Propositional Logic: Semantics and an Example

CPSC 322 – Logic 2

Textbook §5.2

Lecture Overview

1. Recap: Syntax
2. Propositional Definite Clause Logic: Semantics
3. Using Logic to Model the World
4. Proofs

Propositional Definite Clauses: Syntax

Definition (atom)

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Definition (knowledge base)

A knowledge base is a set of definite clauses

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We can use the interpretation to determine the truth value of clauses and knowledge bases:

Definition (truth values of statements)

- A body $b_1 \wedge b_2$ is true in I if and only if b_1 is true in I and b_2 is true in I.
- A rule $h \leftarrow b$ is false in I if and only if b is true in I and h is false in I.
- A knowledge base KB is true in I if and only if every clause in KB is true in I.

Models and Logical Consequence

Definition (model)

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Definition (logical consequence)

If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.

- we also say that g logically follows from KB, or that KB entails g.
- In other words, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.

Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s
I_1	*true*	*true*	*true*	*true*
I_2	*false*	*false*	*false*	*false*
I_3	*true*	*true*	*false*	*false*
I_4	*true*	*true*	*true*	*false*
I_5	*true*	*true*	*false*	*true*

Which interpretations are models?

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I_2	*false*	*false*	*false*	*false*	not a model of KB
I_3	*true*	*true*	*false*	*false*	is a model of KB
I_4	*true*	*true*	*true*	*false*	is a model of KB
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Which of the following is true?

- $KB \models q$, $KB \models p$, $KB \models s$, $KB \models r$

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Which of the following is true?

- $KB \models q$, $KB \models p$, $KB \models s$, $KB \models r$
- $KB \models q$, $KB \models p$, $KB \not\models s$, $KB \not\models r$

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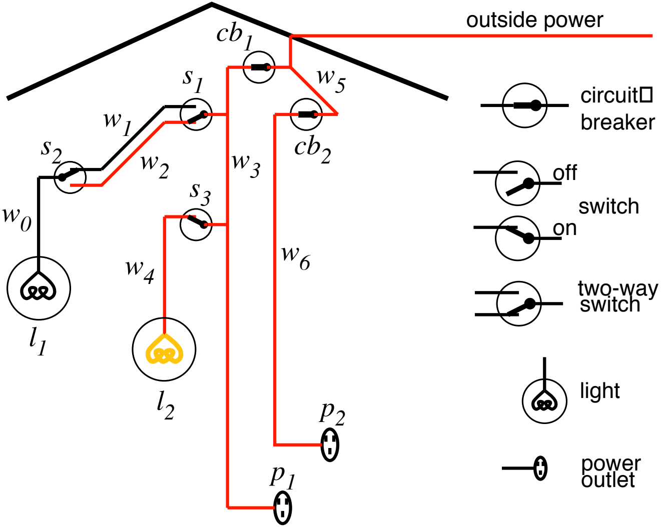
User's view of Semantics

1. Choose a task domain: intended interpretation.
2. Associate an atom with each proposition you want to represent.
3. Tell the system clauses that are true in the intended interpretation: axiomatizing the domain.
4. Ask questions about the intended interpretation.
5. If $KB \models g$, then g must be true in the intended interpretation.
6. The user can interpret the answer using their intended interpretation of the symbols.

Computer's view of semantics

- The computer doesn't have access to the intended interpretation.
  - All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
  - If $KB \models g$ then g must be true in the intended interpretation.
  - If $KB \not\models g$ then there is a model of KB in which g is false. This could be the intended interpretation.

Electrical Environment



Representing the Electrical Environment

$light\_l_1.$

$light\_l_2.$

$down\_s_1.$

$up\_s_2.$

$up\_s_3.$

$ok\_l_1.$

$ok\_l_2.$

$ok\_cb_1.$

$ok\_cb_2.$

$live\_outside.$

$live\_l_1 \leftarrow live\_w_0$

$live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2.$

$live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2.$

$live\_w_1, \leftarrow live\_w_3 \wedge up\_s_1.$

$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1.$

$live\_l_2 \leftarrow live\_w_4.$

$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3.$

$live\_p_1 \leftarrow live\_w_3.$

$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$

$live\_p_2 \leftarrow live\_w_6.$

$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$

$live\_w_5 \leftarrow live\_outside.$

Role of semantics

In user's mind:

- $l2\_broken$: light $l2$ is broken
- $sw3\_up$: switch is up
- $power$: there is power in the building
- $unlit\_l2$: light $l2$ isn't lit
- lit_l1: light $l1$ is lit

In Computer:

$$l2\_broken \leftarrow sw3\_up \wedge power \wedge unlit\_l2.$$

$$sw3\_up.$$

$$power \leftarrow lit\_l1.$$

$$unlit\_l2.$$

$$lit\_l1.$$

Conclusion: $l2\_broken$

- The computer doesn't know the meaning of the symbols
- The user can interpret the symbols using their meaning

Lecture Overview

Proofs

- A proof is a mechanically derivable demonstration that a formula logically follows from a knowledge base.
- Given a proof procedure, $KB \vdash g$ means g can be derived from knowledge base KB.
- Recall $KB \models g$ means g is true in all models of KB.

Definition (soundness)

A proof procedure is sound if $KB \vdash g$ implies $KB \models g$.

Definition (completeness)

A proof procedure is complete if $KB \models g$ implies $KB \vdash g$.