Pyramid Coordinates for Morphing and Deformation

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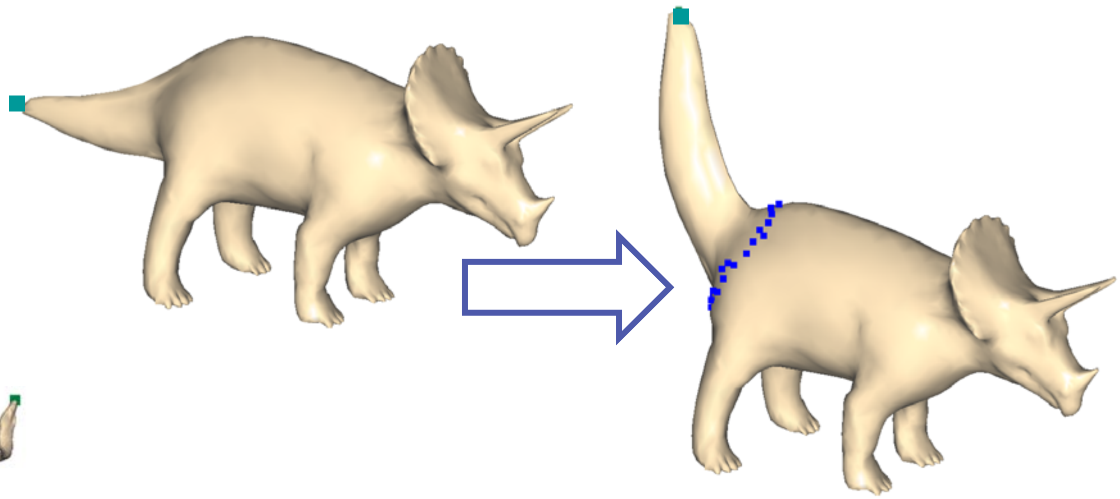
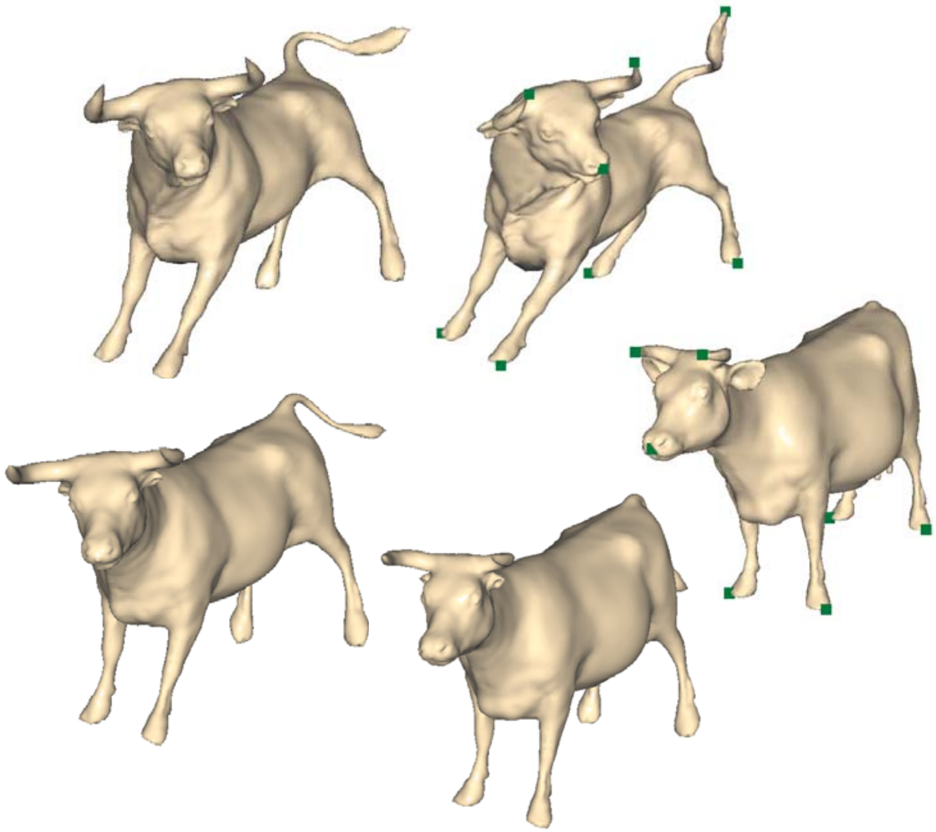
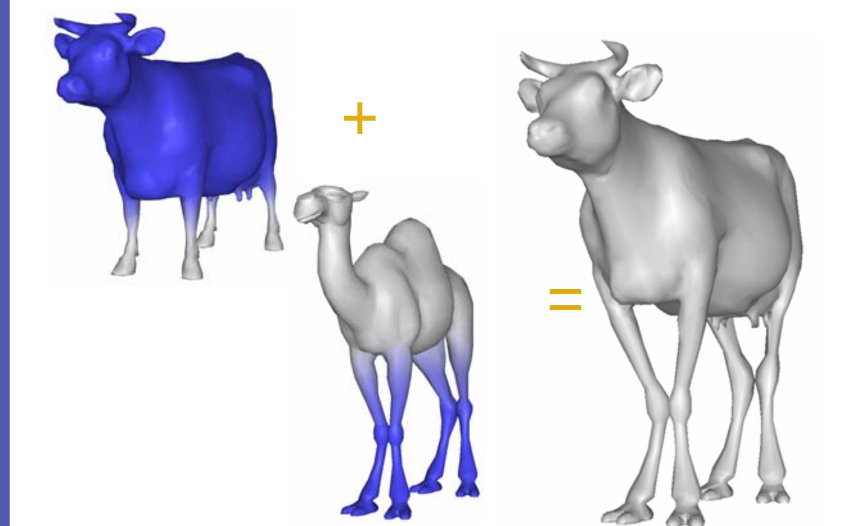
Motivation – Model Editing

- Deformation
- Blending
- Morphing

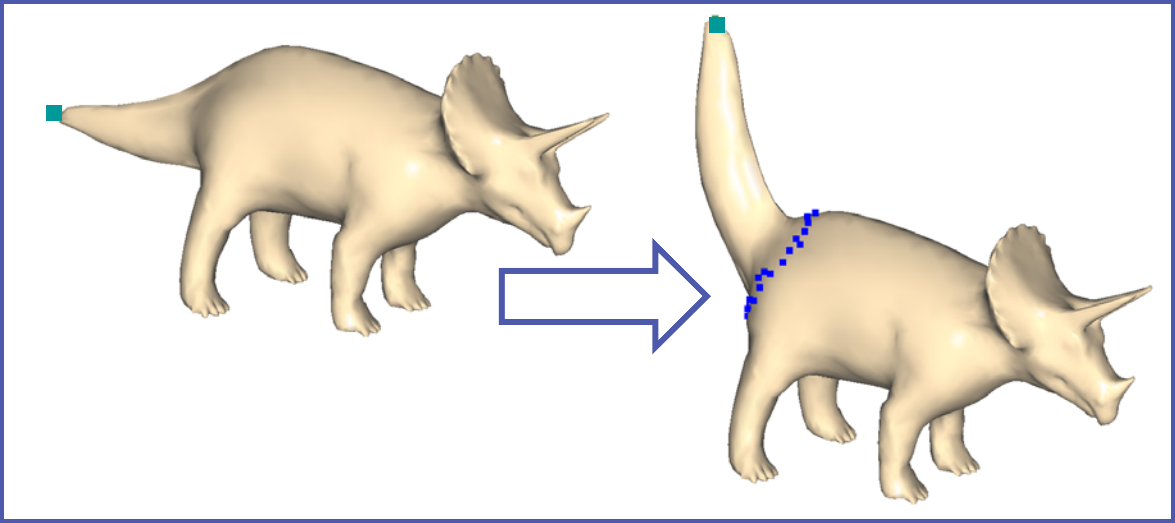


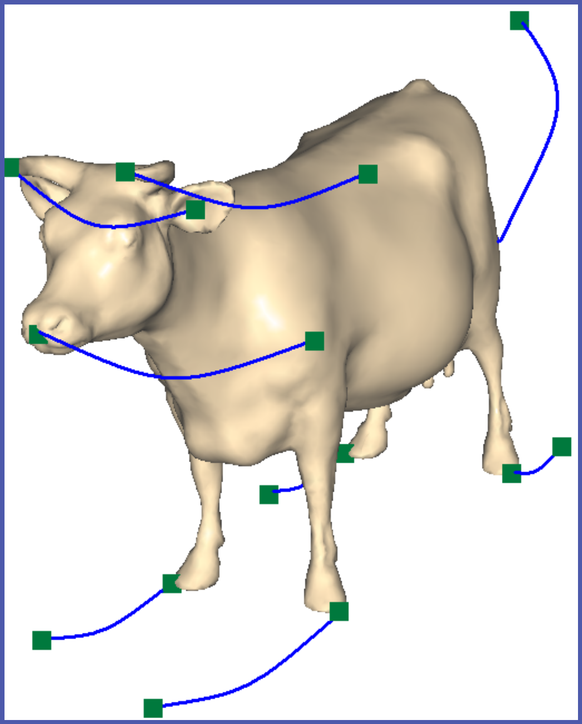




Motivation – Model Editing

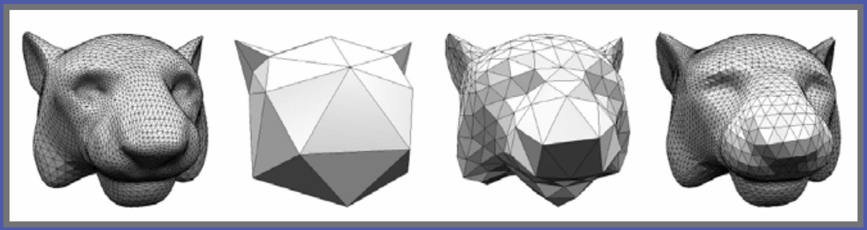
- Simple control mechanism
- Intuitive results





Previous Work

- Multi-resolution: Zorin et al. 97, Kobbelt et al. 99, Guskov et al. 99, Boier-Martin et al. 04, Botsch and Kobbelt 04
  - \+ interactive rates
  - – interpolate absolute coordinates leading to visible artifacts
  - – time consuming preprocessing



- Skeleton based: Yoshizawa et al. 03
  - \+ easy to deform
  - – hard to construct
  - – some deformations cannot be expressed as skeleton modifications

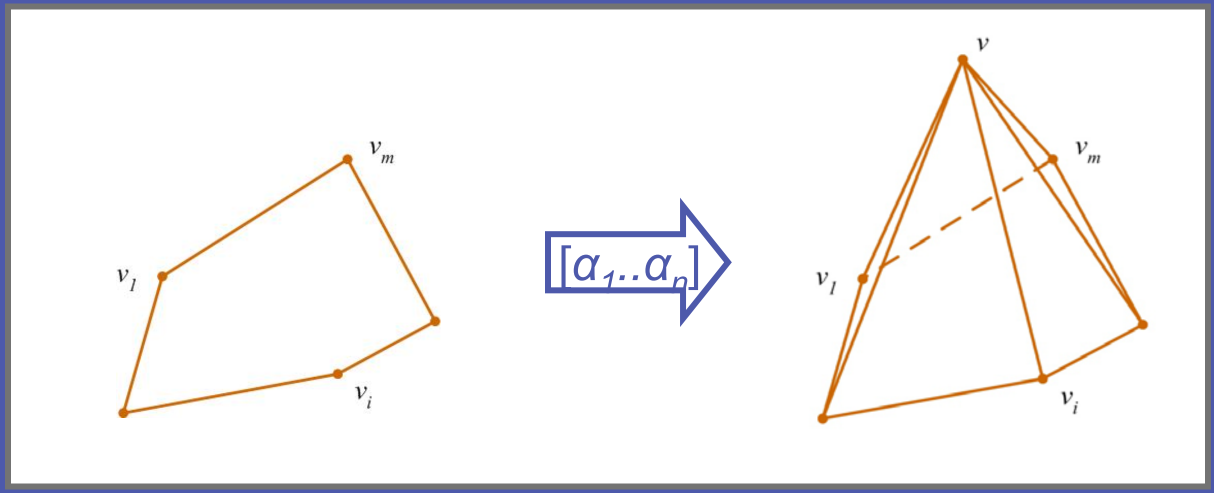
Previous Work

- Local shape representation: Alexa 01, Sorkine et al. 04, Lipman et al. 04, Yu et al. 04

Vertex geometry representarion:

– neigbour vertices $v_1..v_m$

– local coordinates $[\alpha_1..\alpha_n]$

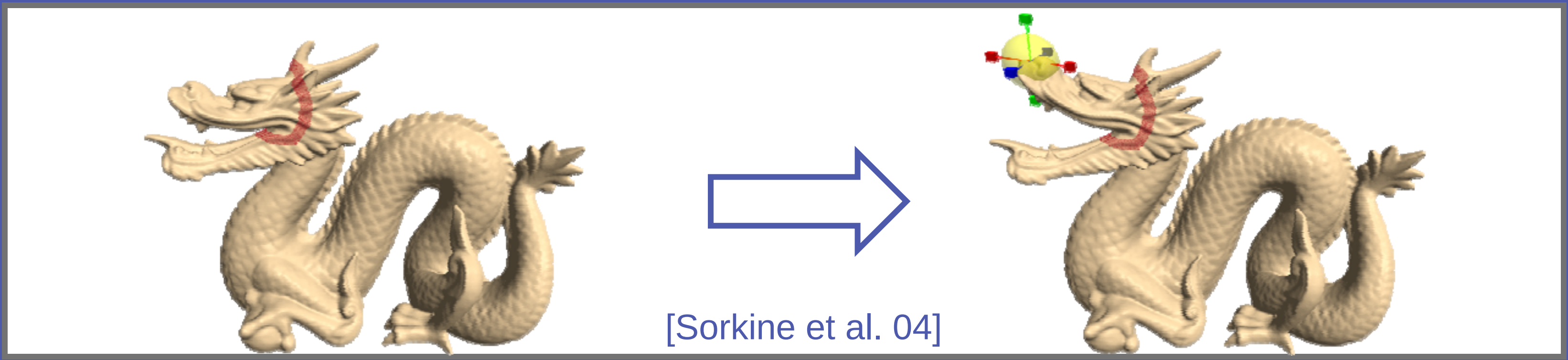


Local implies global

Local Shape Representation

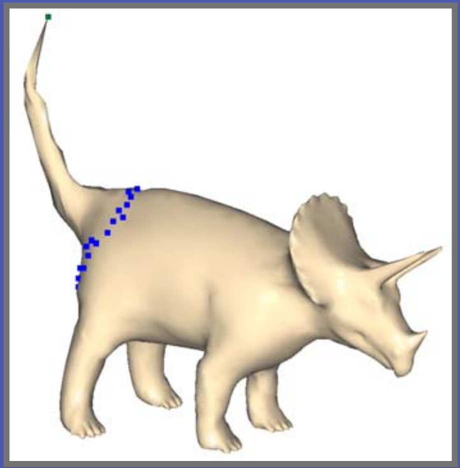
Alexa 01, Sorkine et al. 04, Lipman et al. 04, Yu et al. 04

+ implicitly supports mesh operations
  - editing, blending



[Sorkine et al. 04]

– existing representations not invariant under rigid transformations
  - Partial solution: transformation estimation [Sorkine et al. 04, Lipman et al. 04, Yu et al. 04]



New Local Shape Representation

Desired properties:

- Local shape preservation under deformation
  - implies global
- Invariant under rigid transformations
  - not invariant under shear
  - not invariant under non uniform scale

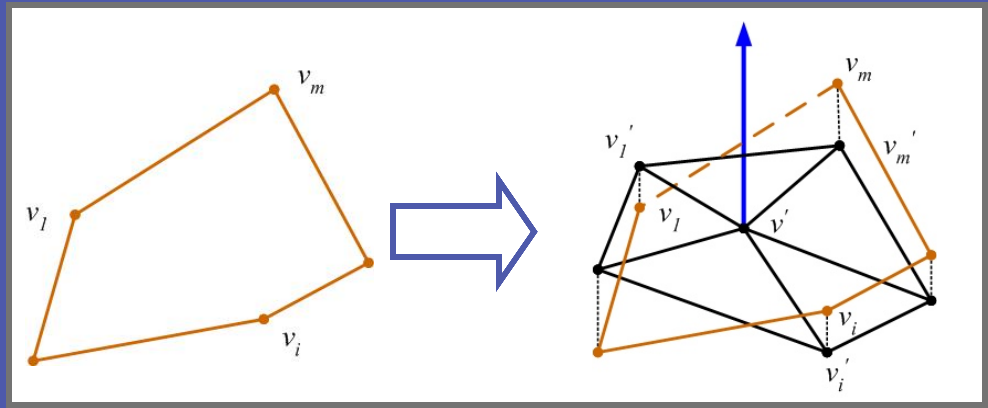
Local Coordinate Frame

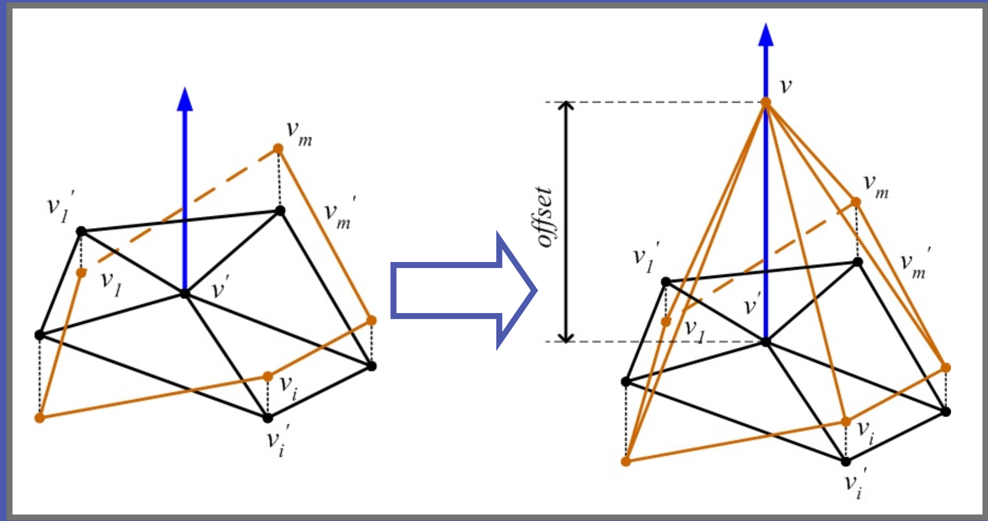
- Define vertex with respect to neighbors
  - Invariant under rigid transformations

- Separate

  1. Tangential component

  2. Normal component

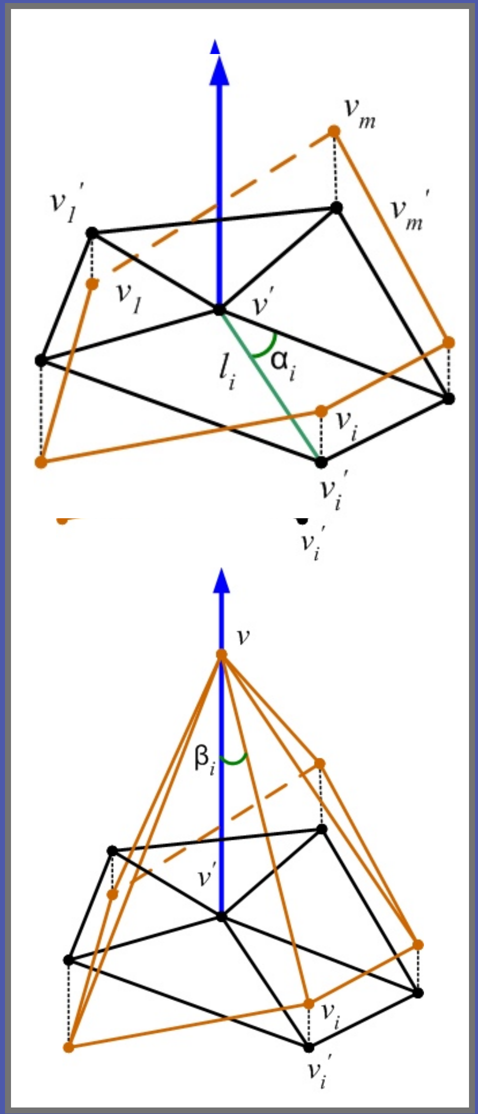




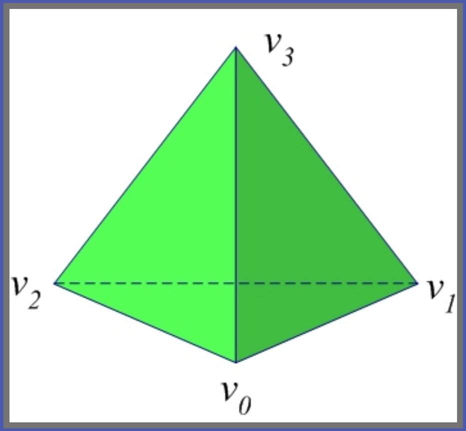
Euclidian to Local

Given v and $v_1..v_m$ compute:

1. Tangential component
   - project the v and $v_1..v_m$ to tangential plane
   - compute α, l
2. Normal component
   - compute β



Euclidian to Local



a) invariant under rigid transformations

b) redundant

Vert	Global Geometry
v_0	*x,y,z*
v_1	*x,y,z*
v_2	*x,y,z*
v_3	*x,y,z*

⇨

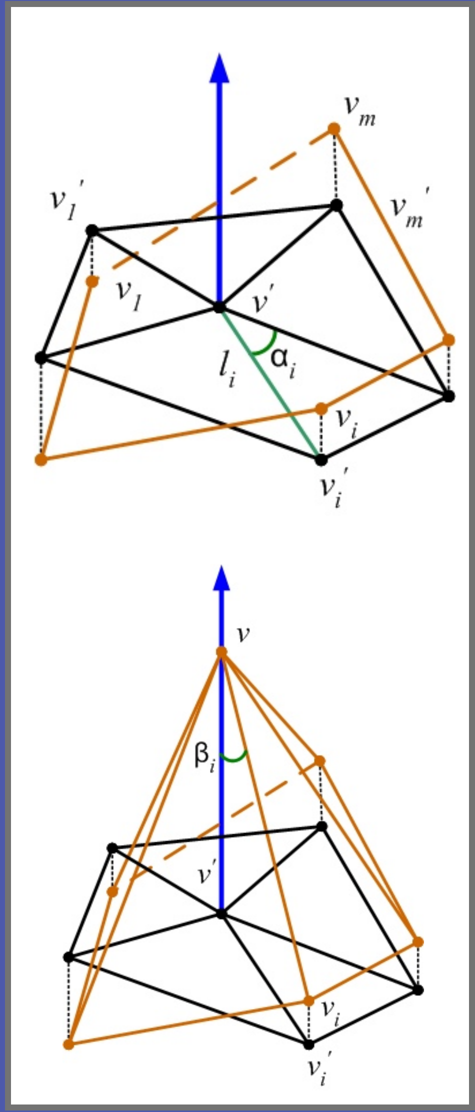
	α	l	β
v_0	*60°, 60°,60°*	*0.58, 0.58, 0.58*	*35.3°, 35.3°, 35.3°*
v_1	.	.	.
v_2	.	.	.
v_3	.	.	.

Local to Euclidian

Given *α, β, l* and $v_1..v_m$ compute:

1. Tangential component
   a) project the $v_1..v_m$ to tangential plane
   b) compute v'

2. Normal component
   a) compute normal offset
   b) compute v



Tangential Component

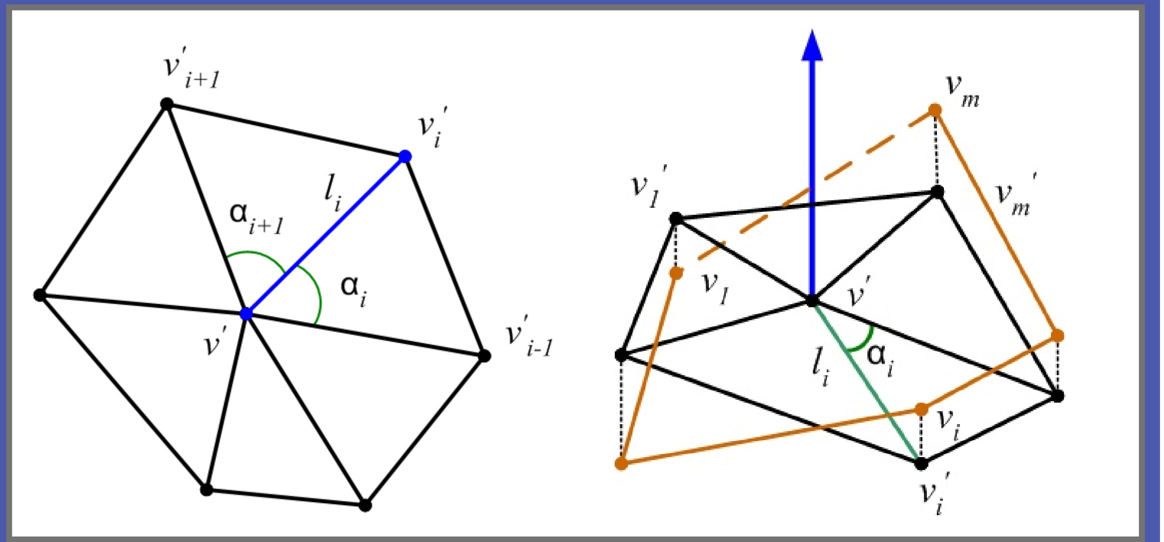
Given *α*, *l* and $v_1..v_m$ compute v':

1. project the $v_1..v_m$ to tangentional plane
2. compute v' using *scaled* mean value coordinates [Floater 03]
    i. Reconstruction
    ii. Shape preservation

$$w_i = \frac{\tan(\alpha_i / 2) + \tan(\alpha_{i+!} / 2)}{l_i}$$

$$w_i = \frac{w_i \, \| v' - v'_i \|}{l_i}$$

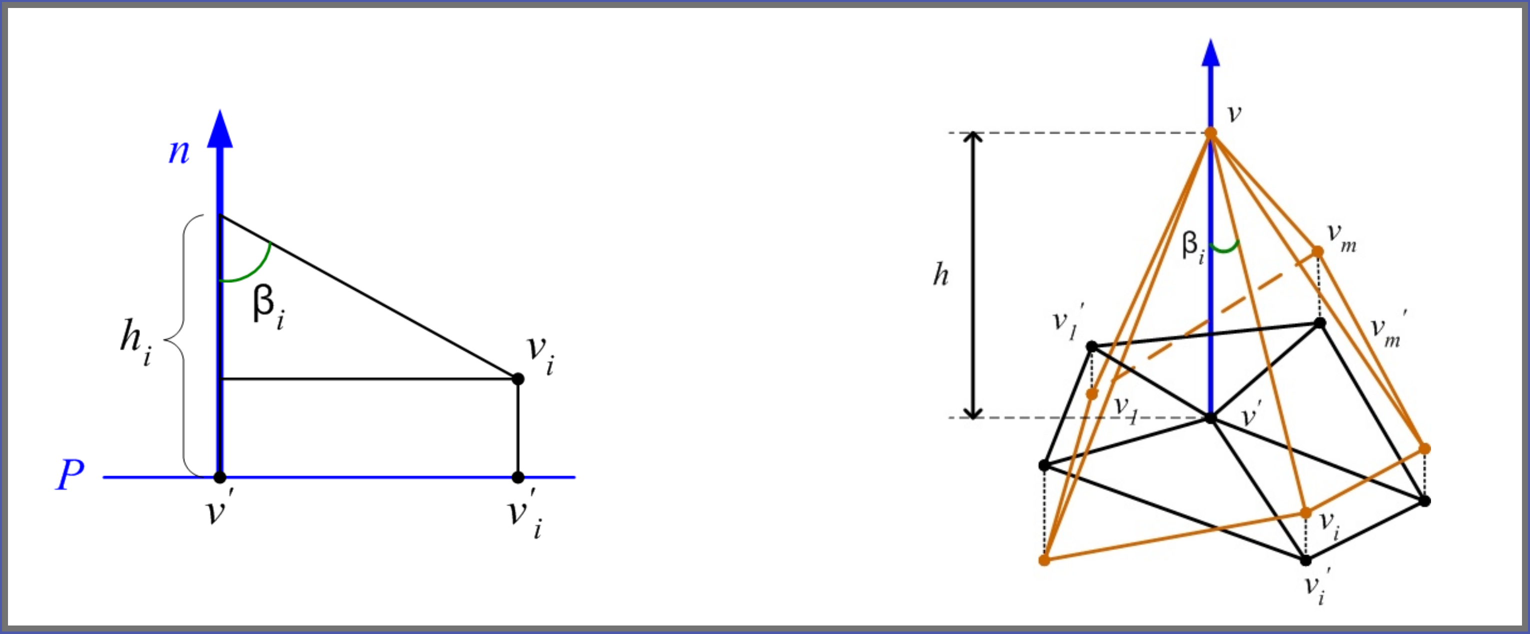
$$v' = \frac{1}{\sum_{i=1}^{m} w_i} \sum_{i=1}^{m} w_i v'_i$$



Normal Component

To compute v given v' and β:

1. calculate a set of offsets h_i along normal n
2. calculate h as an average of h_i
3. calculate v as an offset by h along n



Reconstruction Properties

1. Reconstruction

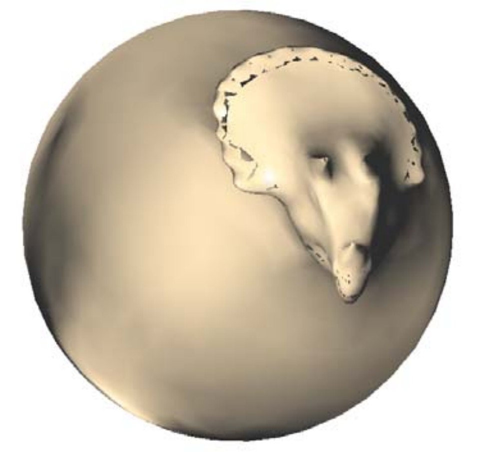
2. Invariance under
   - rigid transformations
   - global scaling

3. Shape preservation
   - local implies global

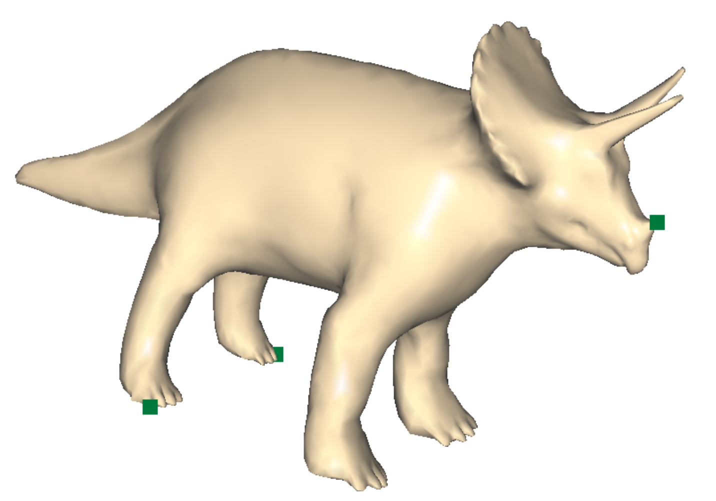
Reconstruction

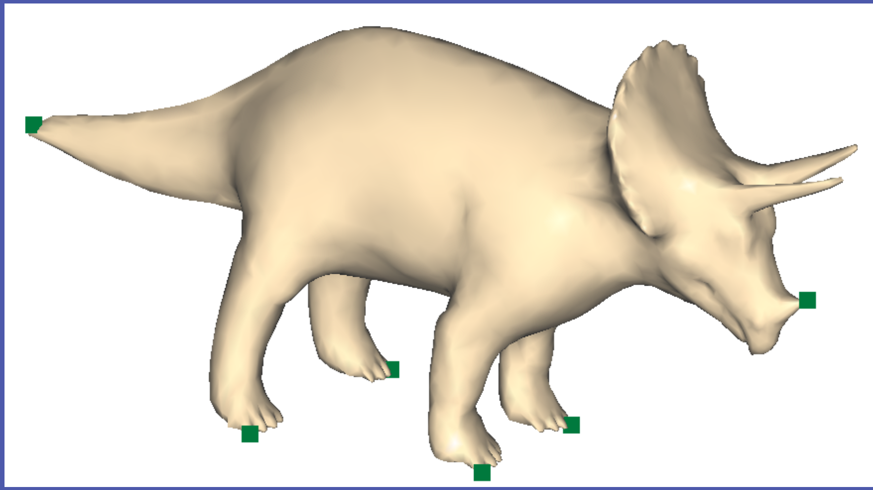
Input: Positions of control vertices V_c

1. For each unconstrained vertex
   a) Compute tangential plane
   b) Reconstruct the vertex



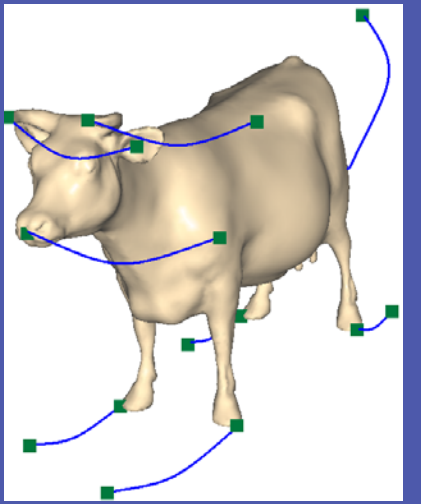
2. Repeat until convergence



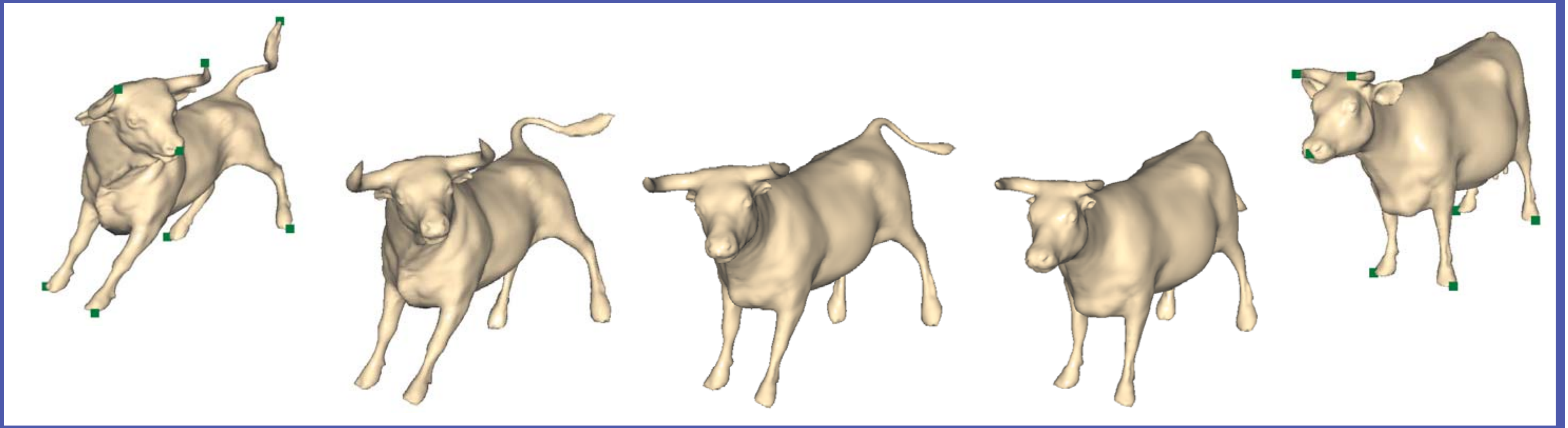


Morphing

- Input:
  - control vertex positions at each time step
  - pyramid coordinates of source & target

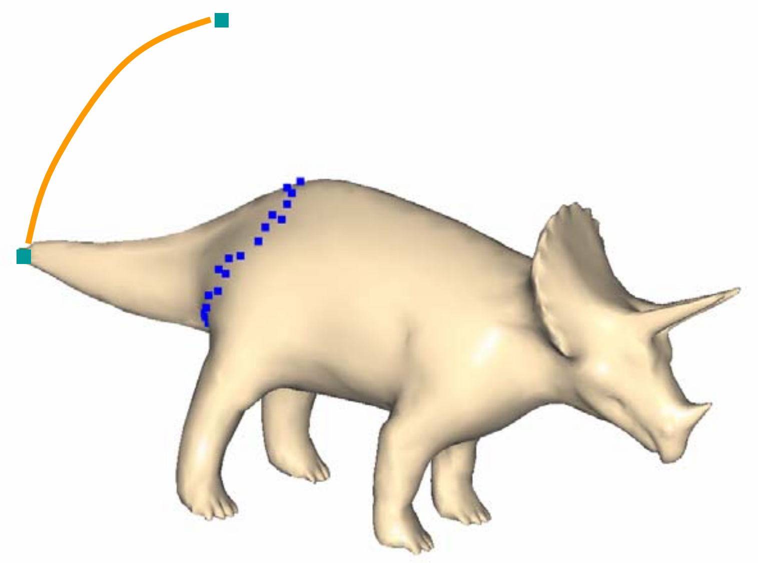


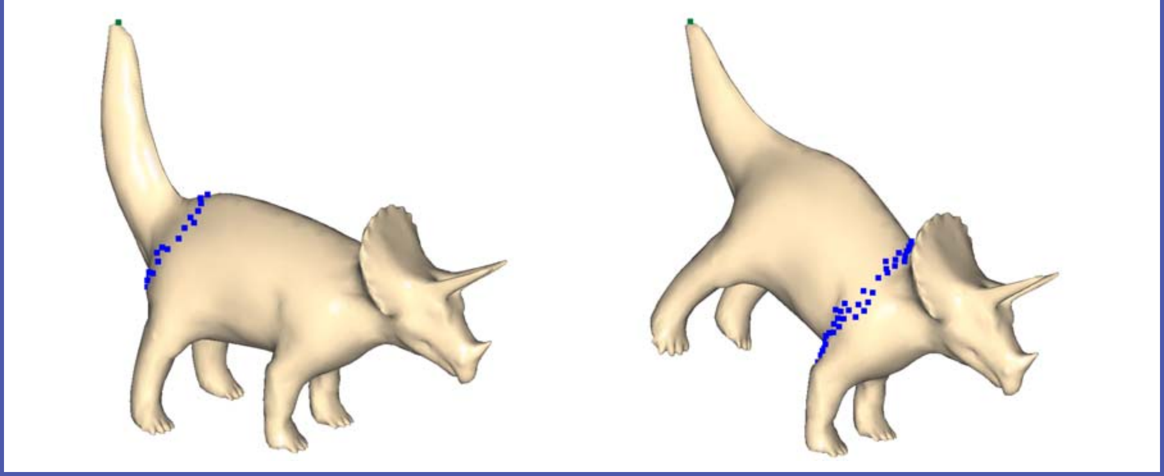
- For each time step
  1. Interpolate source & target pyramid coordinates
  2. Reconstruct using new coordinates



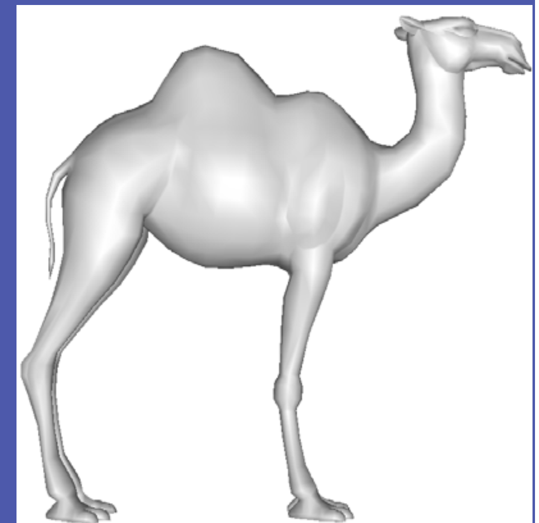
Deformation

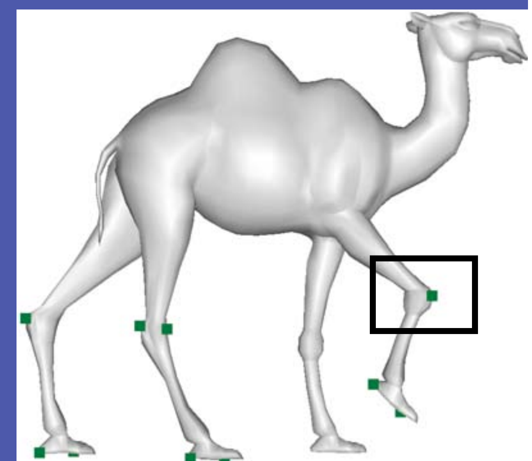
- Input:
  - Control vertices
  - Region of influence
- Impacts model shape!!!



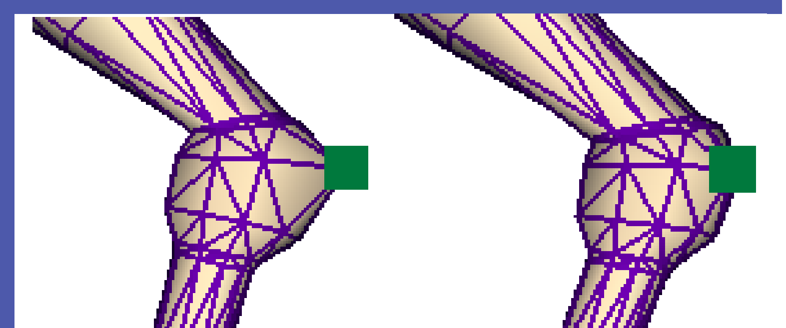


Deformation Algorithm

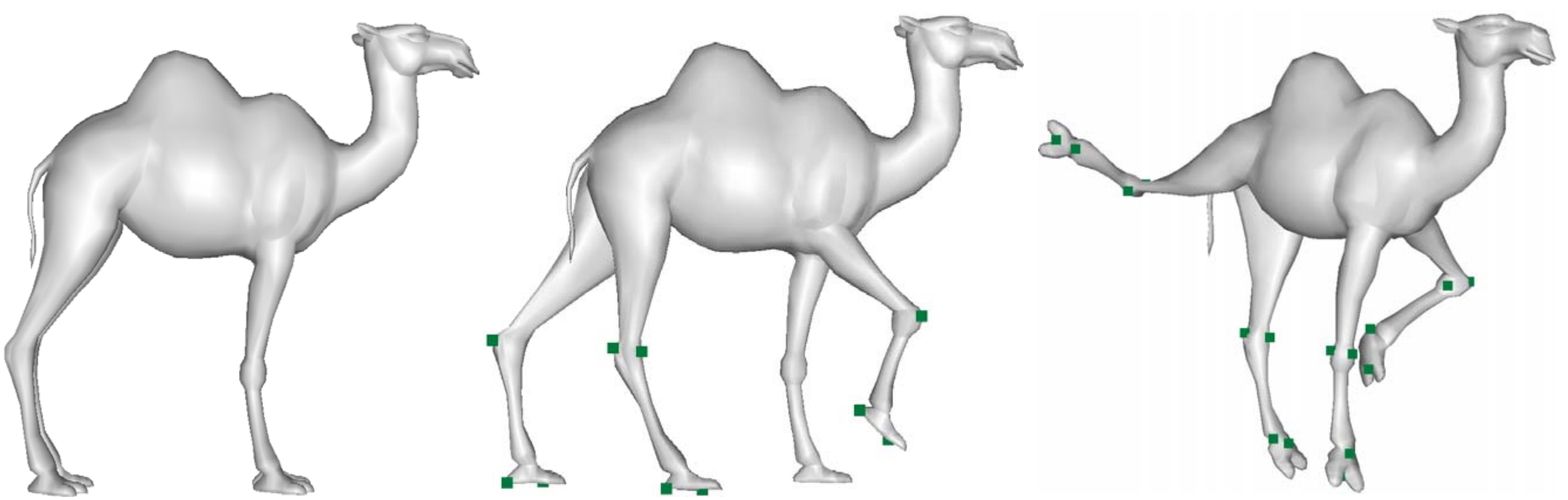




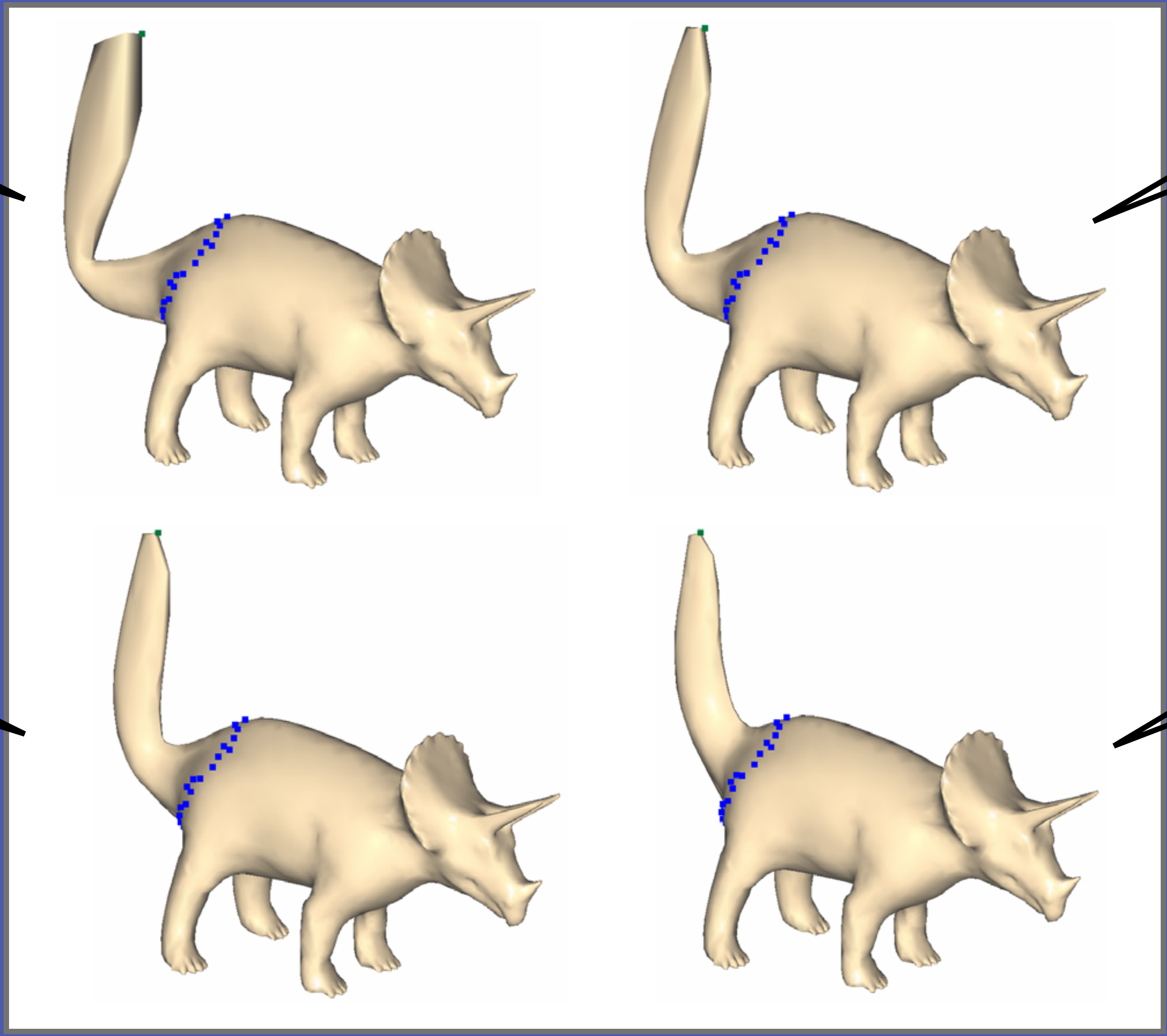
1. Compute influence function
   - 1 at controls & gradually reducing to 0 around them
2. For each unconstrained vertex in the region of influence
   a) Compute tangential plane
   b) Reconstruct the vertex
3. Repeat until convergence
4. Recompute pyramid coordinates as combination of old & new using influence function
5. Repeat from 2.



Results



Algorithm in progress



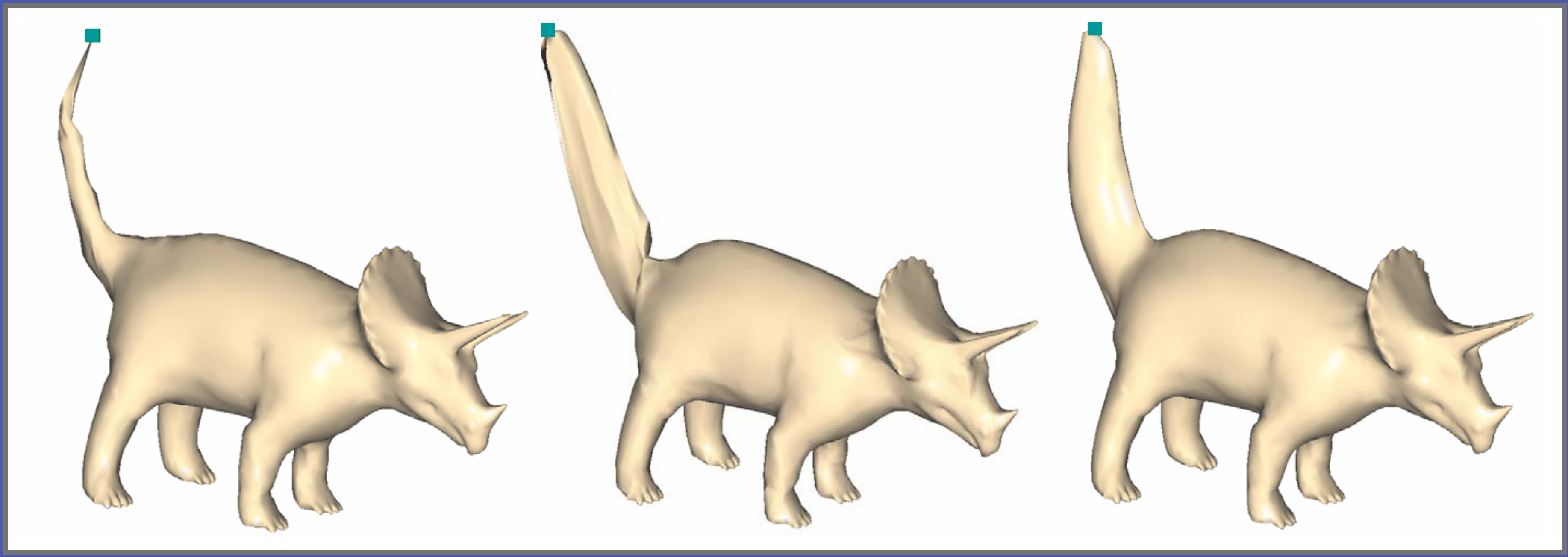
70 iterations

130 iterations

250 iterations

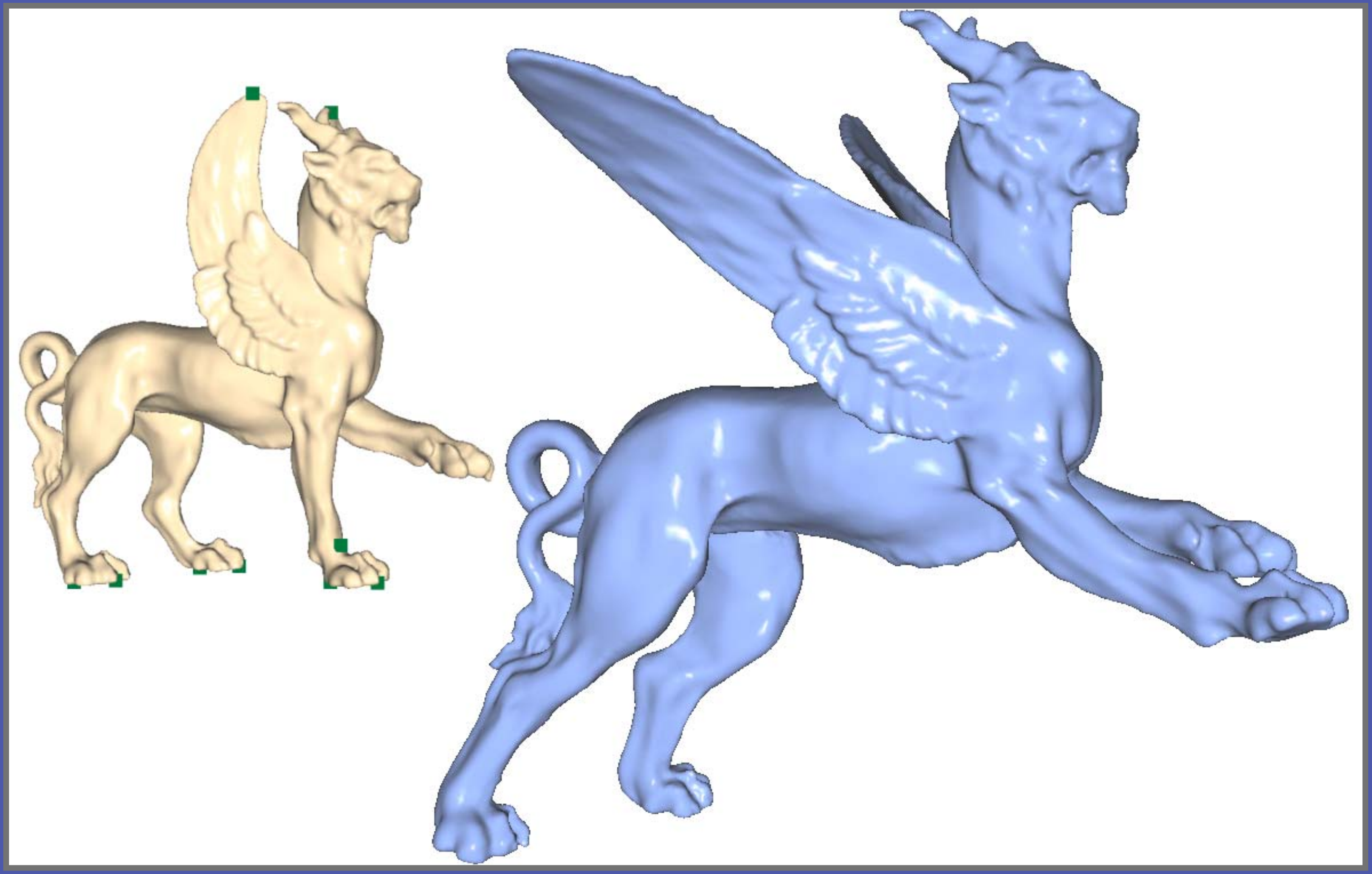
400 iterations

Comparison



[Alexa,01] [Lipman et al.,04] Our

Results – Detail Preservation



Summary

- Novel local coord representation
  - set of editing techniques

- Advantages
  - Local shape preservation
    - implies global preservation
  - Invariant under rigid transformations
    - not invariant under shearing
    - not invariant under non uniform scale

Summary

- Drawback
  - Computation of tangential plane can be unstable on badly shaped meshes
    - apply smoothing as pre-processing
- Future
  - Global formulation
  - Speedup