Reasoning Under Uncertainty: Introduction to Probability

CPSC 322 – Uncertainty 1

Textbook §6.1

Lecture Overview

Syntax of Datalog

Definition (variable)

A variable starts with upper-case letter.

Definition (constant)

A constant starts with lower-case letter or is a sequence of digits.

Definition (term)

A term is either a variable or a constant.

Definition (predicate symbol)

A predicate symbol starts with lower-case letter.

Syntax of Datalog (cont)

Definition (atom)

An atomic symbol (atom) is of the form p or $p(t_1, \ldots, t_n)$ where p is a predicate symbol and t_i are terms.

Definition (definite clause)

A definite clause is either an atomic symbol (a fact) or of the form:

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1 \wedge \cdots \wedge b_m}_{\text{body}}$$

where a and b_i are atomic symbols.

Definition (knowledge base)

A knowledge base is a set of definite clauses.

Formal Semantics

Definition (interpretation)

An interpretation is a triple $I = \langle D, \phi, \pi \rangle$, where

- D, the domain, is a nonempty set. Elements of D are individuals.
- ϕ is a mapping that assigns to each constant an element of D. Constant c denotes individual $\phi(c)$.
- π is a mapping that assigns to each n-ary predicate symbol a relation: a function from D^n into $\{\mathit{TRUE}, \mathit{FALSE}\}$.

Variables

Definition (variable assignment)

A variable assignment is a function from variables into the domain.

- Given an interpretation and a variable assignment, each term denotes an individual and each clause is either true or false.
- A clause containing variables is true in an interpretation if it is true for all variable assignments.
  - Variables are universally quantified in the scope of a clause.
- Now we can use our previous definition of logical entailment.

Lecture Overview

Using Uncertain Knowledge

- Agents don't have complete knowledge about the world.
- Agents need to make decisions based on their uncertainty.
- It isn't enough to assume what the world is like.
  Example: wearing a seat belt.
- An agent needs to reason about its uncertainty.
- When an agent makes an action under uncertainty, it is gambling $\Longrightarrow$ probability.

Numerical Measures of Belief

- Belief in proposition, f, can be measured in terms of a number between 0 and 1 — this is the probability of f.
  - "The probability that f is 0" means that f is believed to be definitely false.
  - "The probability that f is 1" means that f is believed to be definitely true.
- Using 0 and 1 is purely a convention.
- f has a probability between 0 and 1, doesn't mean f is true to some degree, but means you are ignorant of its truth value. Probability is a measure of your ignorance.

Frequentists vs. Bayesians

- Probability is the formal measure of uncertainty. There are two camps:
- Frequentists: believe that probability represents something *objective*, and compute probabilities by counting the frequencies of different events
- Bayesians: believe that probability represents something *subjective*, and understand probabilities as degrees of belief.
  - They compute probabilities by starting with prior beliefs, and then updating beliefs when they get new data.
  - Example: Your degree of belief that a bird can fly is your measure of belief in the flying ability of an individual based only on the knowledge that the individual is a bird.
  - Other agents may have different probabilities, as they may have had different experiences with birds or different knowledge about this particular bird.
  - An agent's belief in a bird's flying ability is affected by what the agent knows about that bird.

Possible World Semantics

Probability is a formal measure of uncertainty.

- A random variable is a variable that is randomly assigned one of a number of different values.
- The domain of a variable X, written $dom(X)$, is the set of values X can take.
- A possible world specifies an assignment of one value to each random variable.
- $w \models X = x$ means variable X is assigned value x in world w.
- Let Ω be the set of all possible worlds.
- Define a nonnegative measure $\mu(w)$ to each world w so that the measures of the possible worlds sum to 1.
- The probability of proposition f is defined by:

$$P(f) = \sum_{w \models f} \mu(w).$$