Uninformed Search

CPSC 322 Lecture 5

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Textbook §2.4

Lecture Overview

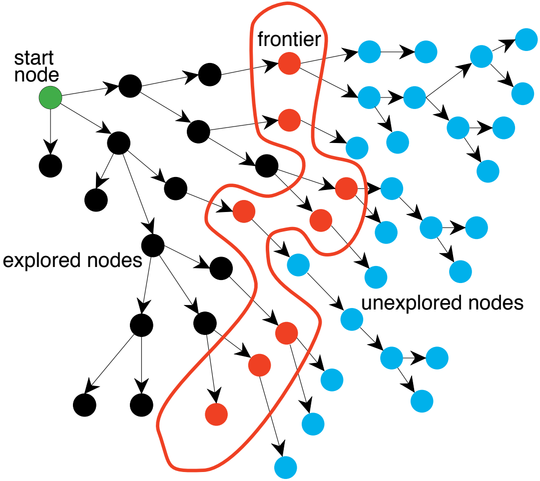
Search

- What we want to be able to do:
  - find a solution when we are not given an algorithm to solve a problem, but only a specification of what a solution looks like
  - idea: search for a solution
- What we need:
  - A set of states
  - A start state
  - A goal state or set of goal states
    - or, equivalently, a goal test: a boolean function which tells us whether a given state is a goal state
  - A set of actions
  - An action function: a mapping from a state and an action to a new state

Search Graphs

- A graph consists of
  - a set N of nodes;
  - a set A of ordered pairs of nodes, called arcs or edges.
- Node n_2 is a neighbor of n_1 if there is an arc from n_1 to n_2.
  - i.e., if $\langle n_1, n_2 \rangle \in A$
- A path is a sequence of nodes $\langle n_0, n_1, \ldots, n_k \rangle$ such that $\langle n_{i-1}, n_i \rangle \in A$.
- Given a start node and a set of goal nodes, a solution is a path from the start node to a goal node.

Problem Solving by Graph Searching



Graph Search Algorithm

Input: a graph,
a set of start nodes,
Boolean procedure $goal(n)$ that tests if n is a goal node.
$frontier := \{\langle s \rangle : s \text{ is a start node}\};$
while *frontier* is not empty:
    select and **remove** path $\langle n_0, \ldots, n_k \rangle$ from *frontier*;
    if $goal(n_k)$
        return $\langle n_0, \ldots, n_k \rangle$;
    for every neighbor n of n_k
        add $\langle n_0, \ldots, n_k, n \rangle$ to *frontier*;
end while

- After the algorithm returns, it can be asked for more answers and the procedure continues.
- Which value is selected from the frontier defines the search strategy.
- The *neighbor* relationship defines the graph.
- The *goal* function defines what is a solution.

Branching Factor

- The forward branching factor of a node is the number of arcs going out of that node
- The backward branching factor of a node is the number of arcs going into the node
- If the forward branching factor of every node is b and the graph is a tree, how many nodes are exactly n steps away from the start node?
  - b^n nodes.
- We'll assume that all branching factors are finite.

Comparing Algorithms

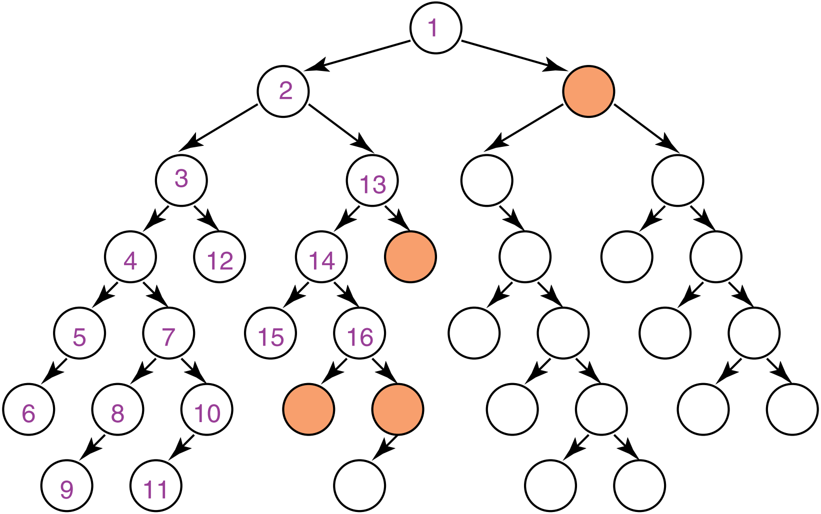
- Completeness
  - if at least one solution exists, the algorithm is guaranteed to find a solution within a finite amount of time
- Time Complexity
  - in terms of the maximum path length m, and the maximum branching factor b, what is the worst-case amount of time that the algorithm will take to run?
- Space Complexity
  - in terms of m and b, what is the worst-case amount of memory that the algorithm must use?

Depth-first Search

- Depth-first search treats the frontier as a stack
- It always selects one of the last elements added to the frontier.

- Example:
  - the frontier is $[p_1, p_2, \ldots, p_r]$
  - neighbours of p_1 are $\{n_1, \ldots, n_k\}$
- What happens?
  - p_1 is selected, and tested for being a goal.
  - Neighbours of p_1 replace p_1 at the beginning of the frontier.
  - Thus, the frontier is now $[n_1, \ldots, n_k, p_2, \ldots, p_r]$.
  - p_2 is only selected when all paths from p_1 have been explored.

Illustrative Graph — Depth-first Search



Analysis of Depth-first Search

- Is DFS complete?
  - Depth-first search isn't guaranteed to halt on infinite graphs or on graphs with cycles.
  - However, DFS *is* complete for finite trees.
- What is the time complexity, if the maximum path length is m and the maximum branching factor is b?
  - The time complexity is $O(b^m)$: must examine every node in the tree.
  - Search is unconstrained by the goal until it happens to stumble on the goal.
- What is the space complexity?
  - Space complexity is $O(bm)$: the longest possible path is m, and for every node in that path must maintain a fringe of size b.

Using Depth-First Search

- When is DFS appropriate?
  - space is restricted
  - solutions tend to occur at the same depth in the tree
  - you know how to order nodes in the list of neighbours so that solutions will be found relatively quickly

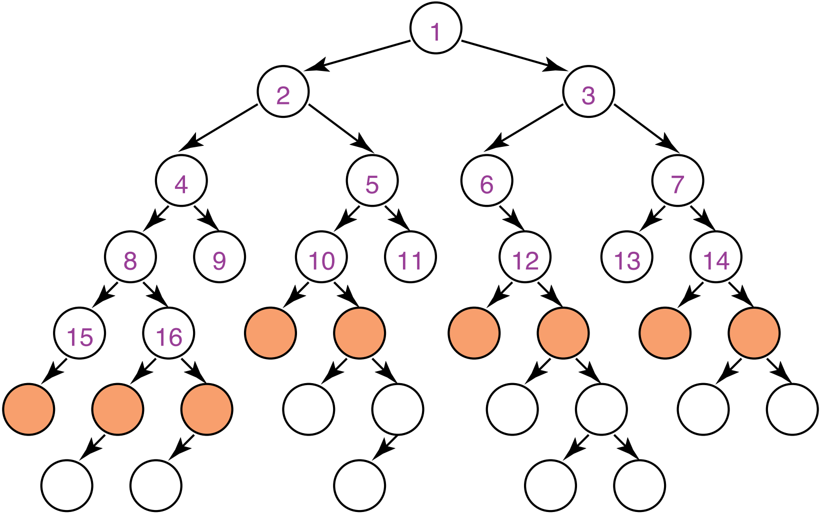
- When is DFS inappropriate?
  - some paths have infinite length
  - the graph contains cycles
  - some solutions are very deep, while others are very shallow

Breadth-first Search

- Breadth-first search treats the frontier as a queue
  - it always selects one of the earliest elements added to the frontier.

- Example:
  - the frontier is $[p_1, p_2, \ldots, p_r]$
  - neighbours of p_1 are $\{n_1, \ldots, n_k\}$
- What happens?
  - p_1 is selected, and tested for being a goal.
  - Neighbours of p_1 follow p_r at the end of the frontier.
  - Thus, the frontier is now $[p_2, \ldots, p_r, n_1, \ldots, n_k]$.
  - p_2 is selected next.

Illustrative Graph — Breadth-first Search



Analysis of Breadth-First Search

- Is BFS complete?
  - Yes (but it wouldn't be if the branching factor for any node was infinite)
  - In fact, BFS is guaranteed to find the path that involves the fewest arcs (why?)
- What is the time complexity, if the maximum path length is m and the maximum branching factor is b?
  - The time complexity is $O(b^m)$: must examine every node in the tree.
  - The order in which we examine nodes (BFS or DFS) makes no difference to the worst case: search is unconstrained by the goal.
- What is the space complexity?
  - Space complexity is $O(b^m)$: we must store the whole frontier in memory

Using Breadth-First Search

- When is BFS appropriate?
  - space is not a problem
  - it's necessary to find the solution with the fewest arcs
  - although all solutions may not be shallow, at least some are
  - there may be infinite paths

- When is BFS inappropriate?
  - space is limited
  - all solutions tend to be located deep in the tree
  - the branching factor is very large

Search with Costs

- Sometimes there are costs associated with arcs.
  - The cost of a path is the sum of the costs of its arcs.

$$cost(\langle n_0, \ldots, n_k \rangle) = \sum_{i=1}^{k} |\langle n_{i-1}, n_i \rangle|$$

- In this setting we often don't just want to find just any solution
  - Instead, we usually want to find the solution that minimizes cost
- We call a search algorithm which always finds such a solution optimal

Lowest-Cost-First Search

- At each stage, lowest-cost-first search selects a path on the frontier with lowest cost.
  - The frontier is a priority queue ordered by path cost.
  - We say "a path" because there may be ties
- When all arc costs are equal, LCFS is equivalent to BFS.
- Example:
  - the frontier is $[\langle p_1, 10\rangle, \langle p_2, 5\rangle, \langle p_3, 7\rangle]$
  - p_2 is the lowest-cost node in the frontier
  - neighbours of p_2 are $\{\langle p_9, 12\rangle, \langle p_{10}, 15\rangle\}$
- What happens?
  - p_2 is selected, and tested for being a goal.
  - Neighbours of p_2 are inserted into the frontier (it doesn't matter where they go)
  - Thus, the frontier is now $[\langle p_1, 10\rangle, \langle p_9, 12\rangle, \langle p_{10}, 15\rangle, \langle p_3, 7\rangle]$.
  - p_3 is selected next.
  - Of course, we'd really implement this as a priority queue.

Analysis of Lowest-Cost-First Search

- Is LCFS complete?
  - not in general: a cycle with zero or negative arc costs could be followed forever.
  - yes, as long as arc costs are strictly positive
- What is the time complexity, if the maximum path length is m and the maximum branching factor is b?
  - The time complexity is $O(b^m)$: must examine every node in the tree.
  - Knowing costs doesn't help here.
- What is the space complexity?
  - Space complexity is $O(b^m)$: we must store the whole frontier in memory.
- Is LCFS optimal?
  - Not in general. Why not?
  - Arc costs could be negative: a path that initially looks high-cost could end up getting a "refund".
  - However, LCFS *is* optimal if arc costs are guaranteed to be non-negative.