CS 536E | Problem Set 2 | 2 Nov 2023
W. Evans | | Due: 16 Nov 2023

Reading:

- Rectangle and Square Representations of Planar Graphs by Felsner.
- Graphs and Geometry (Chapter 6) by Lováz.

You may discuss problems with other people in the class, but you must write up your own solutions. If you do discuss a problem with someone else or you use an outside resource, you must acknowledge them. Do not copy solutions from anyone. A star ($\star$) means that the problem may be somewhat difficult.

1. Suppose G is a plane graph (a graph with a planar embedding) with four vertices of degree 2 on the outer face (the *corners*) and all the other vertices with degree 3 or 4. In class, we talked about the *angle graph* $A(G)$ of G which has a vertex for every face of G (except the outer face) and an edge connecting each such face vertex to the vertices in G of that face. So the vertices in $A(G)$ are the vertices in G (the *white* vertices) plus one face vertex (*black*) for each inner face of G.

   We would like to orient the edges in $A(G)$ so that:
   For all black vertices f, out-deg$(f) = 4$.
   For all white vertices v, out-deg$(v) = 1$ if v is an inner vertex with degree 3 and out-deg$(v) = 0$ otherwise.

   Describe how to construct such an orientation (if it exists) in polynomial time. One way is to use network flow.

2. A *quadrangulation* is a maximal bipartite planar graph. Suppose we are given an quadrangulation Q with a planar embedding. Color the vertices in one partition white and in the other black, and call the two black vertices on the outer face s and t. A *separating decomposition* of Q is an orientation and coloring of its edges with colors red and blue such that:

   (a) Vertex s is incident to only incoming red edges.
   (b) Vertex t is incident to only incoming blue edges.
   (c) Every vertex $v \neq s, t$ is incident to a nonempty interval of red edges and a nonempty interval of blue edges. If v is white, the first edge of each interval (in clockwise order) is outgoing and if v is black, the last edge of each interval is outgoing. The other edges in each interval are incoming.

   Describe how to obtain a separating decomposition (if it exists). Hint: One way is to modify a Schnyder wood.

3. Write at most one page on the progress you have made on your project.

   The final project will involve writing a less than 10 page paper (five pages is o.k., commented code can count for some of it) and presenting your work to the class (20 minutes with questions). The deadline for the paper and presentation will be December 12 or 14.