CS 500 | Problem Set 4 | 30 Jan 2012
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You may discuss problems with other people in the class, but you must write up your own solutions. Please acknowledge the people you work with and any sources you use (web pages, books, etc.). A star ($\star$) means that the problem may be somewhat difficult.

1. (Exercise 25.3-6 CLRS) Professor Michener claims that there is no need to create a new source vertex s when executing Johnson's reweighting algorithm. He claims that instead we can just use $G' = G$ and let s be any vertex in G. Give an example of a weighted, directed graph G that shows that the professor is wrong. Then show that if G is strongly connected (every vertex is reachable from every other vertex), the professor's claim is correct.

2. Let $A = a_1 a_2 \ldots a_n$ and $B = b_1 b_2 \ldots b_m$ be two strings of characters. Let $A[i]$ be the ith suffix of A, i.e., the string $a_i a_{i+1} \ldots a_n$. Let d_i be the edit distance between $A[i]$ and B. Describe an $O(nm)$-time algorithm to find the minimum value of d_i over all i, $1 \leq i \leq n$.

3. (Problem 5.8.2 in Algorithmics by Brassard and Bratley) Consider the alphabet $\Sigma = \{a, b, c\}$. The elements of Σ have the following multiplication table:

		Right-hand symbol		
		a	b	c
Left-hand	a	b	b	a
symbol	b	c	b	a
	c	a	c	c

   Thus $ab = b$, $ba = c$, and so on. Note that the multiplication defined by this table is neither commutative nor associative.

   Find an efficient algorithm that examines a string $x = x_1 x_2 \ldots x_n$ of characters of Σ and decides whether or not it is possible to parenthesize x in such a way that the value of the resulting expression is a. For example, if $x = bbbba$, your algorithm should return "yes" because $(b(bb))(ba) = a$. (This is not the only parenthesization that works: $(b(b(b(ba)))) = a$.)

4. (Problem 6.15 Algorithms) Suppose two teams, A and B, are playing a match to see who is the first to win n games (for some particular n). We can suppose that A and B are equally competent, so each has a 50% chance of winning any particular game. Suppose they have already played $i+j$ games, of which A has won i and B has won j. Give an efficient algorithm to compute the probability that A will go on to win the match. For example, if $i = n-1$ and $j = n-3$ then the probability that A will win the match is $7/8$, since it must win any of the next three games.

5. (from Problem 15-1 CLRS) A *bitonic tour* of a set of n points in the plane is a tour that starts at the leftmost point, goes strictly left to right to the rightmost point, and then goes strictly right to left back to the starting point. See Figure 1

   Describe an $O(n^2)$-time algorithm for determining a shortest bitonic tour (using Euclidean distance). You may assume that no two points have the same x-coordinate. (Hint: Scan left to right, maintaining optimal possibilities for the two parts of the tour.)

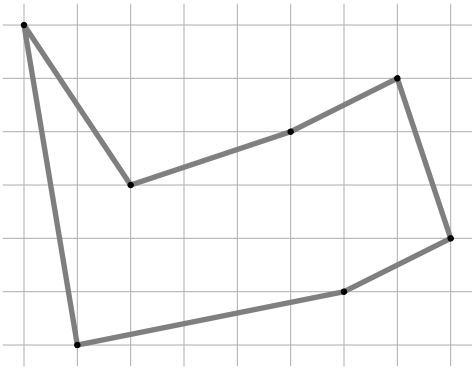

Figure 1: The shortest bitonic tour for a given set of 7 points.