LIGHTING:

Q12. **Three or more visual effects that cannot be modelled when considering light as photons:**

- Diffraction
- Propagation media
- Scattering
- Polarization
- Refraction
- Interference
- Curved light paths
- Continuous wavelength
- Unquantized dispersion
- Unquantized fluorescence

Q13. **Ambient term in Phong illumination model**

**Rationale:** Simulating indirect illumination like in global illumination

**Limitations:** No highlights and hence one cannot make out the shape of the objects in the scene. E.g., sphere appears like a disc.

**Simple ‘Hack’** – No physical phenomena simulation!

Q14. **The Moon is poorly approximated by Phong or Diffuse shading (lighting)**

The Moon is poorly approximated by diffuse lighting as it does not exhibit the cosine fall-off in intensity (typical of Lambertian surfaces) of reflected light with changing surface normal.

The Moon is also not well approximated by Phong lighting as it does not exhibit a sharp specular highlight on its surface. However, the Moon is a fairly specular object reflecting the Sun light with enough intensity to act as a light source in the night sky!
Q.15. SHOW THAT IF THE VIEW VECTOR $V$ LIES IN THE SAME PLANE AS LIGHTING VECTOR $L$, SURFACE NORMAL $N$ AND THE REFLECTED VECTOR $R$, THEN HALFANGLE SATISFIES $2\psi = 0$.

\[ \vec{L}, \vec{N}, \vec{R} \text{ and } \vec{V} \text{ are coplanar:} \]

\[ \text{ANGLE BETWEEN } \vec{L} \& \vec{V} = \theta + \theta + \rho = \delta \]
\[ \text{ANGLE BETWEEN } \vec{L} \& \vec{N} = \theta + \psi \]

\[ \text{FROM DEFINITION OF HALFANGLE } \Rightarrow \theta + \psi = \delta/2 \]
\[ \Rightarrow \theta + \psi = \frac{2\theta + \rho}{2} \]
\[ \Rightarrow 2\theta + 2\psi = 2\theta + \rho \]
\[ \Rightarrow 2\psi = \rho \]

Q.16. IF THE ABOVE VECTORS ARE NOT ALL COPLANAR: COUNTER-EXAMPLE!!

\[ \psi = 0 \]
\[ \vec{L}, \vec{N}, \vec{V} \& \vec{R} \text{ are on the } XZ \text{ plane. } \]
\[ \vec{N} \text{ is orthogonal to the other vector } \vec{R} \]
6.17. Show halfway vector \( \hat{h} \) is the angle at which the surface must be oriented so that the maximum amount of light reaches the viewer.

Viewer receives maximum amount of light when \( \hat{v} = \hat{h} \), i.e., when the viewer is aligned in the mirror reflection direction.

\[ \therefore \text{if } \hat{v} = \hat{h}, \theta = 0 \] (from Q15, assuming coplanarity of vectors hold.

\[ \Rightarrow 2\psi = \theta = 0 \]

\[ \hat{h} = \hat{n} \] (i.e., halfway is aligned with surface normal.

Q18. Under flat shading model, points A, B & C will all have the same color (the color computed at A as specified by the question).

Ambient illumination \( A_{a} = I_{a} \times k_{a} = 0.8 \times 0.2 = 0.16 \)

Diffuse illumination \( A = k_{d} \times I_{l} \times (\hat{n}_{a}, \hat{l}_{a}) \)

\[ \hat{l}_{a} = (0, 2, 0) - (2, 0, 0) = (-2, 2, 0) \] (unnormalized)

\[ \hat{l}_{a} = (-\frac{2}{\sqrt{8}}, \frac{2}{\sqrt{8}}, 0) = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0) \] (normalized)

\[ \overrightarrow{\hat{n}} = \overrightarrow{n}_{a} = \hat{l} \]

\[ \therefore \text{Diffuse illumination} = k_{d} \times I_{l} \times (\overrightarrow{n}_{a}, \hat{l}_{a}) = k_{d} \times I_{l} \times \hat{l} \]

\[ = 0.9 \times 1.0 = 0.9 \]
SPECULAR ILLUMINATION = $k_a \times \mathbf{I}_L \times (\hat{\mathbf{r}}_a \cdot \hat{\mathbf{l}}_a)^n$

$\vec{r}_a = 2(\hat{\mathbf{r}}_a \cdot \hat{\mathbf{l}}_a) \hat{\mathbf{n}}_a - \hat{\mathbf{l}}_a = 2 \times 1 \times \hat{\mathbf{n}}_a - \hat{\mathbf{l}}_a$

$= 2 \hat{\mathbf{n}}_a - \hat{\mathbf{l}}_a$

$= \hat{\mathbf{r}}_a$

$= \vec{r}_a$

$\vec{v} = (6, 2, 0) - (2, 0, 0) = (4, 2, 0)$ (UNNORMALIZED)

$\vec{v} = \left(\frac{4}{\sqrt{20}}, \frac{2}{\sqrt{20}}, 0\right) = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$ (NORMALIZED)

$\therefore \vec{r}_a \cdot \vec{v} = -\frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{5}} + 0 = -0.816227 \leq 0$ (CLAMPED)

$\therefore$ SPECULAR ILLUMINATION = 0

$\therefore$ COMBINED TOTAL ILLUMINATION = AMBIENT + DIFFUSE + SPECULAR

$\mathbf{I}_a = 0.16 + 0.9 + 0 = 1.06$

IN PRACTICE, ILLUMINATION RANGES FROM [0, 1] CORRESPONDING TO [0, 255] BRIGHTNESS VALUES ON SCREEN. HENCE VALUES GREATER THAN 1.0 ARE CLAMPED TO 1.0!
For Gouraud shading model, we calculate the illumination intensity at each vertex (in this case A & C) and interpolate for in-between points (in this case B).

Illumination intensity at A \( I_A \) = 1.06 (from \( \theta \))

Illumination intensity at C \( I_C \) = \( I_a \times k_a + k_d \times I_L (\vec{N}_c \cdot \vec{l})^n \)

\[ \vec{N}_c = (0, 2, 0) - (4, 0, 0) = (-4, 2, 0) \text{ (unnormalized)} \]
\[ = \left( -\frac{4}{\sqrt{20}}, \frac{2}{\sqrt{20}}, 0 \right) \text{ (normalized)} = \left( -\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right) \]

\[ \vec{l}_c = (6, 2, 0) - (4, 0, 0) = (2, 2, 0) \text{ (unnormalized)} \]
\[ = \left( \frac{2}{\sqrt{8}}, \frac{2}{\sqrt{8}}, 0 \right) = \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right) \text{ (normalized)} \]

\[ \vec{N}_c \cdot \vec{l}_c = -\frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{2}} + 0 = -0.316227 \]

\[ = 0 \text{ (clamped)} \]

\( \therefore \text{ diffuse illumination} \ C = 0 \)
SPECULAR ILLUMINATION \( c = k_o \times I_L \times (\hat{\mathbf{r}}_c \cdot \hat{\mathbf{v}}_c) \)

\[
\hat{\mathbf{r}}_c = 2(\hat{\mathbf{r}}_c \cdot \hat{\mathbf{v}}_c)\hat{\mathbf{r}}_c - \hat{\mathbf{v}}_c = 2x0 \times \hat{\mathbf{r}}_c - \hat{\mathbf{v}}_c = \hat{\mathbf{r}}_c
\]

\[
\hat{\mathbf{v}}_c = \hat{\mathbf{r}}_c
\]

\[
\therefore \hat{\mathbf{r}}_c \cdot \hat{\mathbf{v}}_c = \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} + \frac{-1}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = 0.316227
\]

\[
\therefore \text{SPECULAR ILLUMINATION}_c = 0.5 \times 1 \times (0.316) = 5 \times 10^{-16}
\]

\[
\therefore \text{COMBINED TOTAL ILLUMINATION}_c = I_c = \text{AMBIENT} + \text{SPECULAR}
\]

\[
= 0.16 + 0 + 5 \times 10^{-16}
\]

\[
\approx 0.16
\]

ILLUMINATION AT B \( \bar{I}_B = \bar{I}_A + \bar{I}_C \) \( \text{since B BETWEEN} \)

\[
= \frac{1.06 + 0.16}{2} = 0.61
\]
For the Phong shading model, we interpolate the normal at any point on the surface where we wish to apply the illumination. Thereafter we apply the Phong illumination equation to the interpolated normal.

\[
\vec{n}_b = \frac{\vec{n}_a + \vec{n}_c}{2}
\]

\[
\begin{align*}
\vec{n}_b &= \frac{(-2,2,0)+(2,2,0)}{2} \\
&= \left(0,4,0\right) \\
&= \left(0,2,0\right) \text{ (unnormalized)}
\end{align*}
\]

\[
\vec{n}_b = \left(0,1,0\right) \text{ (normalized)}
\]

\[
\vec{\ell}_b = \left(0,2,0\right) - \left(3,0,0\right) = \left(-3,2,0\right) \text{ (unnormalized)}
\]

\[
\vec{\ell}_b = \left(-\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}, 0\right) \text{ (normalized)}
\]

Diffuse illumination \(b\):

\[
\begin{align*}
\vec{I}_b &= \vec{l}_d \times \vec{I}_L \times (\vec{n}_b \cdot \vec{\ell}_b) \\
&= 0.9 \times 1.0 \times \left(\frac{2}{\sqrt{13}}\right) \\
&= 0.499
\end{align*}
\]

Ambient illumination \(b\):

\[
\vec{I}_a = 0.8 \times 0.2 = 0.16
\]
SPECULAR ILLUMINATION \( B = k_d \times I_L \times \left( \vec{I}_B \cdot \vec{V}_B \right)^n \)

\( \vec{V}_B = (6,2,0) - (3,0,0) = (3,2,0) \) (UNNORMALIZED)

\[ = \left( \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}, 0 \right) \] (NORMALIZED)

\( \vec{I}_B = 2 (\vec{N}_B \cdot \vec{I}_B) \vec{N}_B - \vec{I}_B \)

\[ = 2 \left( \frac{2}{\sqrt{13}} \right) (0,1,0) - \left( -\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}, 0 \right) \)

\[ = \left( \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}, 0 \right) \)

\[ \therefore \vec{N}_B \cdot \vec{V}_B = 1 \]

\[ \therefore \text{SPECULAR ILLUMINATION}_B = k_d \times I_L \times 1 \]

\[ = 0.5 \times 1.0 \times 1 \]

\[ = 0.5 \]

\[ \therefore \text{COMBINED TOTAL ILLUMINATION}_B = \vec{I}_B = \text{AMBIENT} + \text{DIFFUSE} + \text{SPECULAR} \]

\[ = 0.16 + 0.499 + 0.5 \]

\[ = 1.159 \]

COMPARSED TO GOURAUD SHADING'S COMPUTED INTENSITY OF 0.61 FOR \( B \), PHONG SHADING RESULTS IN A MUCH BRIGHTER SPECULAR HIGHLIGHT AT \( B \) (OF 1.159 INTENSITY)!