Theorem 1 *Truth telling is a dominant strategy under the Groves mechanism*

Consider a situation where every agent j other than i follows some arbitrary strategy $\hat{v}_j$. Consider agent i's problem of choosing the best strategy $\hat{v}_i$. As a shorthand, we will write $\hat{v} = (\hat{v}_{-i}, \hat{v}_i)$. The best strategy for i is one that solves

$$\max_{\hat{v}_i} \left(v_i(x(\hat{v})) - p(\hat{v})\right) \tag{1}$$

Substituting in the payment function from the Groves mechanism, we have

$$\max_{\hat{v}_i} \left(v_i(x(\hat{v})) - h_i(\hat{v}_{-i}) + \sum_{j \neq i} \hat{v}_j(x(\hat{v})) \right) \tag{2}$$

Since $h_i(\hat{v}_{-i})$ does not depend on $\hat{v}_i$, it is sufficient to solve

$$\max_{\hat{v}_i} \left(v_i(x(\hat{v})) + \sum_{j \neq i} \hat{v}_j(x(\hat{v})) \right). \tag{3}$$

The only way in which the declaration $\hat{v}_i$ influences the maximization above is through the choice of x. Thus, i wants to pick the declaration $\hat{v}_i$ that will lead the mechanism to pick an $x \in X$ which solves

$$\max_{x} \left(v_i(x) + \sum_{j \neq i} \hat{v}_j(x) \right). \tag{4}$$

Under the Groves mechanism,

$$x(\hat{v}) = \arg\max_{x} \left(\sum_{i} \hat{v}_i(x) \right) = \arg\max_{x} \left(\hat{v}_i(x) + \sum_{j \neq i} \hat{v}_j(x) \right). \tag{5}$$

The Groves mechanism will choose x in a way that solves the maximization problem in Equation (4) when i declares $\hat{v}_i = v_i$. Because this argument does not depend in any way on the declarations of the other agents, truthtelling is a dominant strategy for agent i.