Propositional Logic: Semantics and Bottom-Up Proofs

CPSC 322 Lecture 19

February 28, 2007
Textbook §4.2

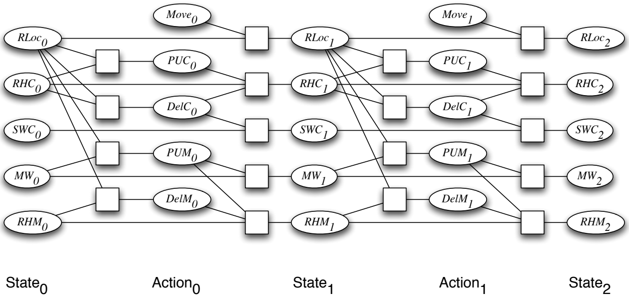
Lecture Overview

1. Recap
2. Semantics
3. Using Logic to Model the World

Planning as a CSP

- We can go forwards and backwards at the same time, if we set up a planning problem as a CSP
- To do this, we need to "unroll" the plan for a fixed number of steps
  - this is called the horizon
- To do this with a horizon of k:
  - construct a variable for each feature at each time step from 0 to k
  - construct a boolean variable for each action at each time step from 0 to $k-1$.

CSP Planning: Robot Example



Do you see why CSP planning is both forwards and backwards?

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An atom is a symbol starting with a lower case letter

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Definition (knowledge base)

A knowledge base is a set of definite clauses

Syntax: Example

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- $ai\_is\_fun \leftarrow get\_good\_grade \wedge not\_too\_much\_work$

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Do any of these statements *mean* anything? Syntax doesn't answer this question.

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We can use the interpretation to determine the truth value of clauses and knowledge bases:

Definition (truth values of statements)

- A body $b_1 \wedge b_2$ is true in I if and only if b_1 is true in I and b_2 is true in I.
- A rule $h \leftarrow b$ is false in I if and only if b is true in I and h is false in I.
- A knowledge base KB is true in I if and only if every clause in KB is true in I.

Models and Logical Consequence

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Definition (logical consequence)

If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.

- we also say that g logically follows from KB, or that KB entails g.
- In other words, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.

Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s
I_1	*true*	*true*	*true*	*true*

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I_1 is a model of KB

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Which of the following is true?

- $KB \models q$, $KB \models p$, $KB \models s$, $KB \models r$

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- $KB \models q$, $KB \models p$, $KB \models s$, $KB \models r$
- $KB \models q$, $KB \models p$, $KB \not\models s$,

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Which of the following is true?

- $KB \models q$, $KB \models p$, $KB \models s$, $KB \models r$
- $KB \models q$, $KB \models p$, $KB \not\models s$, $KB \not\models r$

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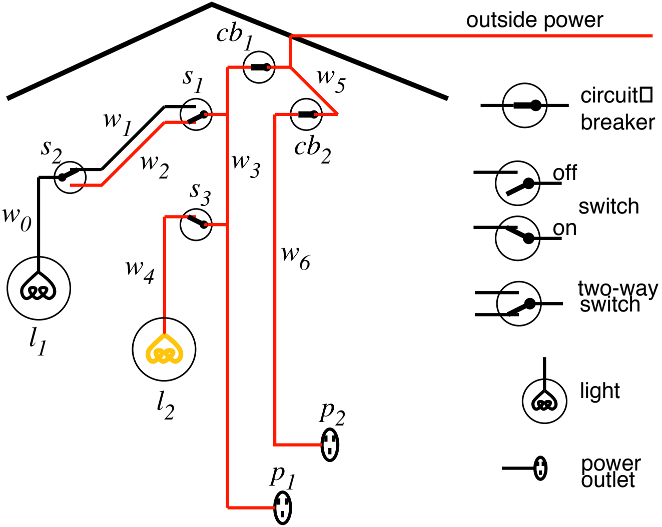
User's view of Semantics

1. Choose a task domain: intended interpretation.
2. Associate an atom with each proposition you want to represent.
3. Tell the system clauses that are true in the intended interpretation: axiomatizing the domain.
4. Ask questions about the intended interpretation.
5. If $KB \models g$, then g must be true in the intended interpretation.
6. The user can interpret the answer using their intended interpretation of the symbols.

Computer's view of semantics

- The computer doesn't have access to the intended interpretation.
  - All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
  - If $KB \models g$ then g must be true in the intended interpretation.
  - If $KB \not\models g$ then there is a model of KB in which g is false. This could be the intended interpretation.

Electrical Environment



Representing the Electrical Environment

$light\_l_1.$

$light\_l_2.$

$down\_s_1.$

$up\_s_2.$

$up\_s_3.$

$ok\_l_1.$

$ok\_l_2.$

$ok\_cb_1.$

$ok\_cb_2.$

$live\_outside.$

$live\_l_1 \leftarrow live\_w_0$

$live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2.$

$live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2.$

$live\_w_1, \leftarrow live\_w_3 \wedge up\_s_1.$

$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1.$

$live\_l_2 \leftarrow live\_w_4.$

$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3.$

$live\_p_1 \leftarrow live\_w_3.$

$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$

$live\_p_2 \leftarrow live\_w_6.$

$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$

$live\_w_5 \leftarrow live\_outside.$

Role of semantics

In user's mind:

- $l2\_broken$: light $l2$ is broken
- $sw3\_up$: switch is up
- $power$: there is power in the building
- $unlit\_l2$: light $l2$ isn't lit
- lit_l1: light $l1$ is lit

In Computer:

$$l2\_broken \leftarrow sw3\_up \wedge power \wedge unlit\_l2.$$

$$sw3\_up.$$

$$power \leftarrow lit\_l1.$$

$$unlit\_l2.$$

$$lit\_l1.$$

Conclusion: $l2\_broken$

- The computer doesn't know the meaning of the symbols
- The user can interpret the symbols using their meaning