# Local Search

CPSC 322 Lecture 13

February 5, 2007
Textbook §3.8

# Lecture Overview

1. Recap
2. Randomized Algorithms
3. Comparing SLS Algorithms
4. SLS Variants

# Stochastic Local Search for CSPs

- Set of Variables: the same as the variables in the CSP
- Neighbour Relation: assignments that differ in the value assigned to one variable, or in the value assigned to the variable that participates in the largest number of conflicts
- Goal is to find an assignment with all constraints satisfied.
  - Scoring function: the number of unsatisfied constraints.
  - We want an assignment with minimum score.

# Hill Climbing

Hill climbing means selecting the neighbour which best improves the scoring function.

- For example, if the goal is to find the highest point on a surface, the scoring function might be the height at the current point.

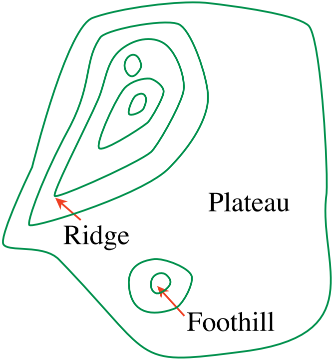
# Problems with Hill Climbing

**Foothills** local maxima that are not global maxima

**Plateaus** heuristic values are uninformative

**Ridge** foothill where a larger neighbour relation would help

**Ignorance of the peak** no way of detecting a global maximum



# Lecture Overview

# Randomized Algorithms

- Consider two methods to find a maximum value:
  - Hill climbing, starting from some position, keep moving uphill & report maximum value found
  - Pick values at random & report maximum value found
- Which do you expect to work better to find a maximum?

# Randomized Algorithms

- Consider two methods to find a maximum value:
  - Hill climbing, starting from some position, keep moving uphill & report maximum value found
  - Pick values at random & report maximum value found
- Which do you expect to work better to find a maximum?
  - hill climbing is good for finding local maxima
  - selecting random nodes is good for finding new parts of the search space
- A mix of the two techniques can work even better

# Stochastic Local Search

- We can bring these two ideas together to make a randomized version of hill climbing.
- As well as uphill steps we can allow for:
  - Random steps: move to a random neighbor.
  - Random restart: reassign random values to all variables.
- Which is more expensive computationally?

# Stochastic Local Search

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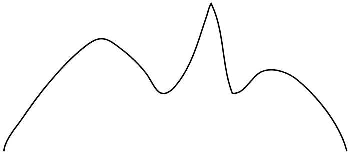
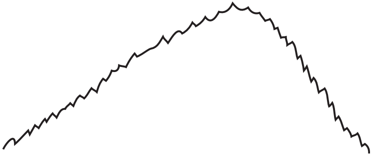
- As well as uphill steps we can allow for:
  - Random steps: move to a random neighbor.
  - Random restart: reassign random values to all variables.
- Which is more expensive computationally?
  - usually, random restart (consider that there could be an extremely large number of neighbors)
  - however, if the neighbour relation is computationally expensive, random restart could be cheaper

# 1-Dimensional Ordered Examples

Two 1-dimensional search spaces; step right or left:





- Which of hill climbing with random walk and hill climbing with random restart would most easily find the maximum?

# 1-Dimensional Ordered Examples

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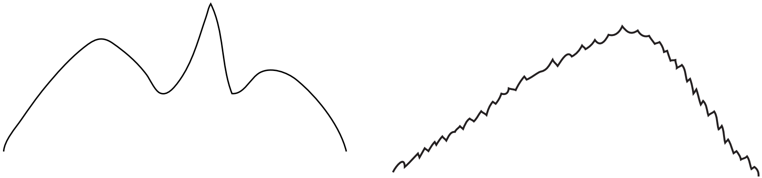


- Which of hill climbing with random walk and hill climbing with random restart would most easily find the maximum?
  - left: random restart; right: random walk
- As indicated before, stochastic local search often involves both kinds of randomization

# Random Walk

Some examples of ways to add randomness to local search for a CSP:

- When choosing the best variable-value pair, randomly sometimes choose a random variable-value pair.
- When selecting a variable followed by a value:
  - Sometimes choose the variable which participates in the largest number of conflicts.
  - Sometimes choose, at random, any variable that participates in some conflict.
  - Sometimes choose a random variable.
  - Sometimes choose the best value for the chosen variable.
  - Sometimes choose a random value for the chosen variable.

# Lecture Overview

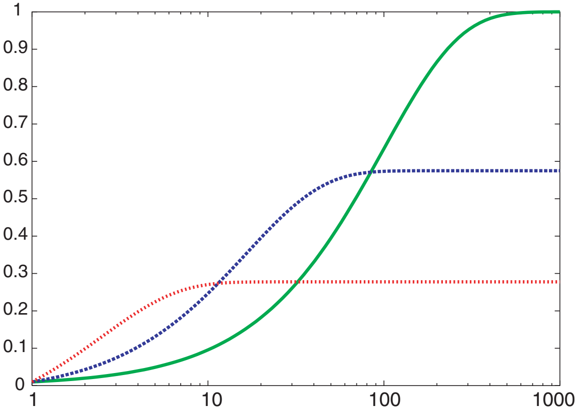
1. Recap
2. Randomized Algorithms
3. **Comparing SLS Algorithms**
4. SLS Variants

# Comparing Stochastic Algorithms

- How can you compare three algorithms when (e.g.,)
  - one solves the problem 30% of the time very quickly but doesn't halt for the other 70% of the cases
  - one solves 60% of the cases reasonably quickly but doesn't solve the rest
  - one solves the problem in 100% of the cases, but slowly?
- Summary statistics, such as mean run time, median run time, and mode run time don't tell the whole story
  - mean: what should you do if an algorithm *never* finished on some runs (infinite? stopping time?)
  - median: an algorithm that finishes 51% of the time is preferred to one that finishes 49% of the time, regardless of how fast it is

# Runtime Distribution

- Plots runtime (or number of steps) and the proportion (or number) of the runs that are solved within that runtime.
  - note the use of a log scale on the $x$ axis



# Lecture Overview

# Variant: Greedy Descent with Min-Conflict Heuristic

This is one of the best techniques for solving CSP problems:

- At random, select one of the variables $v$ that participates in a violated constraint
- Set $v$ to one of the values that minimizes the number of unsatisfied constraints
- This can be implemented efficiently:
  - One data structure stores constraints that are currently violated
  - One data structure stores variables that are involved in violated constraints
  - Selecting the variable to change is a random draw from the second data structure
  - For each of $v$'s values $i$, count the number of constraints that would be violated if $v$ took the value $i$
  - When the new value is set:
    - add all variables that participate in newly-violated constraints
    - check all variables that participate in newly-satisfied constraints to see if they participate in any other violated

# Variant: Simulated Annealing

- Annealing: a metallurgical process where metals are hardened by being slowly cooled.
- Analogy: start with a high "temperature": a high tendency to take random steps
- Over time, cool down: more likely to follow the gradient
- Here's how it works:
    - Pick a variable at random and a new value at random.
    - If it is an improvement, adopt it.
    - If it isn't an improvement, adopt it probabilistically depending on a temperature parameter, $T$.
        - With current node $n$ and proposed node $n'$ we move to $n'$ with probability $e^{(h(n')-h(n))/T}$
    - Temperature reduces over time, according to an annealing schedule

# Tabu lists

- SLS algorithms can get stuck in plateaus (why?)

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- SLS algorithms can get stuck in plateaus (why?)
- To prevent cycling we can maintain a tabu list of the $k$ last nodes visited.
- Don't visit a node that is already on the tabu list.
- If $k = 1$, we don't allow the search to visit the same assignment twice in a row.
- This method can be expensive if $k$ is large.