Search: Advanced Topics and Conclusion

CPSC 322 Lecture 8

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Textbook §2.6

Lecture Overview

1. Recap
2. Branch & Bound
3. A^* Tricks
4. Other Pruning
5. Backwards Search
6. Dynamic Programming

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A^* Search Algorithm

- A^* is a mix of lowest-cost-first and Best-First search.
- It treats the frontier as a priority queue ordered by $f(p)$.
- It always selects the node on the frontier with the lowest estimated total distance.

Analysis of A^*

Let's assume that arc costs are strictly positive.

- Completeness: yes.
- Time complexity: $O(b^m)$
  - the heuristic could be completely uninformative and the edge costs could all be the same, meaning that A^* does the same thing as BFS
- Space complexity: $O(b^m)$
  - like BFS, A^* maintains a frontier which grows with the size of the tree
- Optimality: yes.

Optimal Efficiency of A^*

- In fact, we can prove something even stronger about A^*: in a sense (given the particular heuristic that is available) no search algorithm could do better!

- Optimal Efficiency: Among all optimal algorithms that start from the same start node and use the same heuristic h, A^* expands the minimal number of nodes.
  - problem: A^* could be unlucky about how it breaks ties.
  - So let's define optimal efficiency as expanding the minimal number of nodes n for which $f(n) \neq f^*$, where f^* is the cost of the shortest path.

Why is A^* optimally efficient?

Theorem

A^* is optimally efficient.

- Let f^* be the cost of the shortest path to a goal. Consider any algorithm A' which has the same start node as A^*, uses the same heuristic and fails to expand some node n' expanded by A^* for which $cost(n') + h(n') < f^*$. Assume that A' is optimal.
- Consider a different search problem which is identical to the original and on which h returns the same estimate for each node, except that n' has a child node n'' which is a goal node, and the true cost of the path to n'' is $f(n')$.
  - that is, the edge from n' to n'' has a cost of $h(n')$: the heuristic is exactly right about the cost of getting from n' to a goal.
- A' would behave identically on this new problem.
  - The only difference between the new problem and the original problem is beyond node n', which A' does not expand.
- Cost of the path to n'' is lower than cost of the path found by A'.
- This violates our assumption that A' is optimal.

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Branch-and-Bound Search

- A search strategy often not covered in AI, but widely used in practice
- Uses a heuristic function: like A^*, can avoid expanding some unnecessary nodes
- Depth-first: modest memory demands
  - in fact, some people see “branch and bound” as a broad family that *includes* A^*
  - these people would use the term “depth-first branch and bound”

Branch-and-Bound Search Algorithm

- Follow exactly the same search path as depth-first search
  - treat the frontier as a stack: expand the most-recently added node first
  - the order in which neighbors are expanded can be governed by some arbitrary node-ordering heuristic
- Keep track of a lower bound and upper bound on solution cost at each node
  - lower bound: $LB(n) = cost(n) + h(n)$
  - upper bound: $UB = cost(n')$, where n' is the best solution found so far.
    - if no solution has been found yet, set the upper bound to ∞.
- When a node n is selected for expansion:
  - if $LB(n) \geq UB$, remove n from frontier without expanding it
    - this is called "pruning the search tree" (really!)
  - else expand n, adding all of its neighbours to the frontier

Branch-and-Bound Analysis

- Completeness: no, for the same reasons that DFS isn't complete
    - however, for many problems of interest there are no infinite paths and no cycles
    - hence, for many problems B&B is complete
- Time complexity: $O(b^m)$
- Space complexity: $O(bm)$
    - Branch & Bound has the same space complexity as DFS
    - this is a big improvement over A^*!
- Optimality: yes.

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Other A^* Enhancements

The main problem with A^* is that it uses exponential space. Branch and bound was one way around this problem. Are there others?

- Iterative deepening
- Memory-bounded $A*$

Iterative Deepening

- B & B can still get stuck in cycles
- Search depth-first, but to a fixed depth
    - if you don't find a solution, increase the depth tolerance and try again
    - of course, depth is measured in f value
- Counter-intuitively, the asymptotic complexity is not changed, even though we visit nodes multiple times

Memory-bounded $A*$

- Iterative deepening and B & B use a tiny amount of memory
- what if we've got more memory to use?
- keep as much of the fringe in memory as we can
- if we have to delete something:
    - delete the oldest paths
    - "back them up" to a common ancestor

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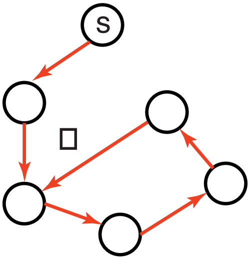
Non-heuristic pruning

What can we prune besides nodes that are ruled out by our heuristic?

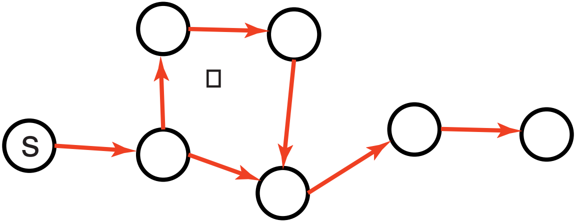
- Cycles
- Multiple paths to the same node

Cycle Checking



- You can prune a path that ends in a node already on the path. This pruning cannot remove an optimal solution.
- Using depth-first methods, with the graph explicitly stored, this can be done in constant time.
- For other methods, the cost is linear in path length.

Multiple-Path Pruning



- You can prune a path to node n that you have already found a path to.
- Multiple-path pruning subsumes a cycle check.
- This entails storing all nodes you have found paths to.

Multiple-Path Pruning & Optimal Solutions

Problem: what if a subsequent path to n is shorter than the first path to n?

- You can remove all paths from the frontier that use the longer path.
- You can change the initial segment of the paths on the frontier to use the shorter path.
- You can ensure this doesn't happen. You make sure that the shortest path to a node is found first.
    - Heuristic function h satisfies the monotone restriction if $|h(m) - h(n)| \leq d(m, n)$ for every arc $\langle m, n \rangle$.
    - If h satisfies the monotone restriction, A^* with multiple path pruning always finds the shortest path to every node
        - otherwise, we have this guarantee only for goals

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Direction of Search

- The definition of searching is symmetric: find path from start nodes to goal node or from goal node to start nodes.
    - Of course, this presumes an explicit goal node, not a goal test.
    - Also, when the graph is dynamically constructed, it can sometimes be impossible to construct the backwards graph
- Forward branching factor: number of arcs out of a node.
- Backward branching factor: number of arcs into a node.
- Search complexity is b^n. Should use forward search if forward branching factor is less than backward branching factor, and vice versa.

Bidirectional Search

- You can search backward from the goal and forward from the start simultaneously.
- This wins as $2b^{k/2} \ll b^k$. This can result in an exponential saving in time and space.
    - The main problem is making sure the frontiers meet.
    - This is often used with one breadth-first method that builds a set of locations that can lead to the goal. In the other direction another method can be used to find a path to these interesting locations.

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Dynamic Programming

Idea: for statically stored graphs, build a table of $dist(n)$ the actual distance of the shortest path from node n to a goal.
This can be built backwards from the goal:

$$dist(n) = \begin{cases} 0 & \text{if } is\_goal(n), \\ \min_{\langle n,m \rangle \in A}(|\langle n,m \rangle| + dist(m)) & \text{otherwise.} \end{cases}$$

This can be used locally to determine what to do.
There are two main problems:

- You need enough space to store the graph.
- The $dist$ function needs to be recomputed for each goal.

Complexity: polynomial in the size of the graph.