Mini Assignment: Incentives of Peer Review Grading

Due in class Wednesday, January 14

Abstract

In this mini-assignment, you'll take a look at the incentive properties of the CPSC 523A peer review grading scheme. You'll also get a sense of the sort of game-theoretic arguments that we'll be making in this class.

1 Background

Students' grades in CPSC 523A will be determined mainly by the instructor; however, they will also depend on peer-review evaluations performed by other students. For example, students will evaluate each other's performance in class presentations. Because this course focuses on systems in which multiple self-interested agents take strategic action to maximize their rewards, it seems sensible to ask whether such peer-review grading will work. Specifically, what will happen if self-interested students are willing to strategically manipulate their peer reviews to maximize their own grades?

We must first introduce a formal model of the peer-review grading scenario. Let $S = \{0, \ldots, N\}$ be the set of participants in CPSC 523A: let 0 denote the instructor, and let $1, \ldots, N$ denote each of the N students in the class. Let α be the fraction of a student's final grade which is determined by the instructor. Let $g : S \times S \setminus \{0\} \mapsto [0, 1]$ be the grading function, where $g(i, j)$ denotes the grade given by participant i to student j. For all $1 \leq i \leq N$, let $g(i, i) = 0$.

Student j's unadjusted final grade is:

$$f_j = \alpha g(0, j) + \sum_{i=1}^{N} \frac{1 - \alpha}{N - 1} g(i, j)$$

Question 1: Argue that student j cannot affect f_j by strategically changing $g(j, \cdot)$.

2 Grading on a Curve

Let μ and σ denote the mean and standard deviation of final grades. Assume that the instructor wants to curve grades so that the mean is μ' and the standard deviation is σ'. He could do this by giving student j the adjusted final grade:

$$f'_j = \frac{\sigma'(f_j - \mu)}{\sigma} + \mu'$$

Question 2a: Argue that j *can* affect f'_j by strategically changing $g(j, \cdot)$.

Question 2b: How should j select values $g(j, \cdot)$ in order to maximize f'_j? Hint: observe that $\sigma \in [0, 1]$ and $\sigma^* \in [0, 1]$ since $\forall i, j \; g(i, j) \in [0, 1]$.

Question 2c: Show that the strategy shown as the answer to question 2a is a *strongly dominant strategy*: i.e., each student is strictly better off following this strategy regardless of the peer-review strategies employed by other students.

Question 2d: What simpler grading system would be equivalent to the situation where every student follows the dominant strategy?

3 Incentive-Compatible Grading

Define

$$\mu_{\sim j} = \frac{(\sum_{i=1}^{n} f_i) - f_j}{N - 1},$$

the mean of unadjusted grades calculated without considering f_j, and define $\sigma_{\sim j}$ analogously. To try to prevent the manipulation of peer-review grades, the instructor calculates curved grades in a new way.

$$f^*_j = \frac{\sigma'(f_j - \mu_{\sim j})}{\sigma_{\sim j}} + \mu'$$

Question 3a: Show that student j cannot affect f^*_j by strategically changing $g(j, \cdot)$.

Question 3b: Note that when each student j receives the grade f^*_j the mean and standard deviation of the grades are not exactly μ' and σ'. Explain why there is no way of choosing f^*_j which simultaneously satisfies the following properties:

1. the mean and standard deviation are exactly μ' and σ';
2. no student j has incentive to strategically change $g(j, \cdot)$;
3. f^*_j is strictly increasing in $g(i, j)$ for all $i \neq j$.