Game Theory intro

CPSC 532A Lecture 3

Lecture Overview

1. Recap
2. Example Matrix Games
3. Pareto Optimality
4. Best Response and Nash Equilibrium

Defining Games

- Finite, n-person game: $\langle N, A, u \rangle$:
  - N is a finite set of n players, indexed by i
  - $A = A_1 \times \ldots \times A_n$, where A_i is the action set for player i
    - $(a_1, \ldots, a_n) \in A$ is an action profile, and so A is the space of action profiles
  - $u = \langle u_1, \ldots, u_n \rangle$, a utility function for each player, where $u_i : A \mapsto \mathbb{R}$

- Writing a 2-player game as a matrix:
  - row player is player 1, column player is player 2
  - rows are actions $a \in A_1$, columns are $a' \in A_2$
  - cells are outcomes, written as a tuple of utility values for each player

Games in Matrix Form

Here's the TCP Backoff Game written as a matrix ("normal form").

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

It's an example of prisoner's dilemma.

Lecture Overview

Games of Pure Competition

Players have exactly opposed interests

- There must be precisely two players (otherwise they can't have exactly opposed interests)
- For all action profiles $a \in A$, $u_1(a) + u_2(a) = c$ for some constant c
  - Special case: zero sum
- Thus, we only need to store a utility function for one player
  - in a sense, it's a one-player game

Matching Pennies

One player wants to match; the other wants to mismatch.

	Heads	Tails
Heads	1	-1
Tails	-1	1

Matching Pennies

One player wants to match; the other wants to mismatch.

	Heads	Tails
Heads	1	-1
Tails	-1	1

Play this game with someone near you, repeating five times.

Rock-Paper-Scissors

Generalized matching pennies.

	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

...Believe it or not, there's an annual international competition for this game!

Games of Cooperation

Players have exactly the same interests.

- no conflict: all players want the same things
- $\forall a \in A, \forall i, j,\ u_i(a) = u_j(a)$
- we often write such games with a single payoff per cell
- why are such games "noncooperative"?

Coordination Game

Which side of the road should you drive on?

	Left	Right
Left	1	0
Right	0	1

Coordination Game

Which side of the road should you drive on?

	Left	Right
Left	1	0
Right	0	1

Play this game with someone near you. Then find a new partner and play again. Play five times in total.

General Games: Battle of the Sexes

The most interesting games combine elements of cooperation *and* competition.

	B	F
B	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

General Games: Battle of the Sexes

The most interesting games combine elements of cooperation *and* competition.

	B	F
B	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

Play this game with someone near you. Then find a new partner and play again. Play five times in total.

Lecture Overview

Analyzing Games

- We've defined some canonical games, and thought about how to play them. Now let's examine the games from the outside
- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?

Analyzing Games

- We've defined some canonical games, and thought about how to play them. Now let's examine the games from the outside
- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?
  - we have no way of saying that one agent's interests are more important than another's
  - intuition: imagine trying to find the revenue-maximizing outcome when you don't know what currency has been used to express each agent's payoff
- Are there situations where we can still prefer one outcome to another?

Pareto Optimality

- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
  - in this case, it seems reasonable to say that o is better than o'
  - we say that o Pareto-dominates o'.

Pareto Optimality

- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
  - in this case, it seems reasonable to say that o is better than o'
  - we say that o Pareto-dominates o'.
- An outcome o^* is Pareto-optimal if there is no other outcome that Pareto-dominates it.

Pareto Optimality

- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
  - in this case, it seems reasonable to say that o is better than o'
  - we say that o Pareto-dominates o'.

- An outcome o^* is Pareto-optimal if there is no other outcome that Pareto-dominates it.
  - can a game have more than one Pareto-optimal outcome?

Pareto Optimality

- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
  - in this case, it seems reasonable to say that o is better than o'
  - we say that o Pareto-dominates o'.
- An outcome o^* is Pareto-optimal if there is no other outcome that Pareto-dominates it.
  - can a game have more than one Pareto-optimal outcome?
  - does every game have at least one Pareto-optimal outcome?

Pareto Optimal Outcomes in Example Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

Pareto Optimal Outcomes in Example Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

	Left	Right
Left	1	0
Right	0	1

Pareto Optimal Outcomes in Example Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

	Left	Right
Left	1	0
Right	0	1

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

Pareto Optimal Outcomes in Example Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

	Left	Right
Left	1	0
Right	0	1

	B	F
B	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

	Heads	Tails
Heads	1	-1
Tails	-1	1

Lecture Overview

Best Response

- If you knew what everyone else was going to do, it would be easy to pick your own action

Best Response

- If you knew what everyone else was going to do, it would be easy to pick your own action
- Let $a_{-i} = \langle a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n \rangle$.
  - now $a = (a_{-i}, a_i)$

- Best response: $a_i^* \in BR(a_{-i})$ iff $\forall a_i \in A_i,\ u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i})$

Nash Equilibrium

- Now let's return to the setting where no agent knows anything about what the others will do
- What can we say about which actions will occur?

Nash Equilibrium

- Now let's return to the setting where no agent knows anything about what the others will do
- What can we say about which actions will occur?

- Idea: look for stable action profiles.
- $a = \langle a_1, \ldots, a_n \rangle$ is a ("pure strategy") Nash equilibrium iff $\forall i,\ a_i \in BR(a_{-i})$.

Nash Equilibria of Example Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

Nash Equilibria of Example Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

	Left	Right
Left	1	0
Right	0	1

Nash Equilibria of Example Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

	Left	Right
Left	1	0
Right	0	1

	B	F
B	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

Nash Equilibria of Example Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

	Left	Right
Left	1	0
Right	0	1

	B	F
B	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

	Heads	Tails
Heads	1	-1
Tails	-1	1

Nash Equilibria of Example Games

	C	D
C	$-1,-1$	$-4,0$
D	$0,-4$	$-3,-3$

	Left	Right
Left	1	0
Right	0	1

	B	F
B	$2,1$	$0,0$
F	$0,0$	$1,2$

	Heads	Tails
Heads	1	-1
Tails	-1	1

The paradox of Prisoner's dilemma: the Nash equilibrium is the only non-Pareto-optimal outcome!