Profit Maximization over Social Networks

Wei Lu
Laks V.S. Lakshmanan
ICDM’12, to appear
Overview

- Background: Social Influence Propagation & Maximization
- Motivation for Profit Maximization
- Proposed Model & Its Properties
- Profit Maximization Algorithms
- Experimental Results
- Conclusions & Discussions
Background
Influence in Social Networks

- We live in communities and interact with friends, families, and even strangers
- This forms social networks
- In social interactions, people may influence each other
Influence Diffusion & Viral Marketing

Word-of-mouth effect

Source: Wei Chen’s KDD’10 slides
Social Network as Directed Graph

- **Nodes**: Individuals in the network
- **Edges**: Links/relationships between individuals
- **Edge weight on** \((i, j)\): Influence weight \(w_{i,j}\)
Linear Threshold Model – Definition

- Each node $v$ chooses an activation threshold $\theta_v$ uniformly at random from $[0, 1]$.
- Time unfolds in discrete steps $0, 1, 2, \ldots$
- At step 0, a set $S$ of seeds are activated.
- At any step $t$, activate node $v$ if
  \[ \sum_{\text{active in neighbor } u} w_{u,v} \geq \theta_v \]
- The diffusion stops when no more nodes can be activated.
- **Influence spread of $S$**: The expected number of active nodes by the end of the diffusion, when targeting $S$ initially.
Linear Threshold Model – Example

Influence spread of \{v\} is 4

Stop!

Source: David Kempe’s slides
Influence Maximization

**Input**
A directed graph representing a social network, with influence weights on edges

**Problem**
Select $k$ individuals such that by activating them, influence spread is maximized.

**Output**

NP-hard

#P-hard to compute exact influence

NP-hard 😞  #P-hard to compute exact influence 😞
Motivation for Profit Maximization
Influence vs. Product Adoption

- Classical models do not fully capture monetary aspects of product adoptions
  - Being influenced ≠ Being willing to purchase
- HP TouchPad significant price drop in 2011 ($499 → $99)
- Worldwide market share of cellphones (as of 2011.7):
  1. Nokia ☹
  2. Samsung (boo…)
  3. LG (…)
  4. Apple iPhone ☻
- iPhone: More expensive in hardware and monthly rate plans (ask Rogers, Telus, or Bell…)
Product Adoption

- Product adoption is a two-stage process (Kalish 85)
- 1\textsuperscript{st} stage: Awareness
  - Get exposed to the product
  - Become familiar with its features
- 2\textsuperscript{nd} stage: Actual adoption
  - Only if \textit{valuation} outweighs \textit{price}
  - Awareness is modeled as being propagated through word-of-mouth: captured by classical models
- OTOH, the 2\textsuperscript{nd} stage is not captured
Valuations for Products

- One’s **valuation** for a product = the maximum amount of money one is willing to pay
  - People do not want to reveal valuations for trust and privacy reasons (Kleinberg & Leighton, FOCS’03)

- **IPV (Independent Private Value) assumption**: The valuation of each person’s is drawn independently at random from a certain distribution (Shoham & Leyton-Brown 09)

- **Price-taker assumption**: Users respond myopically to price, comparing it only with own valuation
Our Contribution

- Incorporate monetary aspects into the modeling of the diffusion process of product adoption
  - Price & user valuations
  - Seeding costs
- LT $\rightarrow$ LT with user valuations (LT-V)
- Profit maximization (ProMax) under LT-V
- Price-Aware GrEedy algorithm (PAGE)
Proposed Model & Problem Definition
Linear Threshold Model with Valuations (LT-V)

- Three node states: **Inactive**, **Influenced**, and **Adopting**
- **Inactive** $\rightarrow$ **Influenced**: same as in LT
- **Influenced** $\rightarrow$ **Adopting**: Only if the valuation is at least the quoted price
- Only *adopting* nodes will propagate influence to *inactive* neighbors
- Model is progressive (see figure)
More about LT-V

- Our LT-V model captures the two-stage product adoption process in (Kalish 1985)
- Only *adopting* nodes propagate influence: Actual adopters can access *experienced-based* features of the product
  - Usability, e.g., Easy to shoot night scenes using Nikon D600?
  - Durability, e.g., How long can iPhone 5’s battery last on LTE?
- Still quite abstract, with room for extensions and refinements (more to come later)
ProMax: Notations

- \( \mathbf{p} = (p_1, p_2, \ldots, p_{|V|}) \): the vector of quoted prices, one per each node
- \( S \): the seed set
- \( \pi: 2^V \times [0,1]^V \rightarrow \mathbb{R} \): the profit function
  - \( \pi(S, \mathbf{p}) \): the expected profit earned by targeting \( S \) and setting prices \( \mathbf{p} \)
Problem Definition

Problem
Select a set $S$ of seeds & determine a vector $p$ of quoted price, such that the $\pi(S, p)$ is maximized under the LT-V model

Input
A directed graph representing a social network, with influence weights on edges

Output
$\$ $\$ $\$
ProMax vs. InfMax

- Difference w/ InfMax under LT
  - Propagation models are different & have distinct properties
  - InfMax only requires “binary decision” on nodes, while ProMax requires to set prices
A Restricted Special Case

- Simplifying assumptions:
  - Valuation distributions degenerate to a single point:
    \[ v_i = p, \forall u_i \in V \]
  - Seeds get the item for free (price = 0)
- Optimal price vector is out of question
- **Restricted ProMax**: Find an optimal seed set \( S \) to maximize
  \[ \pi(S) = p \times (h_L(S) - |S|) - c_a \times |S| \]
  - \( h_L(S) \): expected #adopting nodes under LT-V
  - \( c_a \): acquisition cost (seeding expenses)
A Restricted Special Case

- \( \pi(S) \) is non-monotone, but submodular in \( S \)
  - No need to preset \#seeds to pick (the number \( k \) in InfMax)
  - Simple greedy cannot be applied to get approx. guarantees

- **Theorem**: The restricted ProMax problem is **NP-hard**.
  - Reduction from the **Minimum Vertex Cover** problem

- Aside on Maximizing non-monotone submodular functions:
  Local search approximation algorithms (Feige, Mirrokni, & Vondrak, FOCS’07)
  - Nice, but time complexity too high
  - They assumed an oracle for evaluating the function
Unbudgeted Greedy (U-Greedy)

- Simply grow the seed set $S$ by selecting the node with the largest marginal increase in profit, and stop when no nodes can provide positive marginal gain.

- **Theorem** (Quality guarantee of U-Greedy)
  \[
  \pi(S_g) \geq \left(1 - \frac{1}{e}\right)\pi(S^*) - \Theta(\max\{|S^*|, |S_g|\})
  \]

  - $S_g$: Seed set by U-Greedy
  - $S^*$: optimal seed set

- Proof: Some algebra… omitted…
Properties of LT-V (in general)

- For an arbitrary vector of valuation samples $\mathbf{v} = (v_1, v_2, \ldots, v_{|V|})$, given an instance of the LT-V model, for any fixed vector $\mathbf{p}$ of prices, the profit function $\pi(S, \mathbf{p})$ is submodular in $S$.

- It is $\#\text{P}-\text{hard}$ to compute the exact value of $\pi(S, \mathbf{p})$, given any $S$ and $\mathbf{p}$.
Algorithms for Profit Maximization
ProMax Algorithm: All-OMP

- Given the distribution function (CDF) $F_i$ of $v_i$, the Optimal Myopic Price (OMP) is
  \[ p_i^m = \arg\max_{p \in [0,1]} p \cdot (1 - F_i(p)) \]

- First baseline – All OMP: Offer OMP to all nodes, and select seeds using U-Greedy.
- Ensures max. profit earned solely from a single influenced node
- Ignores network structures and “profit potential” (from influence) of seeds
ProMax Algorithm: Free-For-Seeds (FFS)

- Seeds receive the product for free
- Non-seeds are charged OMP
- Ensure all seeds will adopt & propagate influence
- **Trade-off**: immediate profit from seeds vs. profit potential of seeds (through influence)
  - All-OMP favors the former, good for low influence networks
  - FFS favors the latter, good for high influence networks
- Can we achieve a more balanced heuristic?
Price-Aware GrEedy (PAGE) Algorithm

• The key question: Given a partial seed set, how to determine the price $p_i$ for the next seed candidate $u_i$?

• Consider the marginal profit $u_i$ brings:

$$MP(u_i) = \pi(S \cup \{u_i\}, p_{-i} \oplus p_i) - \pi(S, p_{-i} \oplus p_i^m)$$

• This is a function of $p_i$

• So, let’s find $p_i$ that maximizes $\pi(S \cup \{u_i\}, p_{-i} \oplus p_i)$
Offer $p_i$ to $u_i$ leads to 2 possible worlds:

- $u_i$ accepts, w.p. $1 - F_i(p_i)$.
- $u_i$ does not accept, w.p. $F_i(p_i)$.

Re-write $\pi(S \cup \{u_i\}, p_{-i} \oplus p_i)$ as follows:

$$g_i(p_i) = (1 - F_i(p_i)) \cdot (p_i + Y_1) + F_i(p_i) \cdot Y_0 - c_a$$

- $Y_1$: expected profit earned from other nodes, if $u_i$ accepts
- $Y_0$: -------------------------------, o.w.

Finding the optimal $p_i$ depends on the specific form of $F_i$

We study two cases:

- Normal distribution
- Uniform distribution
Normal Distribution

• CDF:

\[ F_i(p_i) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{p_i - \mu}{\sqrt{2\sigma}} \right) \right] \]

where \( \text{erf}(x) \) is the error function

\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt \]

• No analytical solution can be found, since \( \text{erf}(x) \) has no closed-form expression, and thus neither does \( g_i(p_i) \)

• Numerical method: the **golden section search** algorithm
Aside: Golden Section Search

- Finds the extremum of a function by iteratively narrowing the interval inside which the extremum is known to exist
  - The function must be *unimodal* and *continuous* over the initial interval
  - Terminates when the length of the interval is smaller than a pre-defined number (say, $10^{-6}$)
  - No need to take derivatives (which the Newton’s method will need)
- Performs only one new function evaluation in each step
- Has a constant reduction factor for the size of the interval
Uniform Distribution

- CDF: \( F_i(p_i) = p_i \)
- So, \( g_i(p_i) = -p_i^2 + (1 - Y_1 + Y_0) \cdot p_i + Y_1 - c_a \)
- Easily, we solve for optimal \( p_i \):

\[
p_i^* = \frac{(1 + Y_1 - Y_0)}{2}
\]

- If \(< 0\) or \(>1\), normalize it back to 0 or 1 (respectively)
- **N.B.**, This solution framework can be applied to **any** valuation distributions. Actual solution may be analytical or numerical, depending on the distribution itself.
Experiments: Datasets & Results
Network Datasets

- **Epinions**
  - A who-trusts-whom network from the customer reviews site *Epinions.com*

- **Flixster**
  - A friendship network from social movie site *Flixster.com*

- **NetHEPT**
  - A co-authorship network from *arxiv.org* High Energy Physics Theory section.
Network Datasets

- Statistics of the datasets:

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Epinions</th>
<th>Flixster</th>
<th>NetHEPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodes</td>
<td>11K</td>
<td>7.6K</td>
<td>15K</td>
</tr>
<tr>
<td>Number of edges</td>
<td>119K</td>
<td>50K</td>
<td>62K</td>
</tr>
<tr>
<td>Average out-degree</td>
<td>10.7</td>
<td>6.5</td>
<td>4.12</td>
</tr>
<tr>
<td>Maximum out-degree</td>
<td>1208</td>
<td>197</td>
<td>64</td>
</tr>
<tr>
<td>#Connected components</td>
<td>4603</td>
<td>761</td>
<td>1781</td>
</tr>
<tr>
<td>Largest component size</td>
<td>5933</td>
<td>2861</td>
<td>6794</td>
</tr>
</tbody>
</table>
Influence Weights in Datasets

- **Weighted Distribution (WD)**
  \[ w_{u,v} = \frac{A_{u,v}}{N_v} \]
  - \( A_{u,v} \): #actions \( u \) and \( v \) both have performed (if data is time-stamped, then \( u \) should perform earlier)
  - \( N_v \): total #actions \( v \) has performed

- **Trivalency (TV)**: \( w_{u,v} \) is chosen uniformly at random from \( \{0.001, 0.01, 0.1\} \).

- All weights shall be normalized to ensure \( \forall v \),
  \[ \sum_u w_{u,v} \leq 1 \]
Influence Weights in Datasets

(a) Weighted Distribution

(b) Trivalency

Figure 3: Distribution of influence weights in Flixster
Estimating Valuation Distributions

- Hard to obtain real ones
- Common practice: estimate from historical sales data
Estimating Valuation Distributions

- Besides ratings (1 to 5 stars), users may optionally provide the price they paid.
- At the end of the same review:

```
Recommended: Yes

Amount Paid (US$): 999.00
This Camera is a Good Choice if You Want Something... Flexible Enough for Enthusiasts

Read all comments (21) | Write your own comment
Read all 118 Reviews | Write a Review

Share with your friends /facebook/twitter/pinterest
```
Estimating Valuation Distributions

- We obtain all reviews for Canon EOS 300D, 350D, and 400D DSLR cameras
  - Sequential releases in 3 years $\rightarrow$ approximately the same monetary values
- Remove reviews without price: 276 samples remain
- View rating as utility
  \[ \text{utility} = \text{valuation} - \text{price paid} \]
- Thus, our estimation is
  \[ \text{valuation} = \text{price} \times (1 + \frac{\text{rating}}{5}) \]
Estimating Valuation Distributions

- The fitted normal distribution: $N(0.53, 0.14^2)$
- Figure 4(b): Kolmogorov-Smirnov (K-S) statistics

Figure 4: Statistics of Valuations (Epinions.com)
Experimental Results: Expected Profit Achieved

(a) WD with $c_a = 0.1$

(b) WD with $c_a = 0.001$

(c) TV with $c_a = 0.1$

(d) TV with $c_a = 0.001$

Figure 5: Expected profit achieved (Y-axis) on Epinions graphs w.r.t. $|S|$ (X-axis). (N)/(U) denotes normal/uniform distribution.
Experimental Results: Price Assignment for Seeds

Figure 8: Price assigned to seeds (Y-axis) w.r.t. $|S|$ (X-axis) on Epinions-TV with $\mathcal{N}(0.53, 0.14^2)$. 
Experimental Results: Running Time

- PAGE is more efficient
  - Leveraging lazy-forwarding more effectively
  - Extra overhead for computing price is small (golden search algorithm converges in less than 40 iterations)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Epinions-WD</th>
<th>Flixster-WD</th>
<th>NetHEPT-WD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N$</td>
<td>$U$</td>
<td>$N$</td>
</tr>
<tr>
<td>All-OMP</td>
<td>6.7</td>
<td>2.3</td>
<td>3.0</td>
</tr>
<tr>
<td>FFS</td>
<td>6.3</td>
<td>2.1</td>
<td>2.8</td>
</tr>
<tr>
<td>PAGE</td>
<td>4.8</td>
<td>1.3</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Table 2: Running time in hours (WD weights, $c_a = 0.1$)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Epinions-TV</th>
<th>Flixster-TV</th>
<th>NetHEPT-TV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N$</td>
<td>$U$</td>
<td>$N$</td>
</tr>
<tr>
<td>All-OMP</td>
<td>5.1</td>
<td>2.4</td>
<td>1.4</td>
</tr>
<tr>
<td>FFS</td>
<td>5.5</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>PAGE</td>
<td>4.0</td>
<td>1.0</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 3: Running time in hours (TV weights, $c_a = 0.1$)
Conclusions, Discussions, Related Work
Conclusions

- Extended LT model to incorporate price and valuations & distinguish product adoption from social influence
- Studies the properties of the extended model
- Proposed profit maximization (ProMax) problem & effective algorithm to solve it
Discussions & Future Work

- Make similar extensions to other influence propagation models: IC, LT-C, or even the general threshold model
- Develop fast heuristics to give more efficient & scalable algorithms
- Consider to incorporate other elements into the modeling of product adoption
  - Peoples’ spontaneous interests in product (natural early adopters)
  - Valuation may change over time for some people
  - Valuations may observe externalities
Related Work

- Influence maximization (too many…)
- Revenue/profit maximization in social networks
  - Hartline et al., WWW’08: Influence-and-Exploit
  - Arthur et al., WINE’09
  - Chen et al., WINE’11
  - Bloch & Qurou, working paper, 2011
- Influence vs. product adoption
  - Bhagat, Goyal, & Lakshmanan, WSDM’12: LT model with Colors
Thanks!

Acknowledgements:
Allan Borodin (UofT), Wei Chen (MSR Asia), Pei Lee, Kevin Leyton-Brown, Min Xie, Ruben H. Zamar.