

# Auctions Introduction

## Lecture 18

# Lecture Overview

- 1 Auctions
- 2 Canonical Auctions
- 3 Comparing Auctions
- 4 Second-price auctions

# Motivation

- Auctions are any mechanisms for **allocating resources among self-interested agents**
- Very widely used
  - government sale of resources
  - privatization
  - stock market
  - request for quote
  - FCC spectrum
  - real estate sales
  - eBay

# CS Motivation

- **resource allocation** is a fundamental problem in CS
- increasing importance of studying distributed systems with heterogeneous agents
- markets for:
  - computational resources (JINI, etc.)
  - SETI, etc.
  - autonomous agents
  - P2P systems
  - network bandwidth
- currency needn't be real money, just something scarce
  - that said, real money trading agents are also an important motivation

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# Some Canonical Auctions

- English
- Japanese
- Dutch
- First-Price
- Second-Price
- All-Pay

# English Auction

## English Auction

- auctioneer starts the bidding at some “reservation price”
- bidders then shout out ascending prices
- once bidders stop shouting, the high bidder gets the good at that price

# Japanese Auction

## Japanese Auction

- Same as an English auction except that the auctioneer calls out the prices
  - all bidders start out standing
  - when the price reaches a level that a bidder is not willing to pay, that bidder sits down
    - once a bidder sits down, they can't get back up
  - the last person standing gets the good
- 
- analytically more tractable than English because jump bidding can't occur
    - consider the branching factor of the extensive form game...



# Dutch Auction

## Dutch Auction

- the auctioneer starts a clock at some high value; it descends
- at some point, a bidder shouts “mine!” and gets the good at the price shown on the clock

# First-, Second-Price Auctions

## First-Price Auction

- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- that bidder pays the amount of his bid

## Second-Price Auction

- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- that bidder pays the amount bid by the second-highest bidder

# All-Pay auction

## All-Pay Auction

- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- everyone pays the amount of their bid regardless of whether or not they win

# Auctions as Structured Negotiations

Any negotiation mechanism that is:

- **market-based** (determines an exchange in terms of currency)
- **mediated** (auctioneer)
- **well-specified** (follows rules)

Defined by three kinds of rules:

- rules for bidding
- rules for what information is revealed
- rules for clearing

# Auctions as Structured Negotiations

Defined by three kinds of rules:

- rules for **bidding**
  - who can bid, when
  - what is the form of a bid
  - restrictions on offers, as a function of:
    - bidder's own previous bid
    - auction state (others' bids)
    - eligibility (e.g., budget constraints)
    - expiration, withdrawal, replacement
- rules for what information is revealed
- rules for clearing

# Auctions as Structured Negotiations

Defined by three kinds of rules:

- rules for bidding
- rules for **what information is revealed**
  - when to reveal what information to whom
- rules for clearing

# Auctions as Structured Negotiations

Defined by three kinds of rules:

- rules for bidding
- rules for what information is revealed
- rules for **clearing**
  - when to clear
    - at intervals
    - on each bid
    - after a period of inactivity
  - allocation (who gets what)
  - payment (who pays what)

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# Intuitive comparison of 5 auctions

	English	Dutch	Japanese	1 <sup>st</sup> -Price	2 <sup>nd</sup> -Price
<b>Duration</b>	#bidders, increment	starting price, clock speed	#bidders, increment	fixed	fixed
<b>Info Revealed</b>	2 <sup>nd</sup> -highest val; bounds on others	winner's bid	all val's but winner's	none	none
<b>Jump bids</b>	yes	n/a	no	n/a	n/a
<b>Price Discovery</b>	yes	no	yes	no	no

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- How should agents bid in these auctions?

# Fun Game

- Valuation models:
  - the most important one: IPV
  - valuations are iid draws from some commonly-known distribution
    - do you see how we can write this as a Bayesian game?

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<b>Jump bids</b>	yes	n/a	no	n/a	n/a
<b>Price Discovery</b>	yes	no	yes	no	no
<b>Regret</b>	no	yes	no	yes	no



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# Second-Price

## Theorem

*Truth-telling is a dominant strategy in a second-price auction.*

- In fact, we know this already (do you see why?)
- However, we'll look at a simpler, direct proof.

# Second-Price proof

## Theorem

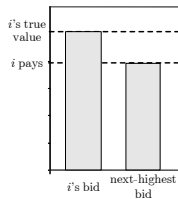
*Truth-telling is a dominant strategy in a second-price auction.*

## Proof.

Assume that the other bidders bid in some arbitrary way. We must show that  $i$ 's best response is always to bid truthfully. We'll break the proof into two cases:

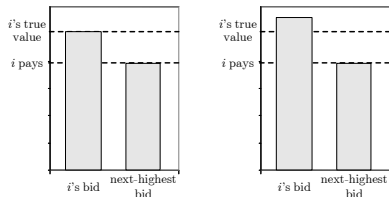
- 1 Bidding honestly,  $i$  would win the auction
- 2 Bidding honestly,  $i$  would lose the auction

# Second-Price proof (2)



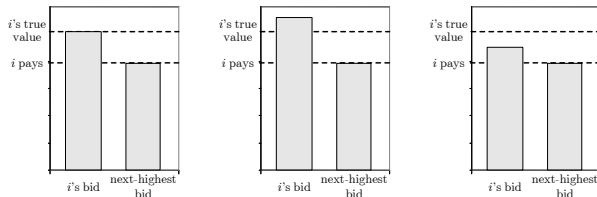
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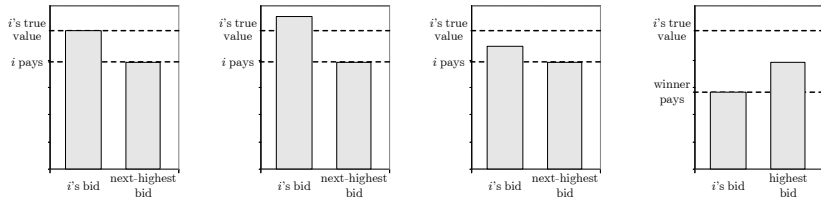
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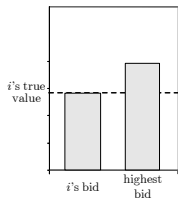
- Bidding honestly,  $i$  is the winner
- If  $i$  bids higher, he will still win and still pay the same amount
- If  $i$  bids lower, he will either still win and still pay the same amount. . .

# Second-Price proof (2)



- Bidding honestly,  $i$  is the winner
- If  $i$  bids higher, he will still win and still pay the same amount
- If  $i$  bids lower, he will either still win and still pay the same amount. . . or lose and get utility of zero.

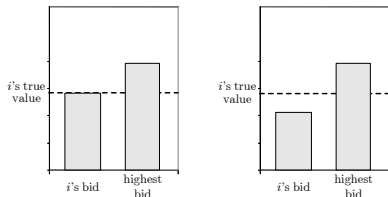
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- Bidding honestly,  $i$  is not the winner

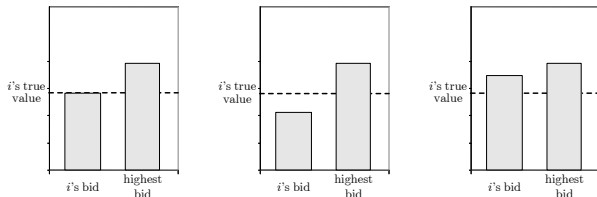


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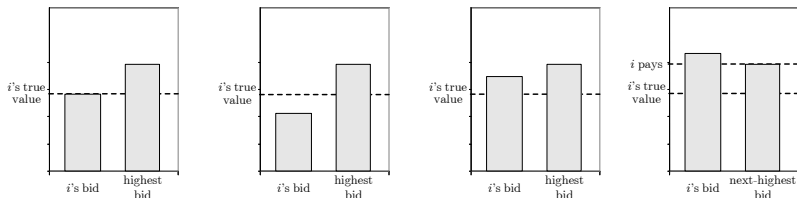
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- Bidding honestly,  $i$  is not the winner
- If  $i$  bids lower, he will still lose and still pay nothing
- If  $i$  bids higher, he will either still lose and still pay nothing...

# Second-Price proof (3)



- Bidding honestly,  $i$  is not the winner
- If  $i$  bids lower, he will still lose and still pay nothing
- If  $i$  bids higher, he will either still lose and still pay nothing... or win and pay more than his valuation.

# English and Japanese auctions

- A much **more complicated** strategy space
  - extensive form game
  - bidders are able to condition their bids on information revealed by others
    - in the case of English auctions, the ability to place jump bids
- intuitively, though, the revealed information doesn't make any difference in the IPV setting.

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- intuitively, though, the revealed information doesn't make any difference in the IPV setting.

## Theorem

*Under the independent private values model (IPV), it is a **dominant strategy** for bidders to bid up to (and not beyond) their valuations in both Japanese and English auctions.*