) \subseteq N$, and a sender $s \in N$. Multicast cost sharing mechanism determines if any agent $i \in R(v)$ should receive the transmission, if yes answers to this question how much she should pay for that($p_i(v)$), and how she is connected to the source. Mechanism defines a tree (T) that provides connection to some agents (destinations) $D(T) \subseteq R(v)$. Each agent with valuation v_i will have a utility[2] of $u_i(v) = v_i - p_i(v)$, if $i \in D(T)$ and $u_i(v) = 0$ if not. Multicast through T has a cost $C(T)$[3], that agents should pay for that.

2.1 Requirements of the mechanism

A multicast cost sharing problem should have these basic properties:

No Positive Transfer (NPT): Mechanism does not pay any user ($p_i(v) \geq 0$)); since the mechanism incurs some cost $C(T)$, participating users should compensate for that.

Voluntary Participation (VP): Users are not charged more than their valuation ($u_i(v) \geq 0$); this means that if the predicted cost for a user is more than her valuation, she should not be obliged to receive the transmission and her utility will be 0.

Costumer Sovereignty (CS): Each user should receive the transmission, if she declares a high enough value $\hat{v}_i$.

A very important requirement of a cost sharing mechanism is its being dominant strategy truthful (also known as *strategy proof* in the literature). A mechanism should be designed in a way that no agent has incentive to misreport her value. A stronger condition than strategy proof is *group strategy proof*, where no group of agents have incentives to cooperate and misreport their value. Some other important requirements are:

Budget Balance (BB): Agents should exactly compensate the network cost. *i.e.* $\sum_{i \in D(T)} p_i(v) = C(T)$. If $C(T) \leq \sum_{i \in D(T)} p_i(v) \leq \beta C(T)$, the mechanism is said to be β-approximate budget balanced (β-BB).

[1] In the problem of cost sharing, network nodes and users are considered the same. Actually any user is connected via a specific node $n \in N$. If a user requests some data form another one, their corresponding network nodes should make a communication path. Without loss of generality each node is associated with a user and visa versa. We will call both of them (users and nodes) the agents.

[2] An agent utility is called her individual welfare in the literature. The overall network welfare is called net worth (NW).

[3] The cost of a multicast tree consists of the cost of the data and the cost of the transmission.

Efficiency: A subset $D(T) \subseteq N$ is efficient, if $NW(T) \geq NW(T'), \forall T' \in \{T' | D(T') \subseteq R(v)\}$. $NW(T)$ (Net Worth of T) is defined as $\sum_{i \in D(T)} v_i - C(T)$. When $\beta NW(T) \geq NW(T'), \forall T' \in \{T' | D(T') \subseteq R(v)\}$ the mechanism is said to be β-efficient. In this case β is also called the *price of anarchy* [9].

Cost Optimality (CO): A mechanism is cost optimal if it adopts the minimum cost tree (T^*) for a specific set of receivers R',
($T^* = \underset{T'}{\operatorname{argmin}}\{C(T') | D(T') = R'\}$).

A classical theory in mechanism design (Green-Laffont;Hurwicz) states that no strategy proof mechanism can be both BB and efficient[4] [9]. VCG mechanisms [9] are well known classes of efficient, truthful mechanisms. *Marginal Cost* (MC) mechanisms are a subclass of VCG mechanisms that are NPT, VP, and CS. In fact all strategies that have the characteristics of MC differ from MC mechanisms only in number of users with zero utility [6]. MC selects the largest efficient set of receivers among these mechanisms; assuming $R^*(v)$ to be this set, MC basically charges each user i its marginal cost:

$$p_i(v) = C(R^*(v) - C(R^*(v_{-i})) \tag{1}$$

When the cost function is non-decreasing, non-negative, and sub-modular[5] MC mechanisms are the only strategy proof, efficient mechanisms. As mentioned before MC is not budget balanced.

Considering budget balance, *Shapley value Mechanisms* have the lowest price of anarchy among all mechanisms that satisfy NPT, VP, and CS. While MC is not group strategy proof, the Shapley value mechanism has this property. When the cost function is non-decreasing and sub-modular, Shapley value mechanism incurs a cross monotonic[6] cost sharing method. This means that for such cost functions payments of a Shapley value mechanism are in the core[7]. Shapley payments are computed from:

$$p(R, x_i) = \sum_{Q \subseteq R\{x_i\}} \frac{|Q|!(|R| - |Q| - 1)!}{|R|!} (C(T(Q \cup \{x_i\})) - C(T(Q))) \tag{2}$$

The Shapley value mechanism simple drops the agents that have declared a value less than their payment defined by the Shapley value. Sub-modular, non-decreasing cost functions support the entire class of cross monotonic cost functions; Shapely value is one of them. If no cross monotonic payment function

[4]With a different but equivalent definition for efficiency, Roughgarden *et al.* in [8] has shown that approximate efficiency and approximate-BB are achievable simultaneously. They have defined efficiency as "social cost minimization", social cost is the sum of the values of all the agents minus the net worth.

[5]This means that (1) $C(\emptyset) = 0$, (2)$Q \subseteq R \Rightarrow C(Q) \leq C(R)$ (3) $C(Q \cup R) \leq C(Q) + C(R) - C(Q \cap R)$, since cost is a negative utility, sub-modularity of the cost function equals convexity of the game.

[6]$f(R, x_i)$ is cross monotonic if $Q \subseteq R \Rightarrow f(Q, x_i) \geq f(R, x_i) \;\; \forall x_i \in Q$.

[7]Payments of a mechanism are in the core when no group of agents can obtain a better welfare by collusion.

exists for a mechanism, the Shapley value does not result in a strategy proof BB mechanism. On the other hand if the cost function is not sub-modular, MC mechanism will not yield the largest efficient set, and so will not be efficient. From the above discussion, it is clear that if a cost function is non-decreasing and sub-modular, then MC results in an efficient, strategy proof mechanism, and Shapley value results in a BB one. In [6], it is assumed that a global fixed tree is given, and each multicast tree is computed by the union of the pathes in the global tree. This gives rise to the desired sub-modular non-decreasing cost function. The problem is that the mentioned mechanism do not produce a cost optimal tree that is the minimum Steiner tree. However, the problem of finding a Steiner tree is NP-hard. In this section the classical results in the wired networks and the concepts of multicast cost sharing were discussed, in the next two section we try to extend the results to the wireless case.

3 Wireless Networks, General Communication Graphs

In this section, we limit ourselves to the case where the power assignments are induced by a universal broadcast tree from the communication graph $G = (N, c)$, where $c : N^2 \rightarrow \Re$ is the cost function. We can prove that any universal tree (T), results in a non-decreasing sub-modular cost function [2]. So the Shapley value results in a BB strategy proof mechanism and the MC results in an efficient one.

Penna *et al.* ([7]), proved that if an oracle provides a tree containing the optimal multicast one, the cost sharing mechanism admits a strategy proof, efficient mechanism that satisfies NPT, VP, CS, and CO. The mechanism (that is a VCG mechanism) is similar to [6], and proceeds with $O(1)$ messages per link, in a bottom-up distributed manner. It computes the optimal tree T_i at each node $i \in N$, knowing the valuation of its children $ch(i)$ upon receipt of message μ^j from $j \in ch(i)$. T_i includes (i, j) if j and all the nodes that are connected if j is so, can compensate the cost of link (i, j):

$$\mu^i_j \sum_{k \in ch(i) | c_k \leq c_j} \mu^k - c(i,j) \geq 0 \tag{3}$$

Node i sends $\mu^i = v_i + max\{\mu^i_j, 0\}$ to its parent $p(i)$. Payments of the mechanism are computed in a top-down distributed manner again with $O(1)$ message per link, exactly like VCG payments. The algorithm is distributed since, if a node opts out, all of its children are disconnected, and payments can be computed from parents down to children with one message per link.

Instead of the universal tree containing the optimal multicast tree (provided by the oracle!), Penna *et al.* showed that a pre-computed shortest path tree (computed polynomially) can be used. The result would be a strategy proof mechanism satisfying NPT, VP, and $O(n)$-CO. If the resulted multicast tree T satisfies $|D(T)| \leq \gamma(NW(T)/C(T) - 1)$, it will also be $D(T)$-efficient. However

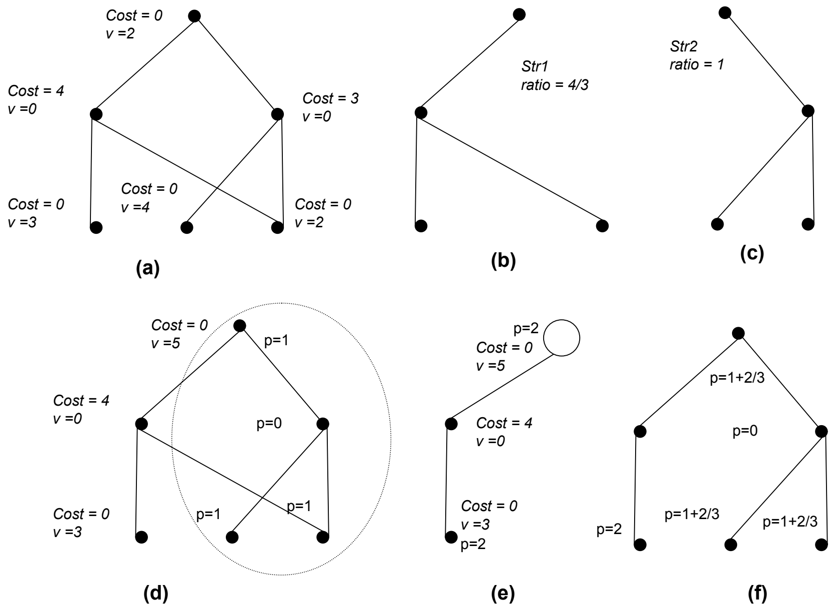

Figure 1: (a) Network graph, nodes that should be connected have positive utilities, (b and c) 3^+ stars with (c) being the minimum ratio one, (d) all the terminals can pay they ratio and star is shrunk to a new super node, (e) optimal connection of the remaining terminals, (f) final payments: terminals in the super node should share the new cost as well.

they prove that in general. no polynomial algorithm can guarantee β-efficiency, unless P=NP.

Bilò *et al.* in [2] on the other hand investigated the existence of β-BB mechanisms for wireless networks defined over communication graphs. They proved that such mechanisms exist for $\beta = 3\ln(|D(T)| + 1)$. They adopted a method for changing the edge weighted graph to a node weighted one. Following notes can be extracted from [2].

Note 1: Every Minimum Energy Multicast Tree (MEMT) problem can be converted to a minimum Node Weighted Steiner tree (NWST) problem.

Note 2: Every ρ approximation to the minimum NWST provides a 2ρ approximation to the MEMT problem.

Note 3: There exist a simple $1.5\ln(k)$ approximation algorithm for NWST (k is the number of receivers in NWST), this means that a $3\ln(k)$ approximation solution exists for MEMT.

Based on the $1.5\ln(k)$ approximation algorithm for NWST, Biló *et al.* proposed a $1.5ln(k)$-BB mechanism for node weighted graphs. The proposed mechanism is strategy proof but not group strategy proof. The mechanism starts by finding subtrees that contain at least three terminals (nodes that should be connected in the tree) and have at most one node with a degree more than two (these subtrees are known as 3^+ stars), ratio of a star Str is defined as: $ratio(Str) = C(Str)/n_{Str}$ where n_{Str} is the number of terminals in the star. The algorithm adopts the minimum ration 3^+ star , if terminals in this star can compensate their cost share (which is the ratio of the star), the star is shrunk as new terminal representing all of its terminals. Algorithm continues with the next minimum ratio star in the resulted network and repeats until only two terminals are left and then connects them optimally. If terminals of a star can not compensate its cost they are disconnected form the whole network. Fig. 1 shows how the algorithm works with a simple example.

Bilò *et al.* ([2]) proved that their proposed algorithm is $1.5\ln(k)$-CO, and is strategy proof, they also proved that the mechanism is not group strategy proof. Based on this algorithm authors proposed a $3\ln(k)$-BB, strategy proof algorithm for multicast transmission. The algorithm starts with setting $R(V) = N$
s, and transforms the MEMT problem to NWST, executes the previously mentioned algorithm, changes back to the MEMT, and omits some receivers[8]. The algorithm re-runs with the new set of receivers until both NWST and MEMT have the same set of receivers. The algorithm meets NPT, VP, CS, and is $3\ln(|D(T)|+1)$-BB. In the next section mechanism design in a more practical model for wireless networks, is studied.

4 Wireless Networks in Euclidian Space

As mentioned in the introduction, wireless networks in practice are usually modeled as complete graphs $G = (N, c)$, in a d-dimensional euclidian space. where $c(i,j) = \gamma dist(i,j)^{\alpha}$. $\gamma > 0$, is a constant that represents the minimal reception power of a node and $1 \leq \alpha \leq 6$, is the power attenuation factor, and $dist(i,j)$ is the euclidian distance between $i, j \in N$. Lets first consider the case where, $\alpha = 1$ or $d = 1$, and then study the more complicated (and practical) scenario of $\alpha > 1$ and $d > 1$.

4.1 $\alpha = 1$ or $d = 1$

Bilò *et al.* in [2] showed that the cost functions in these cases are non-decreasing and sub-modular. As a result, efficiently computable NPT, VP, CS, and strategy proof mechanisms exist, among which MC is efficient, and Shapley value mechanism is budget balanced.

[8]NWST does not distinguish between senders and the receivers, directions in MEMT are dictated by the sender.

It can be easily shown that in case of $\alpha = 1$ (i.e. required transmission power changes linearly with distance), the optimal cost $C^*(R)$ for a set of receivers $R \subseteq N \backslash s$, is simply defined by the farthest receiver in R, i.e. $\gamma * \max\{dist(s,j)|j \in R\}$. This cost is $O(n)$ computable.

In the case where $d = 1$ (i.e. network nodes lie on a line), it is shown in [2] that, once the power assignment of the sender s, is known optimal cost for the any other nodes $j \in R$, is calculated from its very next farther receiver from s, if j is the last node in the platoon, it is assigned a power of 0. The optimal power assignment for the sender is also $O(n)$ computable, testing any $dist(s,j)$.

From the above, we can understand that the Shapely value mechanism is $O(n)$ computable. The largest efficient set for the MC mechanism is $O(n)$ computable when $\alpha = 1$; for the case $d = 1$, the largest efficient set is determined by testing any pair of nodes as the two ends of the platoon, thus $O(n^2)$ computable. In the next section the more practical case of $\alpha > 1$ and $d > 1$ is studied.

4.2 $\alpha > 1$ and $d > 1$

Let us start with two negative results:
For any $\alpha > 1$ and $d > 1$, the problem of finding the largest efficient set is NP-hard ([7]).
For any $\alpha > 1$ and $d > 1$, there exist instances of wireless networks for which the core is empty([2]).

As we mentioned before, if a cross monotonic function exists, the core is non-empty; thus the emptiness of the core precludes exitance of such a function that precludes the assumption that the cost function is non-decreasing and submodular. So, in general, BB or efficient strategy proof mechanisms do not exist for wireless networks, when $\alpha > 1$ and $d > 1$. We shall continue with some notes from [2] and [7], until we come to a positive result!

- If $R(v) = N \backslash s$, for any $d \geq 1$ and $\alpha \geq d$ there exists a constant c^d_α, such that the cost of the directed minimum spanning tree (MST) (directions defined by the s in the undirected MST) is at most c^d_α times of the core of the optimal tree. This is not true in general for $R \subset N \backslash s$ ([7]).

- It is shown in [2] that, for the general case of($R \subseteq N \backslash s$ if there exists a $j \in R$ such that, $c(k,j) \leq c$,[9] directed MST, has a cost of at most $\gamma(3^d - 1)c^\alpha$ ([2]).

- Cost of the directed minimum Steiner tree (T) for the above case satisfies: $(T) \leq (3^d - 1)C^*(R)$, where $C^*(R)$ is the cost of the optimal tree ([2]).

From the above Bilò *et al.* in [2] proved the following theorem:
For ever $\alpha \geq d$ there exist a family of efficiently computable $2(3^d - 1)$-BB group strategy proof mechanisms for wireless multicast problems in any d-dimensional euclidian space.

[9] This assumption is in general true for every network in a bordered area, the important assumption that authors have not mentioned is the power limitation, i.e. can two farthest nodes in a network communicate using their maximum power or not?

In the next section we discuss some properties of these mechanisms.

5 Discussion and open research areas

Bilò *et al.* in [2], did an extensive study of the multicast cost sharing mechanisms for the wireless networks modeled by a complete graph. An important issue in wireless networks is their distributed nature. Some nodes may not be able to communicate directly due to power limitations and so the complete graph model is not suitable. Mechanisms proposed in [2], that were efficiently computable, where run globally, ad hoc implementation of these algorithms may not be possible. The study of distributed multicast cost sharing mechanisms for wireless networks with non-complete graph models where each node only has its n-hop neighborhood information is a practical and challenging open research issue.

Other issue in the mechanism design for wireless multicast cost sharing, rises from the NPT. A desired property of multicast cost sharing in wired networks was that mechanism should not pay any agent. For the energy constrained wireless networks this property seems not suitable. When the receivers are a subset of the nodes, all the proposed mechanisms require some mediator nodes that do not receive the data, pay nothing, but help in forwarding the data. A power limitation necessitates that these nodes do not cooperate. This requires that the mechanism pays some positive amount to these nodes for the service they provide. Another open research area is to investigate the existence of approximate-BB or efficient mechanisms that consider this extra payments.

Another issue is that powers were assigned according to the receiver's maximum cost outgoing edge, but most of the protocols that are commonly used in wireless ad hoc networks assume symmetric links between wireless nodes. If a node is sending some data to its child, the child should be able to communicate with the parent as well. Mechanisms mentioned above charge nodes based on their ingoing edges and pay them based on their outgoing edges. Investigating the existence of mechanisms that take into account the symmetric property of each link can be a practical continuation of research in this area.

6 Conclusion

In this report, the desired properties of mechanism design for multicast cost sharing in wireless networks were discussed. Wireless networks studied were modeled as complete graphs $G(N, c)$. We mentioned that when an optimal universal tree is given, budget balance and efficient truthful cost sharing mechanisms exist and are efficiently computable. For the practical case where the costs are power of link lengths, and no global tree is given, we mentioned that in general budget balance and efficiency are not attainable. Tn the case where the attenuation factor is greater than the dimension d we narrated the existence of approximate budget balanced strategy proof mechanisms. At last we brought up some problems and open research issues from the studied papers.

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