Gradient Sampling for Improved Action Selection and Path Synthesis

Ian M. Mitchell

Department of Computer Science
The University of British Columbia

November 2016

mitchell@cs.ubc.ca
http://www.cs.ubc.ca/~mitchell

Copyright 2016 by Ian M. Mitchell
This work is made available under the terms of the Creative Commons Attribution 4.0 International license
http://creativecommons.org/licenses/by/4.0/

Shortest Path via the Value Function

- Assume isotropic holonomic vehicle

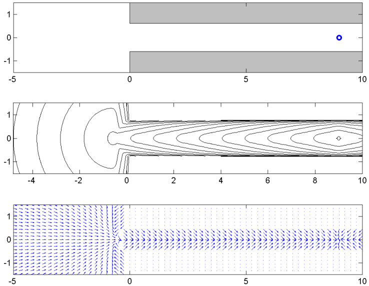
$$\frac{d}{dt}x(t) = \dot{x}(t) = u(t), \quad \|u(t)\| \leq 1.$$

- Plan paths to target set $\mathcal{T}$ optimal by cost metric

$$\psi(x_0) = \inf_{x(\cdot)} \int_{t_0}^{t_f} c(x(s))\, ds,$$
$$t_f = \operatorname{argmin}\{s \mid x(s) \in \mathcal{T}\}.$$

- Value function $\psi(x)$ satisfies Eikonal equation

$$\begin{aligned} \|\nabla\psi(x)\| &= c(x), && \text{for } x \in \Omega \setminus \mathcal{T}; \\ \psi(x) &= 0, && \text{for } x \in \mathcal{T}. \end{aligned}$$



Top: Goal location (blue circle) and obstacles (grey).
Middle: Contours of value function.
Bottom: Gradients of value function (subsampled grid).

Path Extraction from Value Function

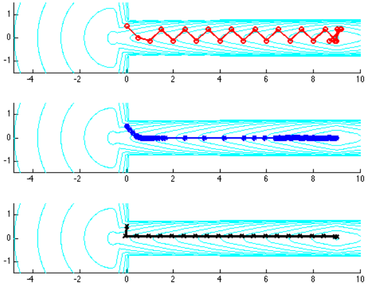
- Given the value function, optimal state feedback action

$$u^*(x) = \frac{\nabla \psi(x)}{\|\nabla \psi(x)\|}.$$

- Typical robot makes decisions on a periodic cycle with period δt so path is given by

$$\begin{aligned} t_{i+1} &= t_i + \Delta t, \\ x(t_{i+1}) &= x(t_i) + \Delta t \; u^*(x(t_i)). \end{aligned}$$

- Even variable step integrators for $\dot{x}(t) = u^*(x(t))$ struggle

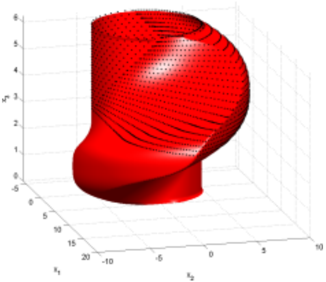


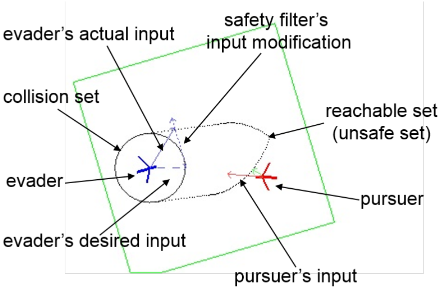
Top: Fixed stepsize explicit (forward Euler).
Middle: Adaptive stepsize implicit (`ode15s`).
Bottom: Sampled gradient algorithm.

Not Just for Static HJ PDEs!

Time dependent HJ PDEs arise in:

- Finite horizon optimal control.
- Reachable sets and viability kernels for formal verification of continuous and hybrid state systems.





(See game of two identical vehicles / softwalls slide deck.)

Outline

Outline

1. Motivation: Value Functions and Action / Path Synthesis
2. Background: Gradient Sampling and Particle Filters
3. Gradient Sampling Particle Filter
4. Dealing with Stationary Points
5. Concluding Remarks

Gradient Sampling for Nonsmooth Optimization I

Gradient sampling algorithm [Burke, Lewis & Overton, SIOPT 2005]

- Evaluate gradient at k random samples within ϵ-ball of current point $x(t_i)$

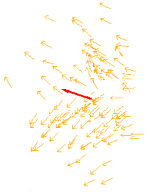
$$x^{(k)}(t_i) = x(t_i) + \epsilon\, \delta x^{(k)},$$
$$p^{(k)}(t_i) = \nabla \psi(x^{(k)}(t_i)).$$

- Determine consensus direction

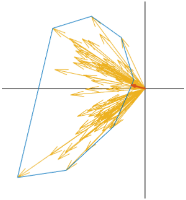
$$p^*(t_i) = \operatorname*{argmin}_{p \in \mathcal{P}(t_i)} \|p\|$$
$$\mathcal{P}(t_i) = \operatorname{conv}\{p^{(1)}(t_i), \ldots, p^{(K)}(t_i)\}.$$

$\mathcal{P}(t_i)$ approximates the Clarke subdifferential at $x(t_i)$.

Gradient samples (yellow) and consensus direction (red).



Plotted in state space.



Plotted in gradient space. Convex hull (blue) also shown.

Gradient Sampling for Nonsmooth Optimization II

Gradient sampling algorithm [Burke, Lewis & Overton, SIOPT 2005]

If $\|p^*(t_i)\| = 0$

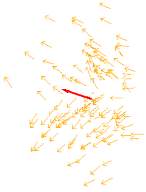
- There is a Clarke ϵ-stationary point inside the sampling ball.
- Shrink ϵ and resample.

If $\|p^*(t_i)\| \neq 0$

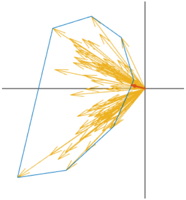
- Choose step length s by Armijo line search along $p^*(t_i)$.
- Set new point

$$x(t_{i+1}) = x(t_i) - s\frac{p^*(t_i)}{\|p^*(t_i)\|}.$$

Gradient samples (yellow) and consensus direction (red).



Plotted in state space.



Plotted in gradient space. Convex hull (blue) also shown.

Particle Filters

Monte Carlo localization (MCL) [Thrun, Burgard & Fox, *Probabilistic Robotics*, 2005] is often used to estimate current state for mobile robots.

- State estimate is a collection of weighted samples

$$\{(w^{(k)}(t), x^{(k)}(t))\}.$$

- Predict: Draw new sample state $x^{(k)}(t_{i+1})$ when action $u(t_i)$ is taken

$$x^{(k)}(t_{i+1}) \sim p(x(t_{i+1}) \mid x^{(k)}(t_i), u(t_i)).$$

- Correct: Update weights $w^{(k)}(t_{i+1})$ when sensor reading arrives

$$w^{(k)}(t_{i+1}) = p(\text{sensor reading} \mid x^{(k)}(t_{i+1}))\, w^{(k)}(t_i),$$

- Resample states and reset weights regularly.

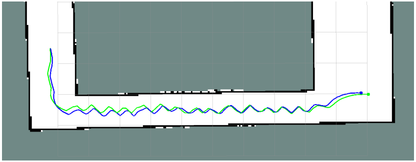
We always work with particle cloud after resampling (when all weights are unity).

Outline

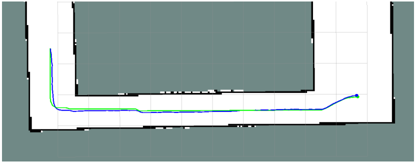
1. Motivation: Value Functions and Action / Path Synthesis
2. Background: Gradient Sampling and Particle Filters
3. Gradient Sampling Particle Filter
4. Dealing with Stationary Points
5. Concluding Remarks

The Gradient Sampling Particle Filter (GSPF)

- Sample the gradients at the particle locations.
- If $\|p^*(t_i)\| \neq 0$, then $p^*(t_i)$ is a consensus descent direction for current state estimate.



Choosing action by steepest descent on the expected state.

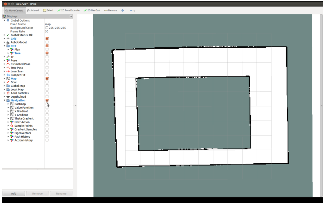


Choosing action by GSPF.

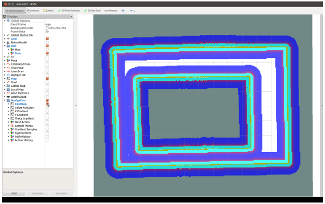
Simulated traversal of a narrow corridor.
Estimated (blue) and true (green) paths shown.

Narrow Corridor Simulation

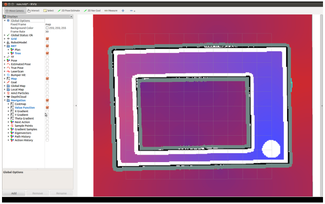
Run in ROS (Robot Operating System) with simulated motion and sensor noise (ROS/Gazebo) and particle filter localization (ROS/AMCL).



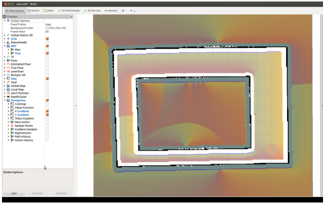
Occupancy grid constructed with laser range finder(s) and SLAM package.



Cost function $c(x)$ (higher cost is darker blue).



Value function $\psi(x)$ (higher value is red).

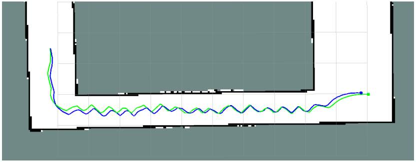


Vertical component of $\nabla\psi(x)$ (yellow is downward, brown is upward).

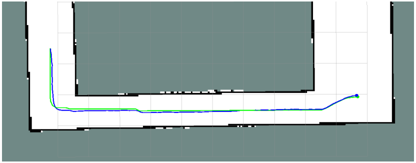
Narrow Corridor Simulation

Compare paths taken when choosing action by AMCL expected state (roughly the mean of particle locations) or GSPF.

- Chattering remains even as step size reduced.



Choosing action by steepest descent on the expected state.



Choosing action by GSPF.

Visualizations shown in the videos:

- **Main window:** Estimated state and path (blue), actual state and path (green), particle cloud (pink), action choices (red arrows), sensor readings (red dots), vertical component of $\nabla\psi(x)$ (background).
- **Lower left window:** Gradient space visualization of gradient samples (yellow) and action choice (red).

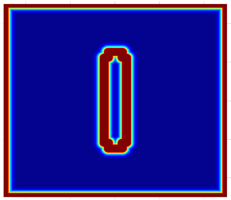
Outline

Finite Wall Scenario

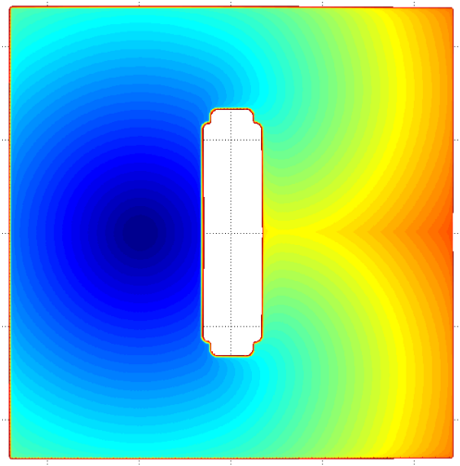
If $\|p^*(t_i)\| = 0$ there is no consensus direction.

Finite wall scenario displays the two typical types of stationary points:

- Minimum (left side): Path is complete(?)
- Saddle point (right side): Seek a descent direction.



Cost.



Value approximation.

Classify the Stationary Point

Quadratic ansatz for value function in neighborhood of samples

$$\bar{\psi}(x) = \tfrac{1}{2}(x - x_c)^T \mathrm{A}(x - x_c)$$

- Fit to the existing gradient samples

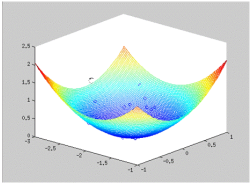
$$\nabla\bar{\psi}(x) = \mathrm{A}(x - x_c).$$

- Solve by least squares

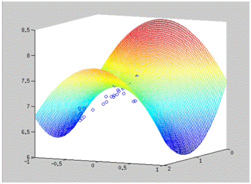
$$\min_{\mathrm{A},b} \|p^{(k)}(t_i) - \mathrm{A}x^{(k)}(t_i) - b\|$$

  and set $x_c = \mathrm{A}^{-1}b$.

- Examine eigenvalues $\{\lambda_j\}_{j=1}^d$ of A
  - If all $\lambda_j > 0$, local minimum.
  - If any $\lambda_j < 0$, corresponding eigenvectors are descent directions.

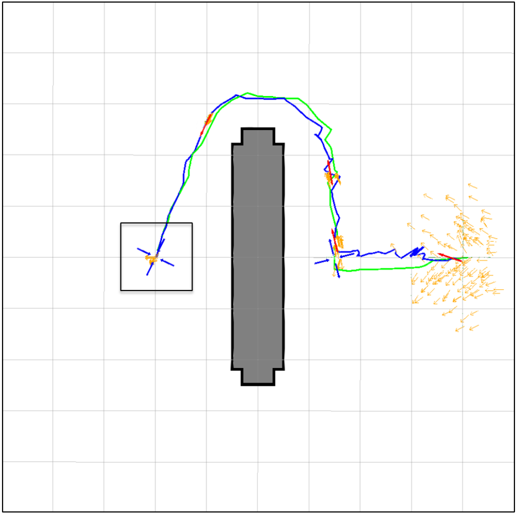


$\bar{\psi}(x)$ at minimum.

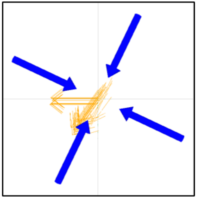


$\bar{\psi}(x)$ at saddle.

Classification Experiments: Minimum

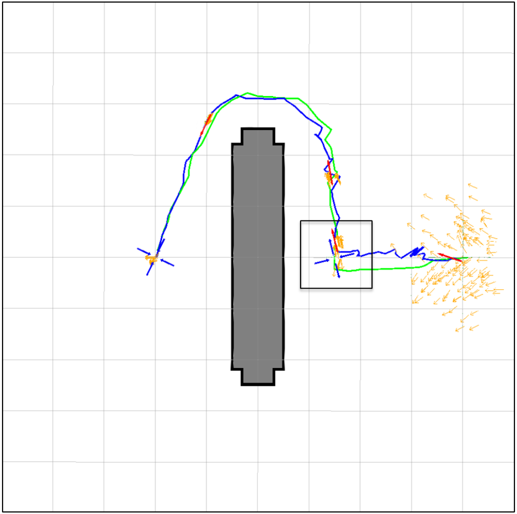


State space view of path.

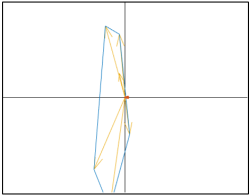


State space gradient samples (gold) and eigenvectors of Hessian of $\bar{\psi}(x)$ (blue). Inward pointing eigenvector arrow pairs correspond to positive eigenvalues.

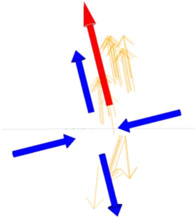
Classification Experiments: Saddle



State space view of path.



Gradient space convex hull

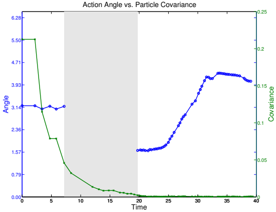


State space eigenvectors of Hessian of $\bar{\psi}(x)$.

Resolve the Stationary Point

Three potential responses to detection of a stationary point

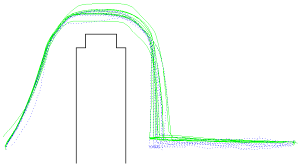
- Stop: If it is a minimum and localization is sufficiently accurate.
- Reduce sampling radius: Collect additional sensor data to improve localization estimate.
  - Rate and/or quality of sensing can be reduced when consensus direction is available.
  - Localization should be improved by independent sensor data.
- Vote: If it is a saddle and improved localization is infeasible.
  - Let v be the eigenvector associated to a negative eigenvalue and $\alpha = \sum_k \operatorname{sign}(-v^T p^{(k)})$.
  - Travel in direction $\operatorname{sign}(\alpha) v$.



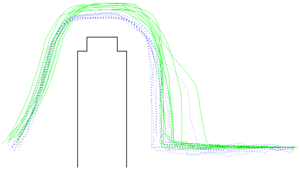
Decreasing localization covariance (green) allows for identification of a consensus direction (blue). In grey region, no consensus direction was found.

Finite Wall Simulation

Generate paths for finite wall scenario using each of the two resolution procedures for the saddle point.



Resolution by using an improved sensor.



Resolution by voting.

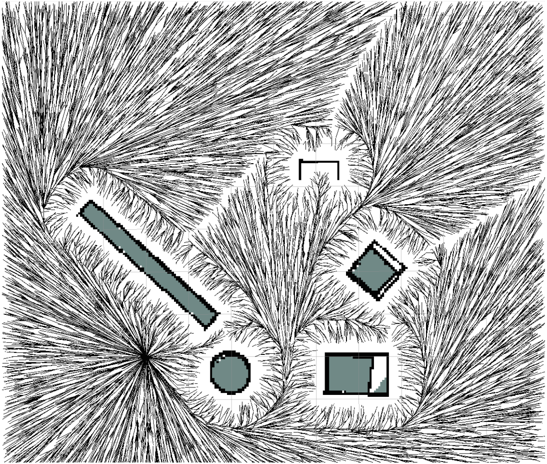
Visualizations shown in the videos:

- **Main window:** Estimated state and path (blue), actual state and path (green), particle cloud (pink), action choices (red arrows), gradient samples (yellow), sensor readings (red dots) and eigenvectors of the Hessian approximation (blue arrows).
- **Lower left window:** Gradient space visualization of gradient samples (yellow) and action choice (red).

Outline

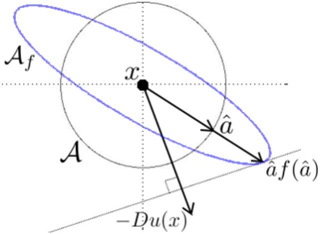
1. Motivation: Value Functions and Action / Path Synthesis
2. Background: Gradient Sampling and Particle Filters
3. Gradient Sampling Particle Filter
4. Dealing with Stationary Points
5. Concluding Remarks

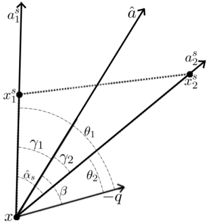
Gradient Sampling is not just for Value Functions

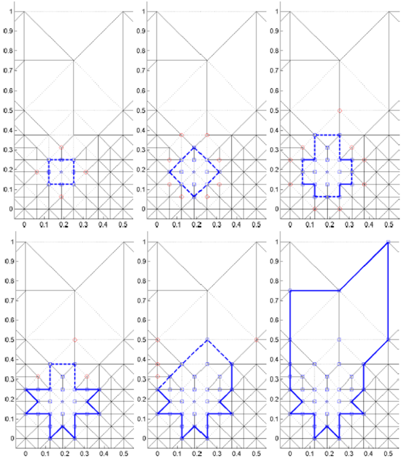


Actions synthesized by nearest neighbor lookup on RRT* tree.

No Time For. . .

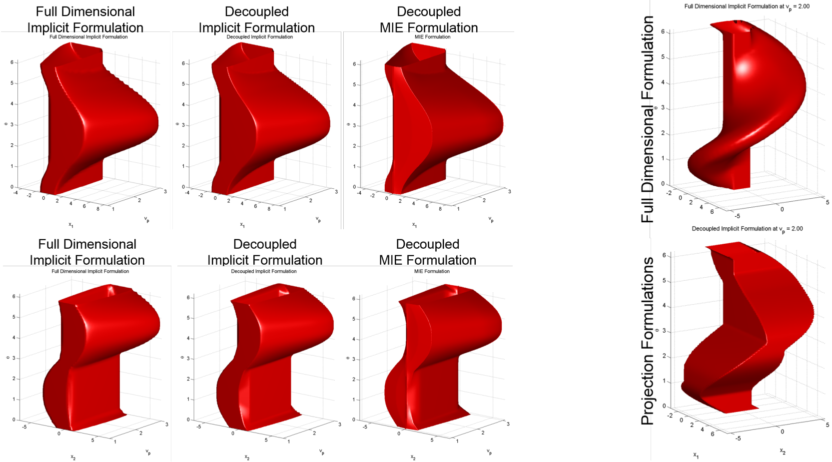






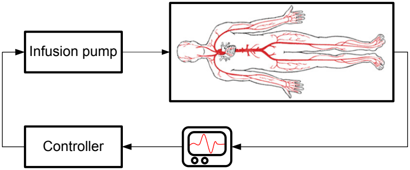
Monotone acceptance ordered upwind method [Alton & Mitchell, JSC 2012].

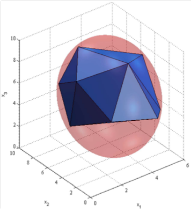
No Time For. . .

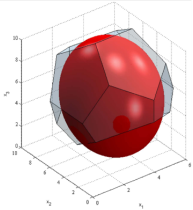


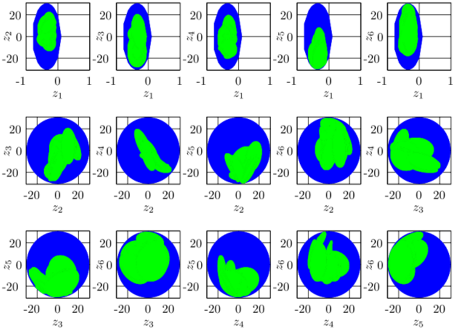
Mixed implicit explicit formulation of reach sets to reduce computational dimension [Mitchell, HSCC 2011].

No Time For...





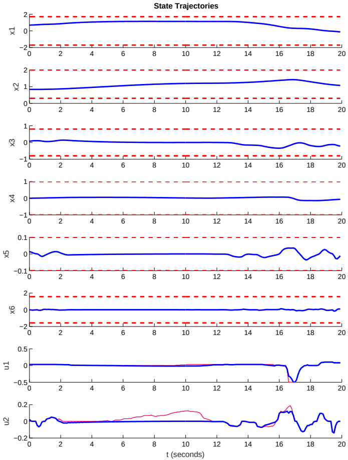




Parametric approximations of viability and discriminating kernels with applications to verification of automated anesthesia delivery [Maidens et al, Automatica 2013].

No Time For...

Shared control smart wheelchair for older adults with cognitive impairments [Viswanathan et al, Autonomous Robots 2016].



Ensuring safety for human-in-the-loop flight control of low sample rate indoor quadrotors [Mitchell et al, CDC 2016].

Conclusions

Gradient sampling particle filter (GSPF)

- Utilizes natural uncertainty in system state to reduce chattering due to non-smooth value function and/or numerical approximation.
- Easily implemented on existing planners and state estimation.
- To appear in [Traft & Mitchell, CDC 2016].

Future work

- Nonholonomic dynamics.
- Convergence proof.
- Scalability to more particles.
- Set-valued actions.
- Human-in-the-loop control and interface.