University of British Columbia

CPSC 414 Computer Graphics

Visibility

Week 9, Fri 31 Oct 2003

News

- extra office hours
  - Thu 5:30-6:30
  - Friday 11-1:30, 4:30-5:30
  - Mon 10:30-12:30, 1-3
  - (normal lab hours: Thu 12-1, Fri 10-11)
- don’t use graphics remotely!
  - or else console person can’t use graphics
  - reboot if you have this problem
- this week’s labs:
  - picking, texturing details

University of British Columbia

CPSC 414 Computer Graphics

Rotation Methods recap

Representing 3 Rotational DOFs

- 3x3 Matrix (9 DOFs)
  - Rows of matrix define orthogonal axes
- Euler Angles (3 DOFs)
  - Rot x + Rot y + Rot z
- Axis-angle (4 DOFs)
  - Axis of rotation + Rotation amount
- Quaternion (4 DOFs)
  - 4 dimensional complex numbers

Rotation Matrices Won't Interpolate

- interpolate linearly from +90 to -90 in y

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

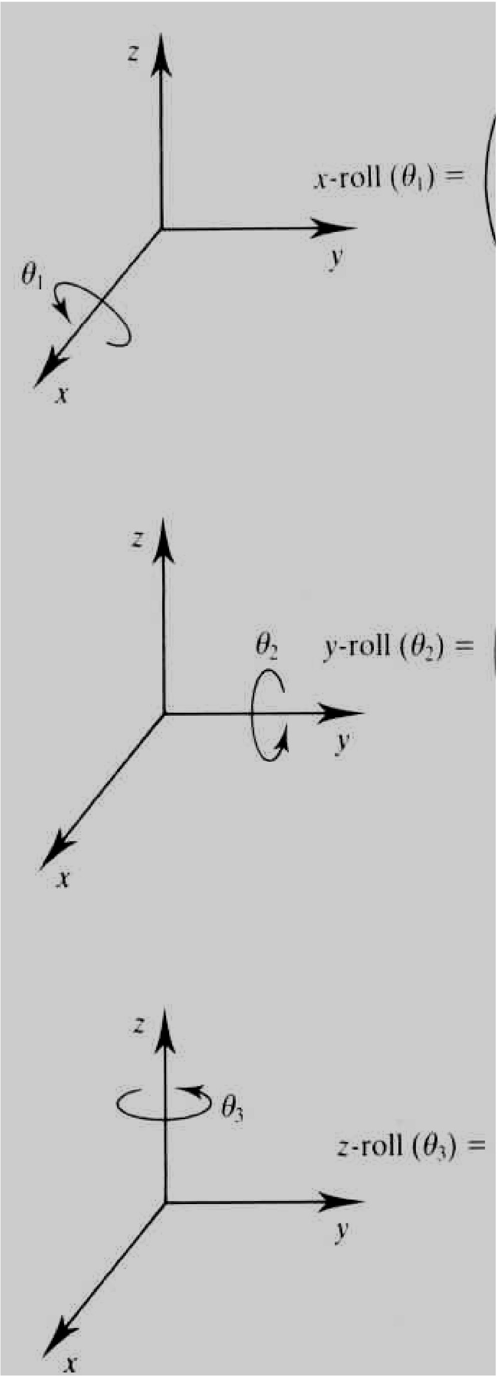
- halfway through component interpolation

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

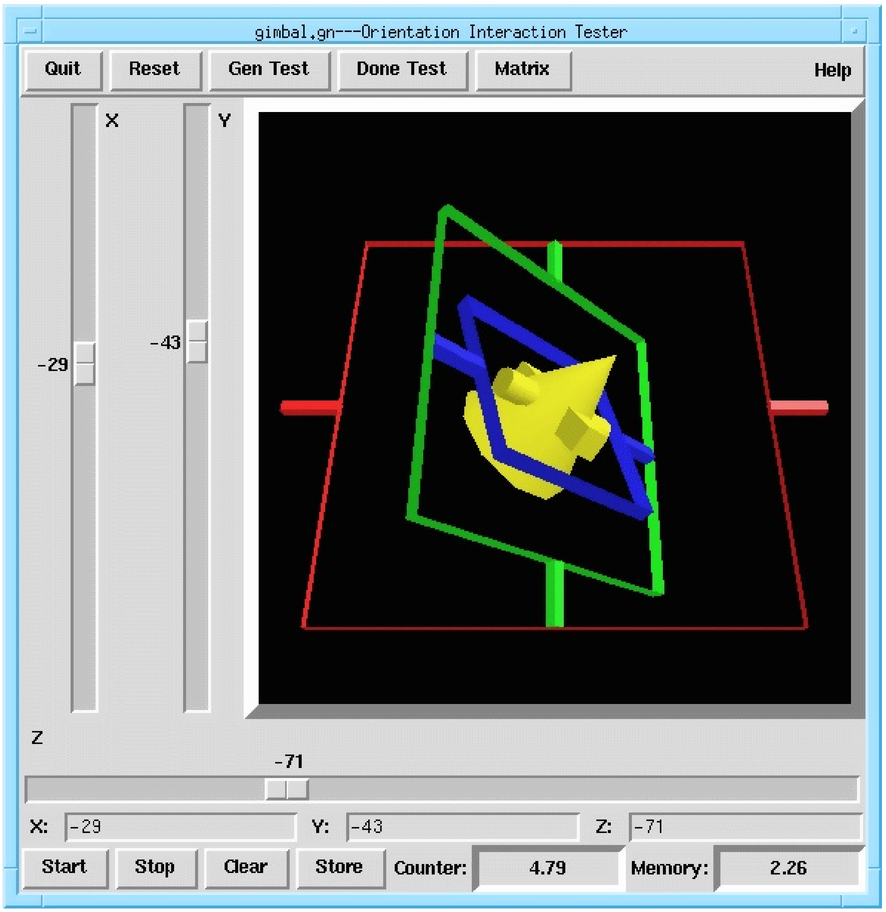
  – problem 1: not a rotation matrix anymore!
    - not orthonormal, x flattened out

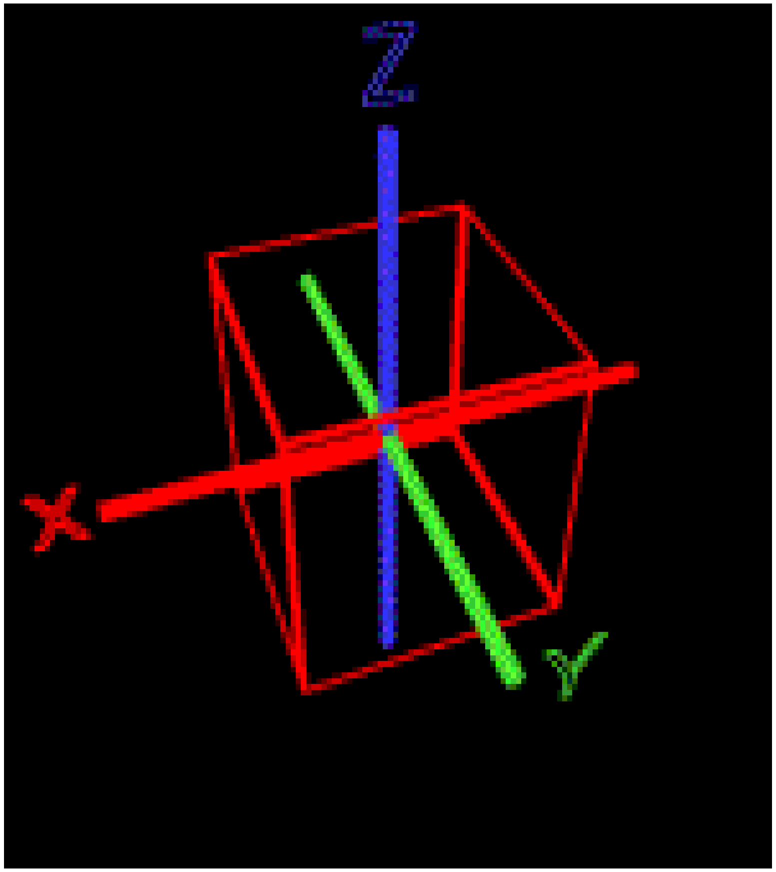
Euler Angles Have Gimbal Lock

- keep rotation angle for each axis
- problem 2: gimbal lock
  - occurs when two axes are aligned
- second and third rotations have effect of transforming earlier rotations
  - if Rot y = 90 degrees, Rot z == -Rot x



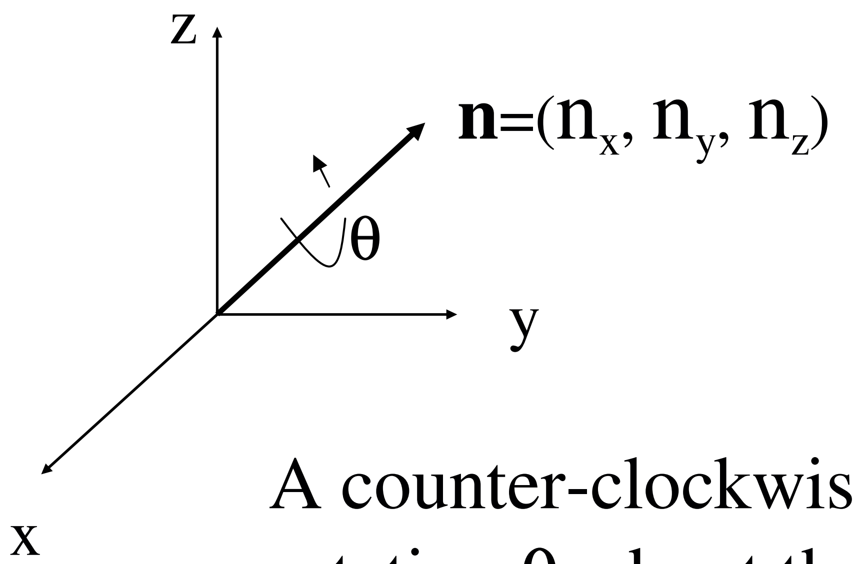
Gimbal Lock





http://www.anticz.com/eularqua.htm

Axis-angle Won't Concatenate



A counter-clockwise (right-handed) rotation θ about the axis specified by the unit vector $\mathbf{n}=(n_x, n_y, n_z)$

- problem 3
  – no easy way to determine how to concatenate

Quaternions

- quaternion is a 4-D unit vector q = [x y z w]
  - lies on the unit hypersphere $x^2 + y^2 + z^2 + w^2 = 1$
- for rotation about (unit) axis v by angle θ
  - vector part = (sin θ/2) *v* = *[x y z]*
  - scalar part = (cos θ/2) = *w*
- rotation matrix $$\begin{bmatrix} 1-2y^2-2z^2 & 2xy+2wz & 2xz-2wy \\ 2xy-2wz & 1-2x^2-2z^2 & 2yz+2wx \\ 2xz+2wy & 2yz-2wx & 1-2x^2-2y^2 \end{bmatrix}$$
- quaternion multiplication $q_1 * q_2 =$

$[\mathbf{v_1}, w_1] * [\mathbf{v_2}, w_2] = [(w_1 v_2 + w_2 v_1 + (v_1 \times v_2)), w_1 w_2 - v_1 . v_2]$

Rotation Methods Summary

- 3x3 matrices
  - good: simple. bad: drifting, can’t interpolate
- Euler angles
  - good: can interpolate, no drift
  - bad: gimbal lock
- axis-angle
  - good: no gimbal lock, can interpolate
  - bad: can’t concatenate
- quaternions
  - good: solve all problems. bad: complex

University of British Columbia

CPSC 414 Computer Graphics

Visibility

Rendering Pipeline

- modeling transformations
- viewing transformations
- projection transformations
- clipping
- scan conversion
- lighting
- shading

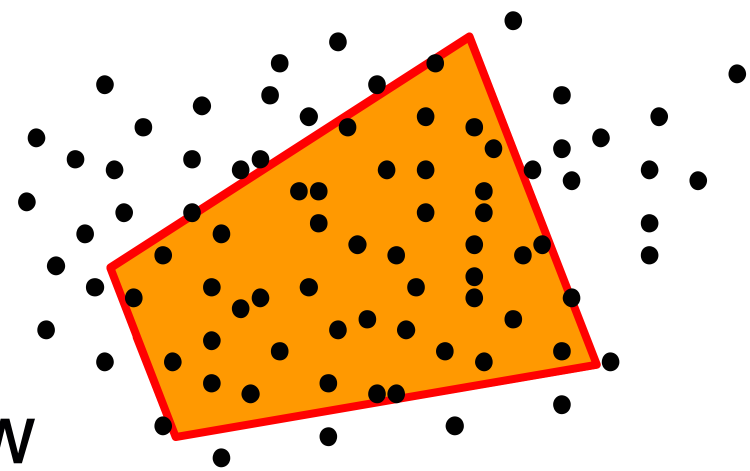
- we now know everything about how to draw a polygon on the screen, except *visible surface determination*

Invisible Primitives

- *why might a polygon be invisible?*
  - polygon outside the *field of view / frustum*
  - polygon is *backfacing*
  - polygon is *occluded* by object(s) nearer the viewpoint
- for efficiency reasons, we want to avoid spending work on polygons outside field of view or backfacing
- for efficiency and correctness reasons, we need to know when polygons are occluded

View Frustum Clipping

- remove polygons entirely outside frustum
  - note that this includes polygons “behind” eye (actually behind near plane)
- pass through polygons entirely inside frustum
- modify remaining polygons to include only portions intersecting view frustum

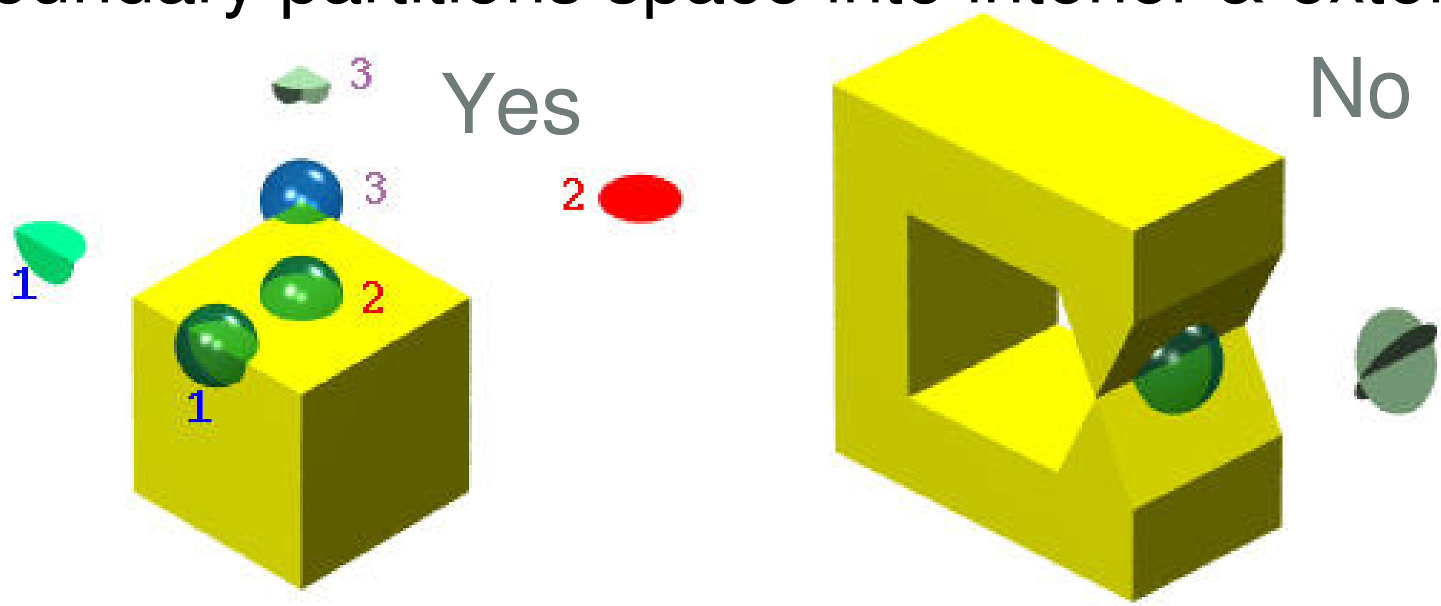


Back-Face Culling

- most objects in scene are typically "solid"
- rigorously: orientable closed manifolds
  - orientable: must have two distinct sides
    - cannot self-intersect
    - a sphere is orientable since has two sides, 'inside' and 'outside'.
    - a Mobius strip or a Klein bottle is not orientable
  - closed: cannot "walk" from one side to the other
    - sphere is closed manifold
    - plane is not

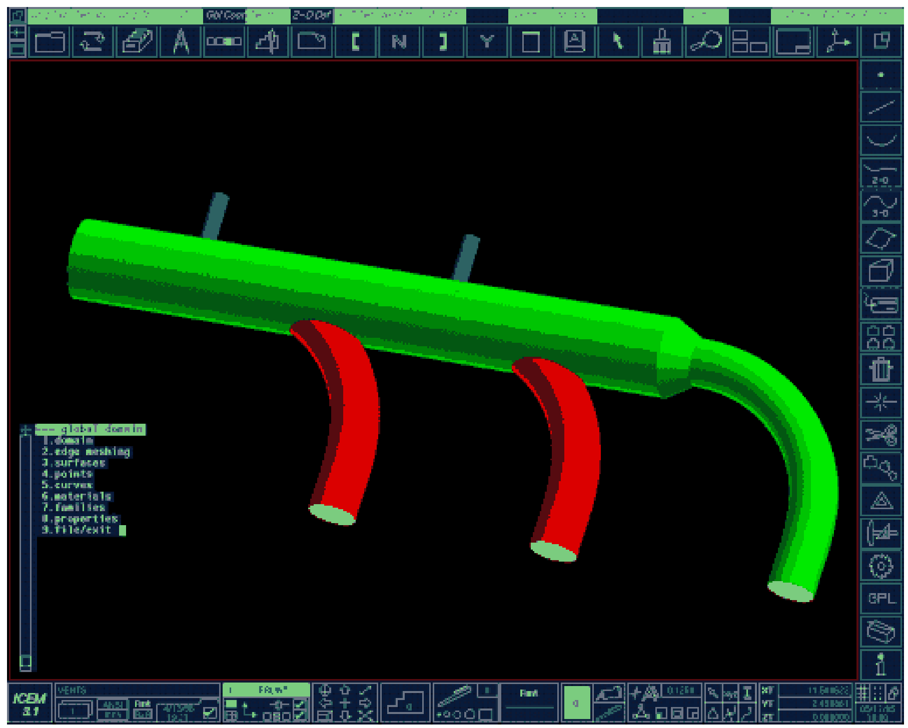
Back-Face Culling

- most objects in scene are typically “solid”
- rigorously: orientable closed manifolds
  - manifold: local neighborhood of all points isomorphic to disc
  - boundary partitions space into interior & exterior



Manifold

- examples of *manifold* objects:
  - sphere
  - torus
  - well-formed CAD part

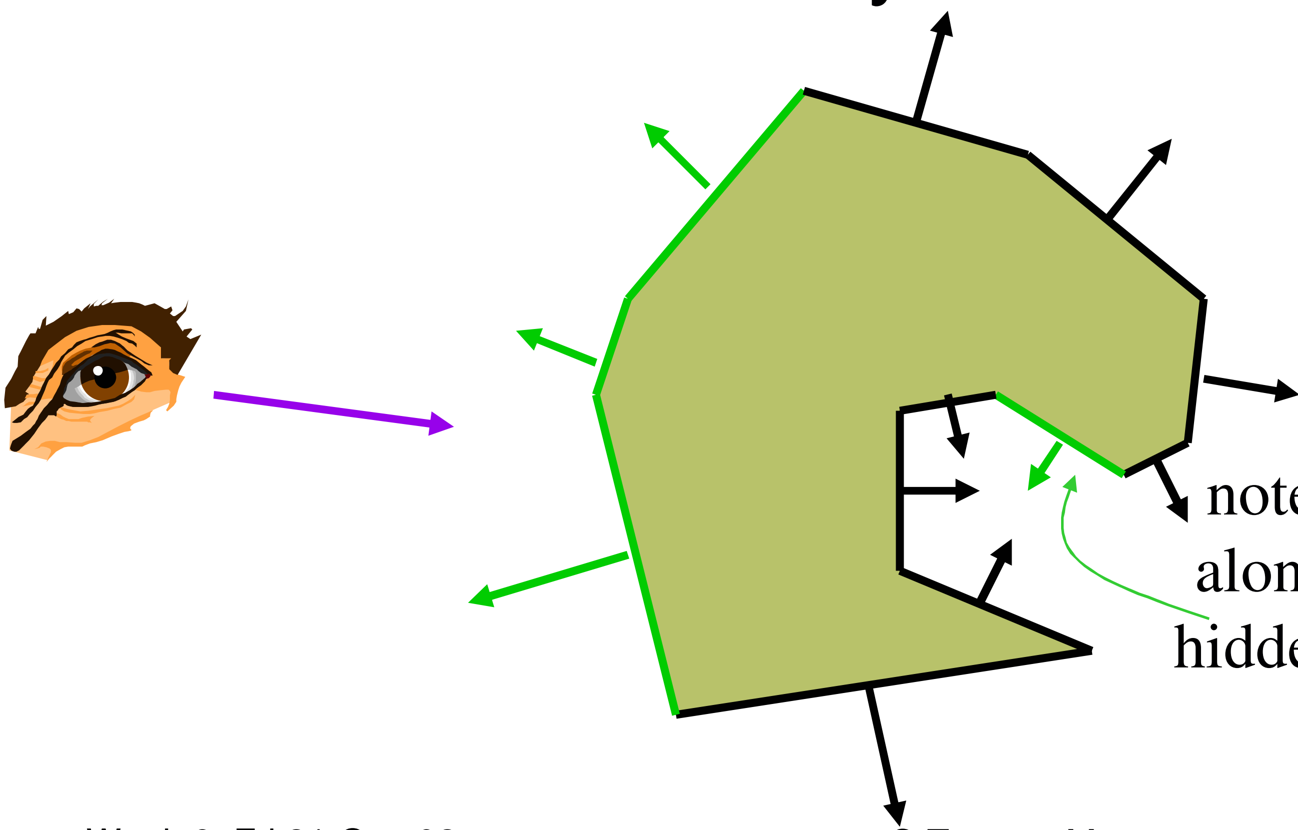


Back-Face Culling

- examples of non-manifold objects:
  - a single polygon
  - a terrain or height field
  - polyhedron w/ missing face
  - anything with cracks or holes in boundary
  - one-polygon thick lampshade

Back-Face Culling

- on the surface of a closed manifold, polygons whose normals point away from the camera are always occluded:

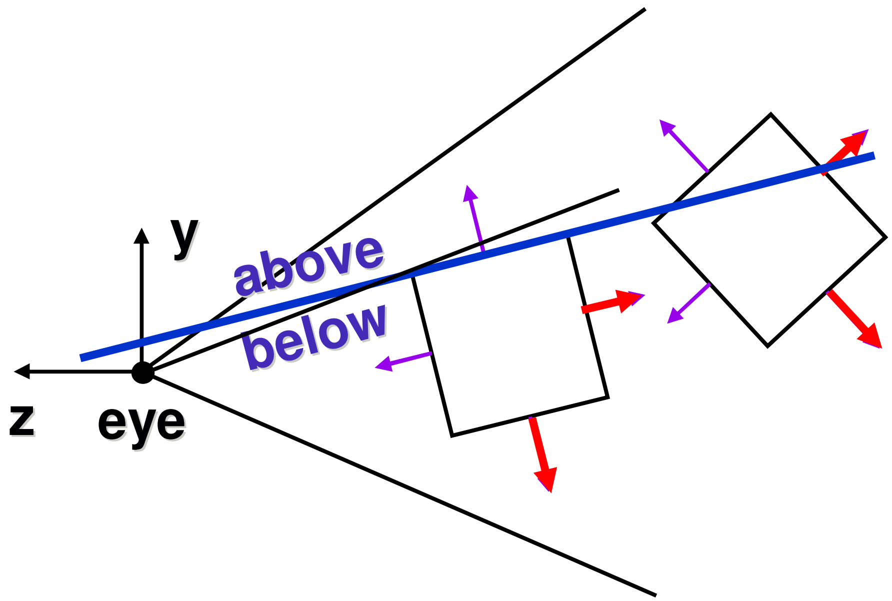


note: backface culling alone doesn’t solve the hidden-surface problem!

Back-Face Culling

- not rendering backfacing polygons improves performance
  - by how much?
    - reduces by about half the number of polygons to be considered for each pixel

Back-face Culling: VCS



first idea:

cull if $N_Z < 0$

works, but sometimes misses polygons that should be culled

better idea:

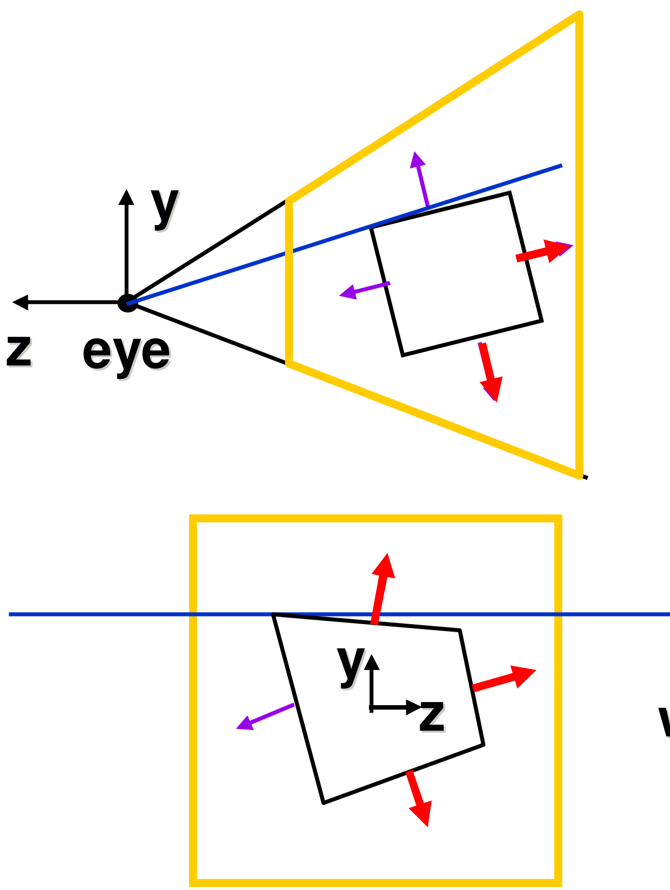
cull if eye is below polygon plane

Back-face Culling: NDCS

VCS

NDCS

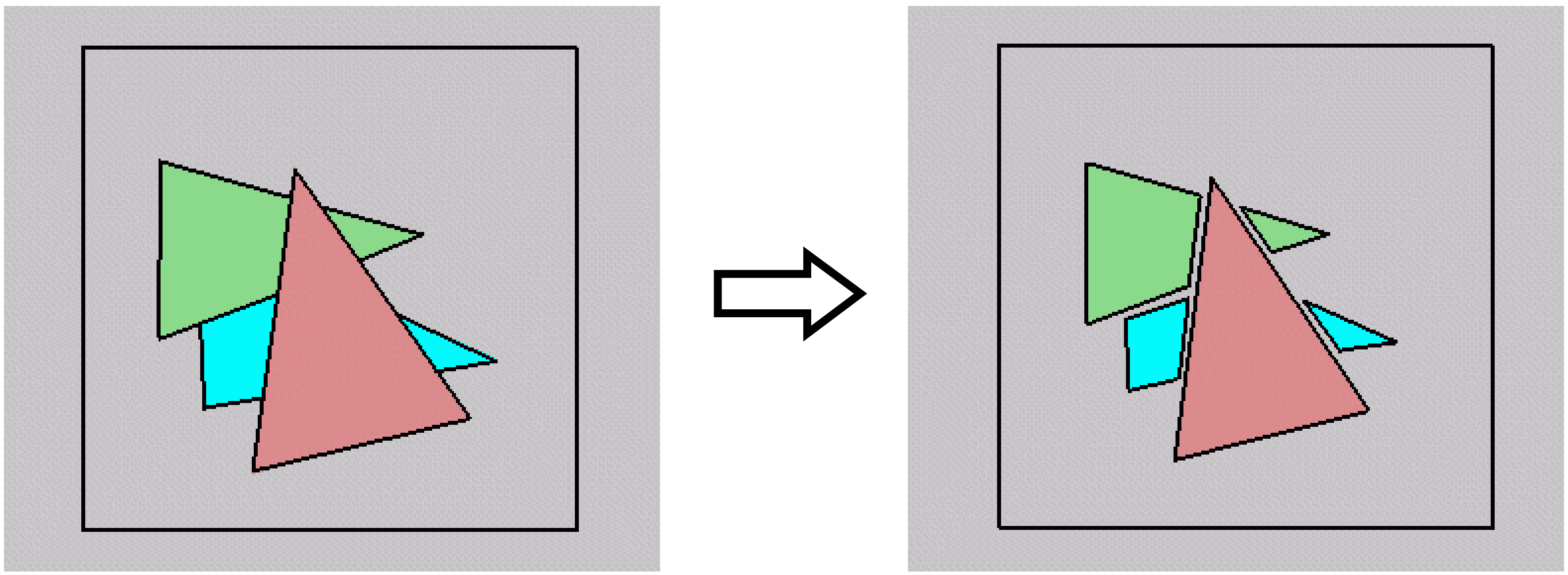
eye



works to cull if $N_Z > 0$

Occlusion

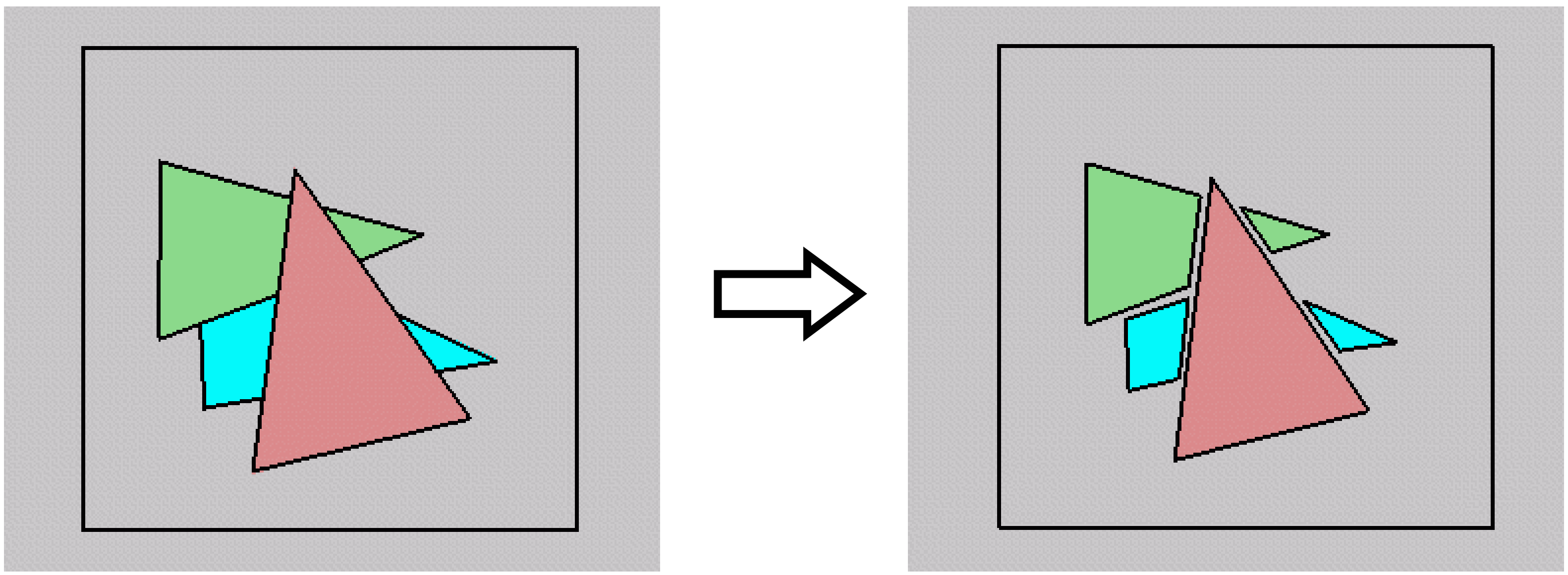
- for most interesting scenes, some polygons overlap



- to render the correct image, we need to determine which polygons occlude which

Painter's Algorithm

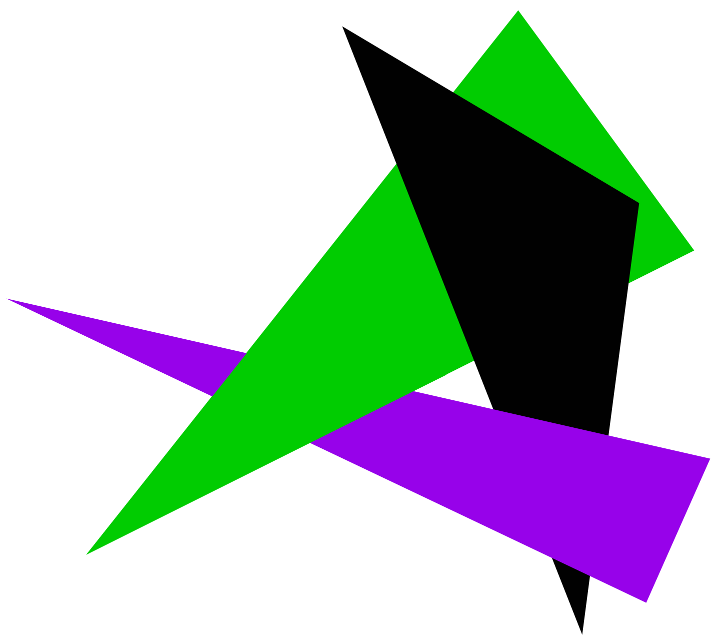
- simple: render the polygons from back to front, "painting over" previous polygons



  - draw blue, then green, then orange
- will this work in the general case?

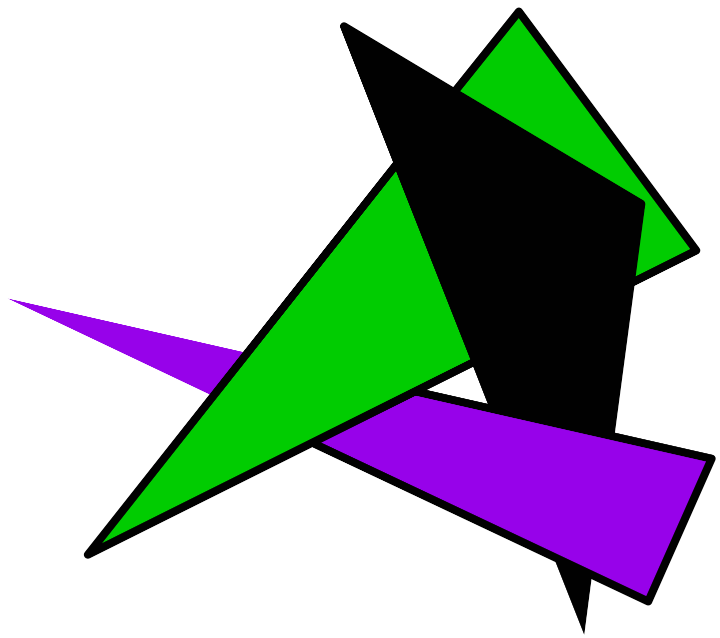
Painter's Algorithm: Problems

- *intersecting polygons* present a problem
- even non-intersecting polygons can form a cycle with no valid visibility order:



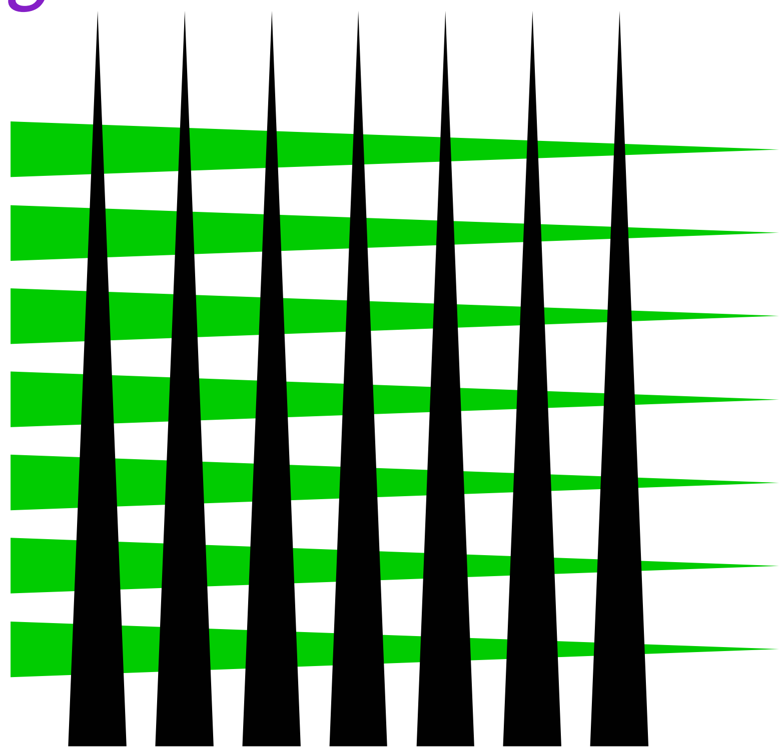
Analytic Visibility Algorithms

- early visibility algorithms computed the set of visible polygon *fragments* directly, then rendered the fragments to a display:



Analytic Visibility Algorithms

- *what is the minimum worst-case cost of computing the fragments for a scene composed of* n *polygons?*
- answer:
  $O(n^2)$



Analytic Visibility Algorithms

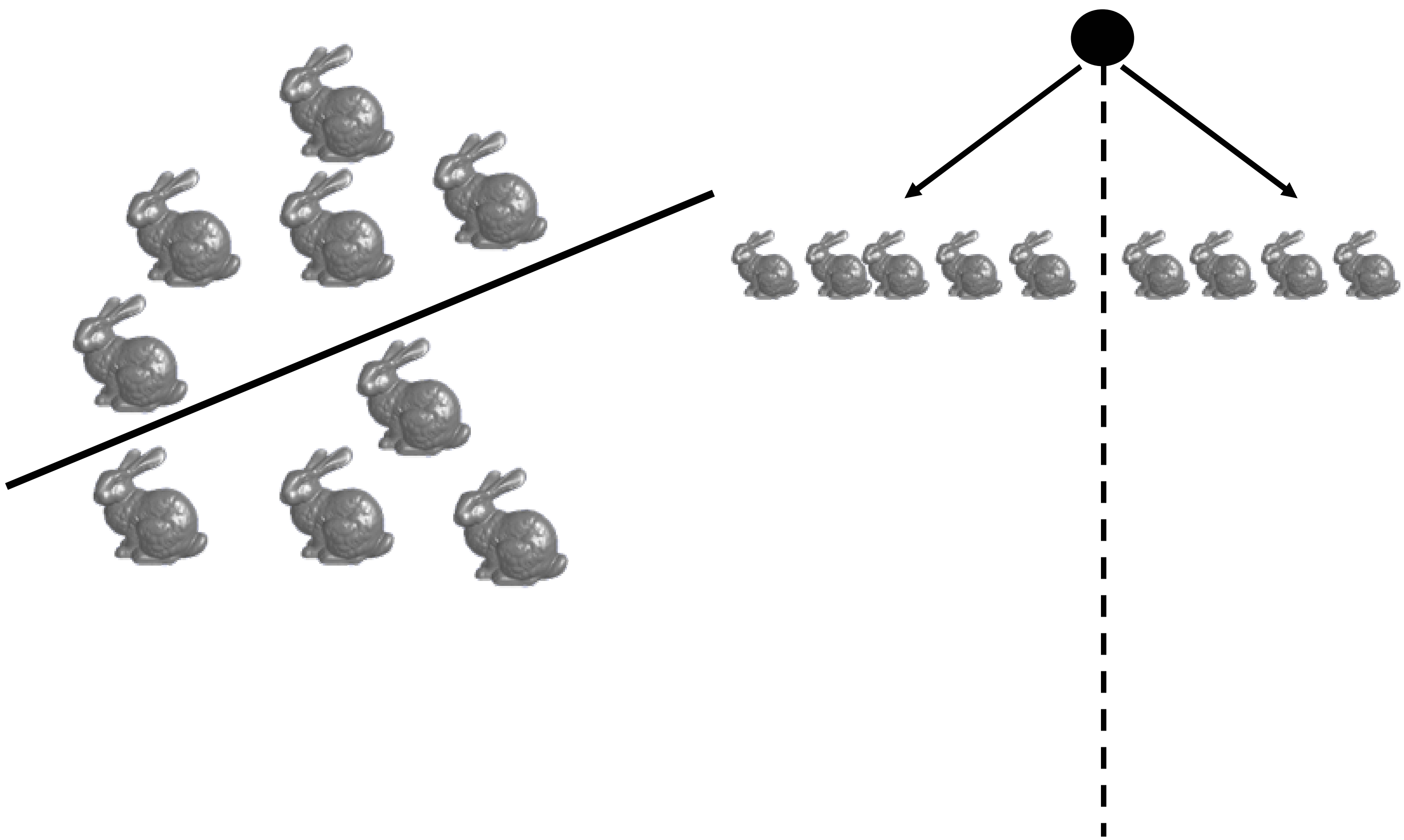
- so, for about a decade (late 60s to late 70s) there was intense interest in finding efficient algorithms for hidden surface removal
- we'll talk about two:
  - *Binary Space-Partition (BSP) Trees*
  - *Warnock's Algorithm*

Binary Space Partition Trees (1979)

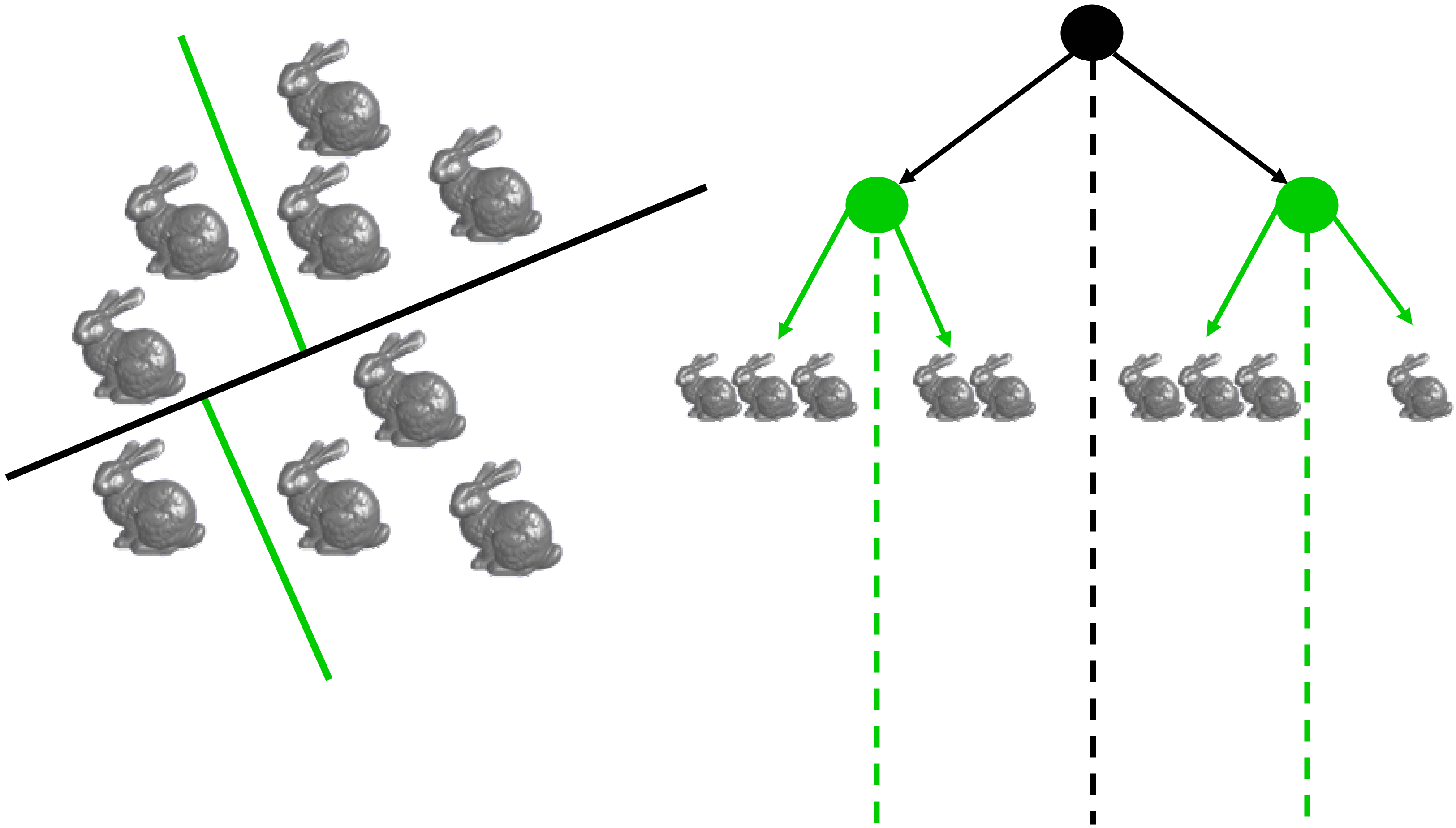
- BSP tree: organize all of space (hence partition) into a binary tree
  - *preprocess*: overlay a binary tree on objects in the scene
  - *runtime*: correctly traversing this tree enumerates objects from back to front
  - idea: divide space recursively into half-spaces by choosing *splitting planes*
    - splitting planes can be arbitrarily oriented

BSP Trees: Objects

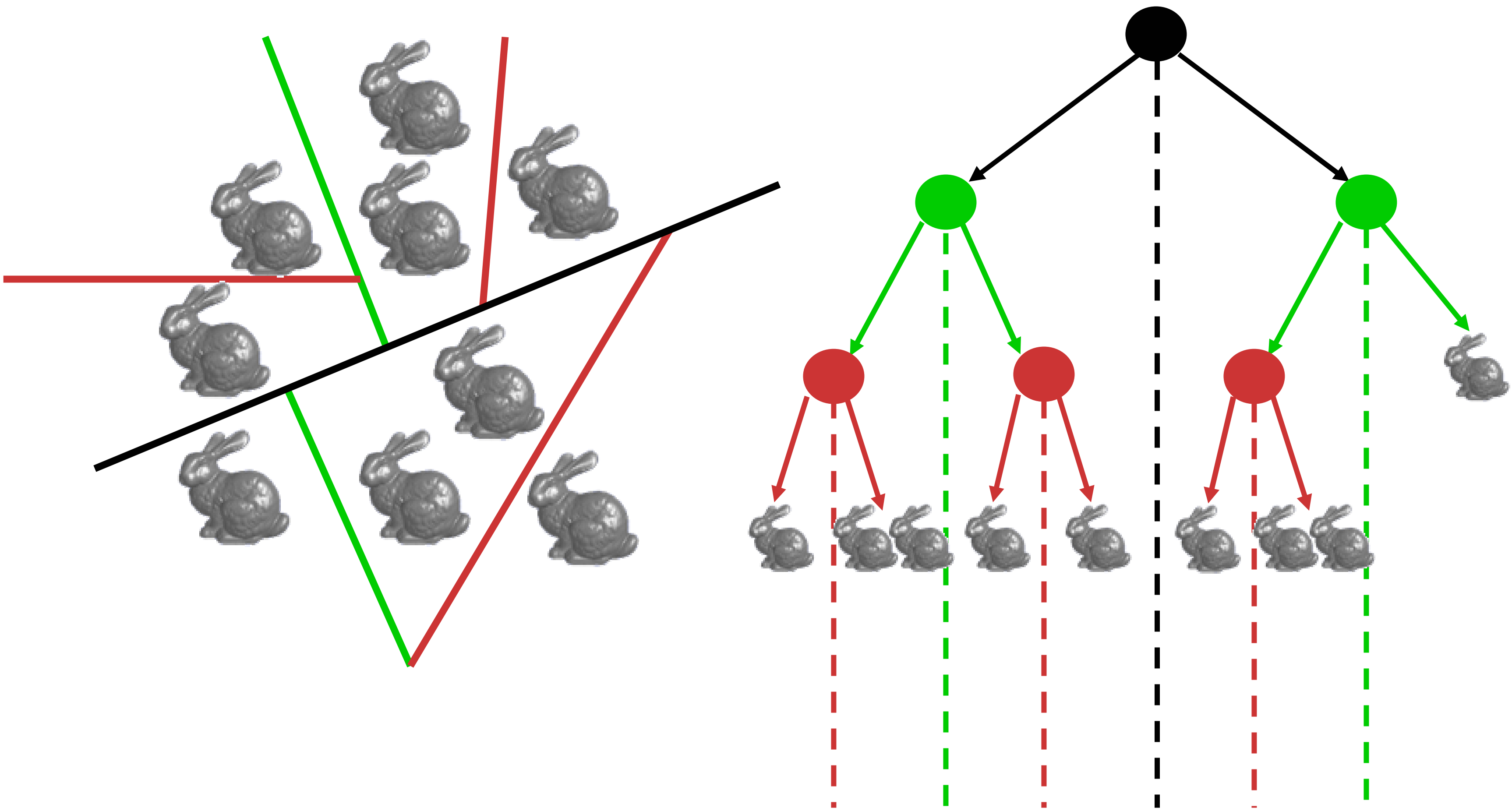
BSP Trees: Objects



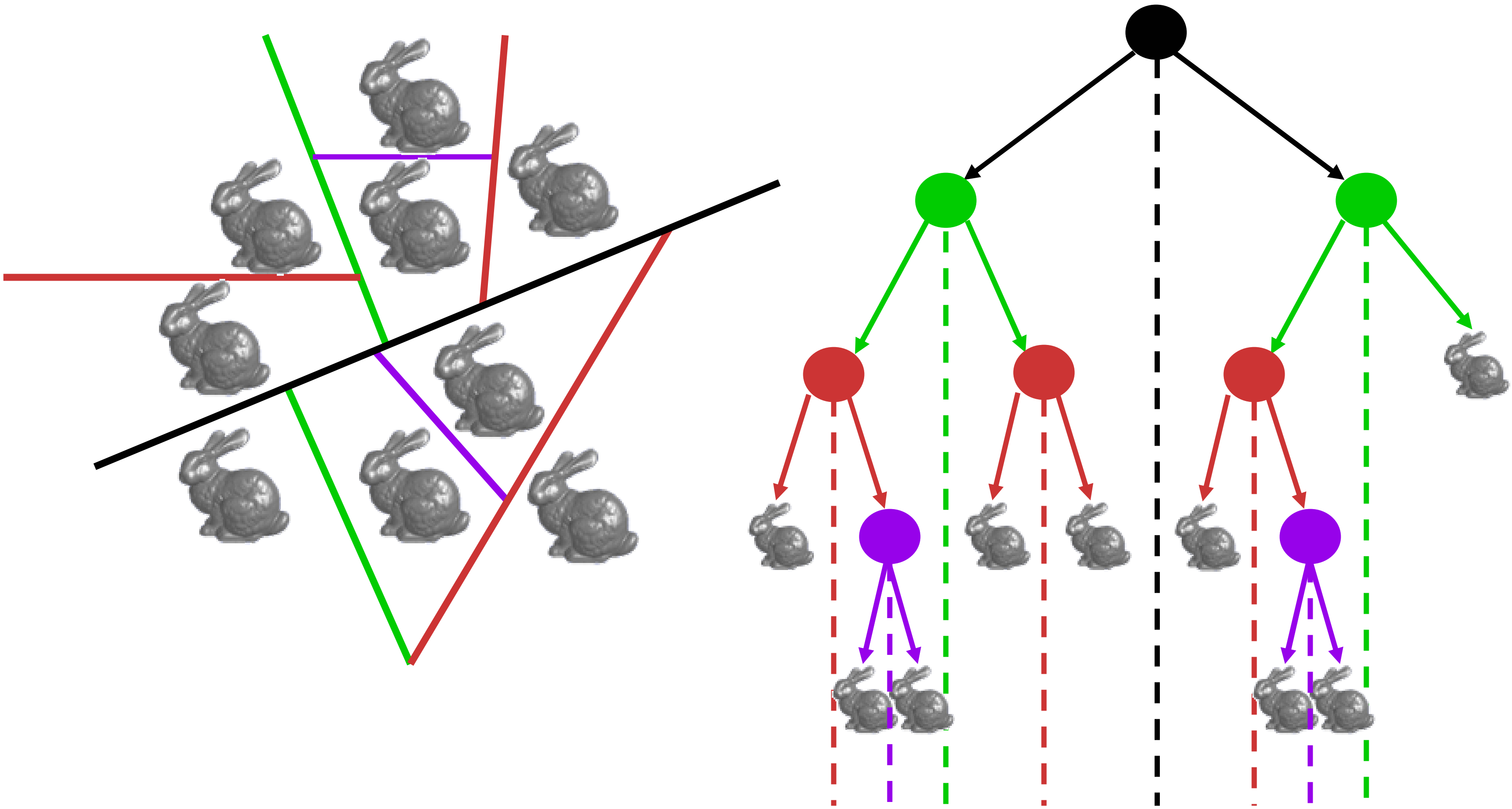
BSP Trees: Objects



BSP Trees: Objects



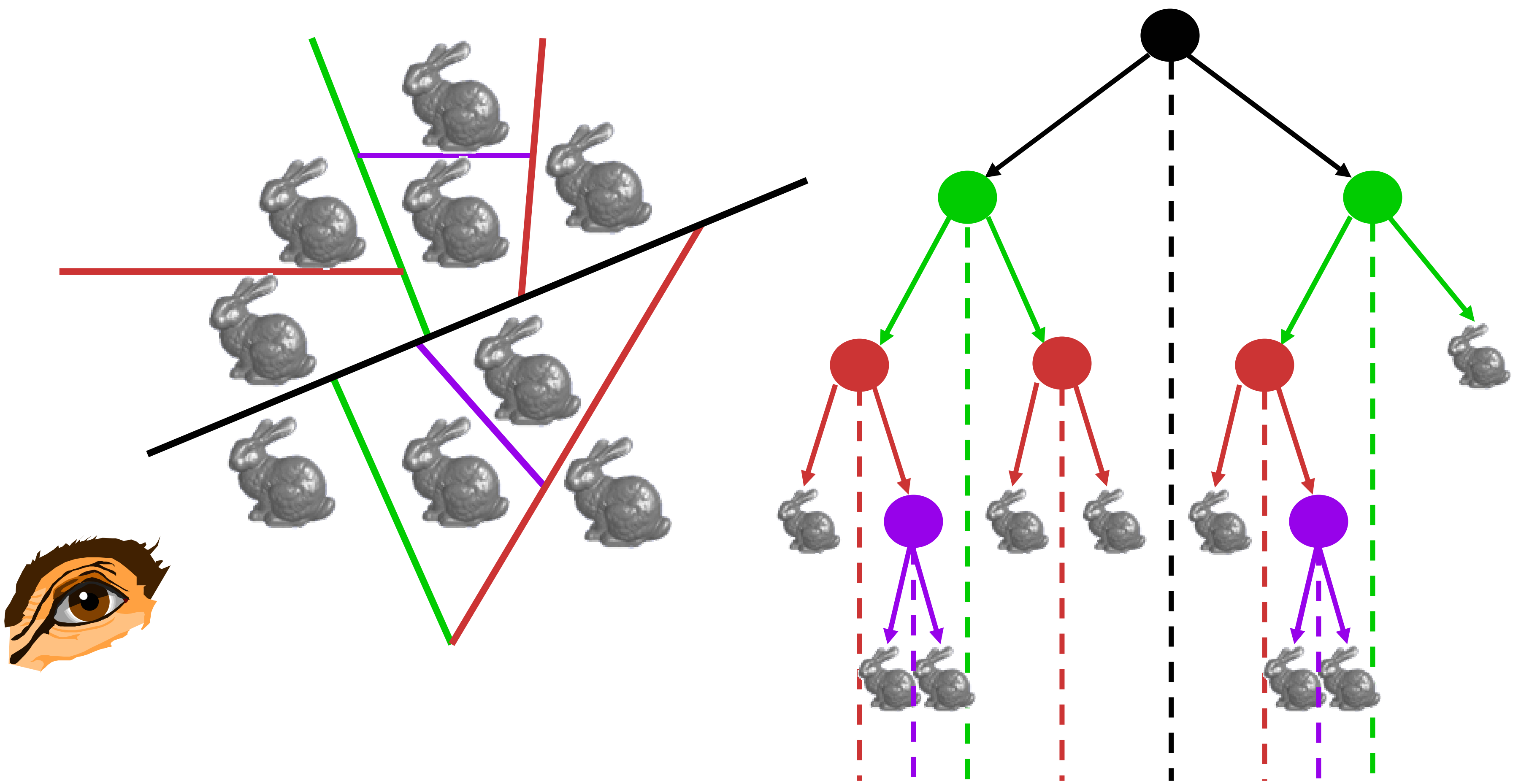
BSP Trees: Objects



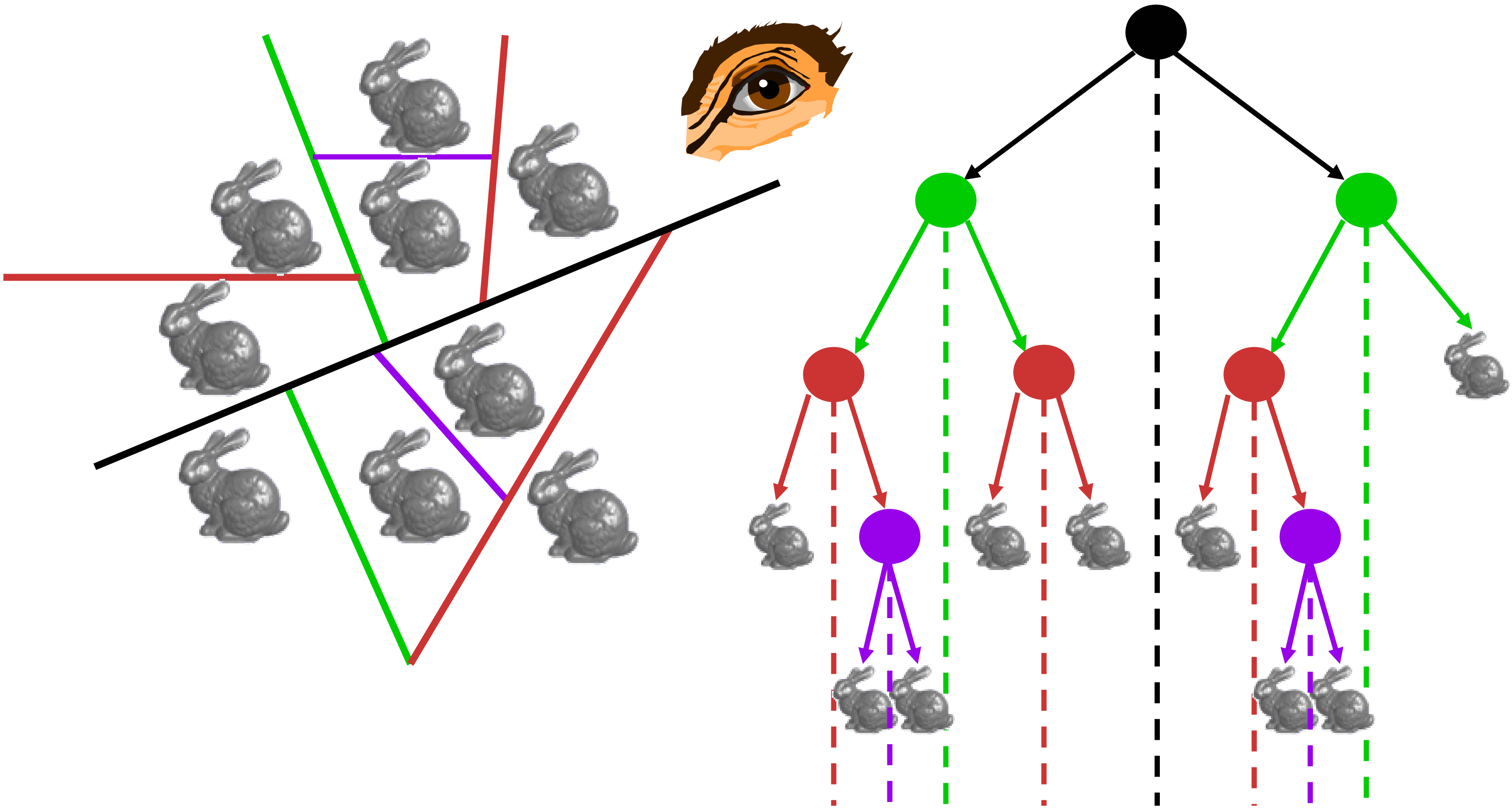
Rendering BSP Trees

```
renderBSP(BSPtree *T)
  BSPtree *near, *far;
  if (eye on left side of T->plane)
     near = T->left; far = T->right;
  else
     near = T->right; far = T->left;
  renderBSP(far);
  if (T is a leaf node)
     renderObject(T)
  renderBSP(near);
```

BSP Trees: Objects



BSP Trees: Objects



Polygons: BSP Tree Construction

- split along the plane defined by any polygon from scene
- classify all polygons into positive or negative half-space of the plane
  - if a polygon intersects plane, split polygon into two and classify them both
- recurse down the negative half-space
- recurse down the positive half-space

Polygons: BSP Tree Traversal

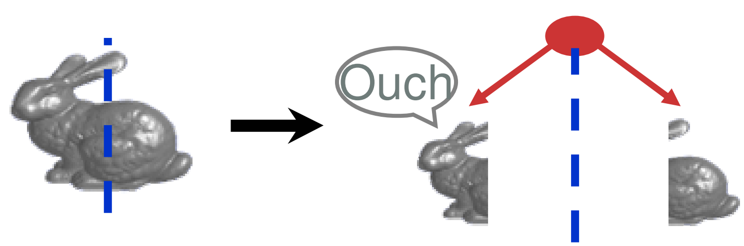
- query: given a viewpoint, produce an ordered list of (possibly split) polygons from back to front:

```
BSPnode::Draw(Vec3 viewpt)
   Classify viewpt: in + or - half-space of node->plane?
   /* Call that the "near" half-space */
       farchild->draw(viewpt);
       render node->polygon; /* always on node->plane */
       nearchild->draw(viewpt);
```

- intuitively: at each partition, draw the stuff on the farther side, then the polygon on the partition, then the stuff on the nearer side

Discussion: BSP Tree Cons

- no bunnies were harmed in my example
- but what if a splitting plane passes through an object?
  - split the object; give half to each node



BSP Demo

- nice demo:

  http://symbolcraft.com/graphics/bsp

Summary: BSP Trees

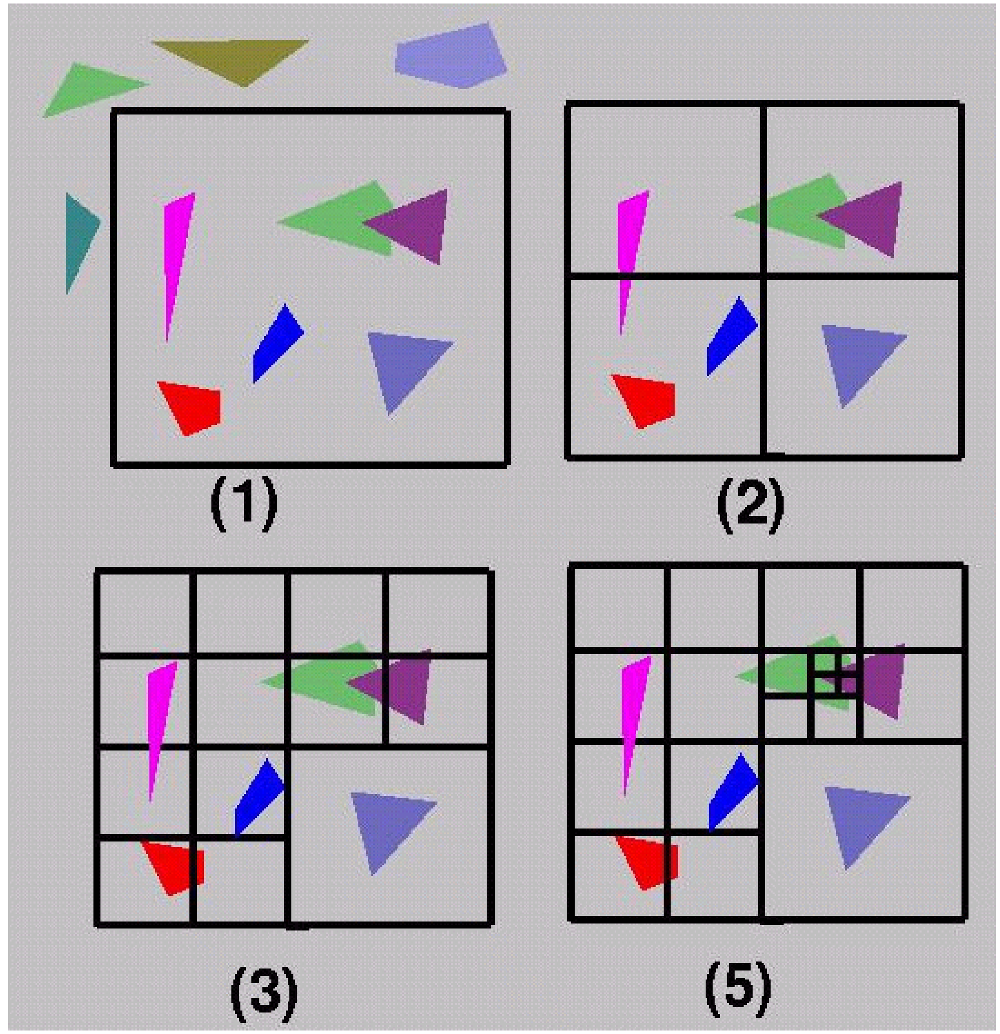
- pros:
  - simple, elegant scheme
  - only writes to framebuffer (no reads to see if current polygon is in front of previously rendered polygon, i.e., painters algorithm)
    - thus very popular for video games (but getting less so)
- cons:
  - computationally intense preprocess stage restricts algorithm to static scenes
  - slow time to construct tree: O(n log n) to

Warnock's Algorithm (1969)

- elegant scheme based on a powerful general approach common in graphics: *if the situation is too complex, **subdivide***
  - start with a *root viewport* and a list of all primitives (polygons)
  - then recursively:
    - clip objects to viewport
    - if number of objects incident to viewport is zero or one, visibility is trivial
    - otherwise, subdivide into smaller viewports, distribute primitives among them, and recurse

Warnock's Algorithm

- what is the terminating condition?
- how to determine the correct visible surface in this case?



Warnock’s Algorithm

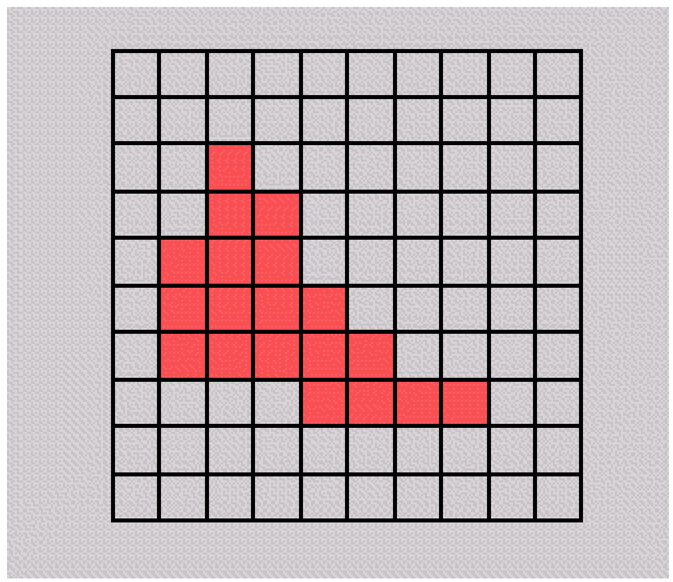
- pros:
  - very elegant scheme
  - extends to any primitive type
- cons:
  - hard to embed hierarchical schemes in hardware
  - complex scenes usually have small polygons and high depth complexity
    - thus most screen regions come down to the single-pixel case

The Z-Buffer Algorithm

- both BSP trees and Warnock's algorithm were proposed when memory was expensive
  - example: first 512x512 framebuffer > $50,000!
- Ed Catmull (mid-70s) proposed a radical new approach called z-buffering.
- the big idea: resolve visibility independently at each pixel

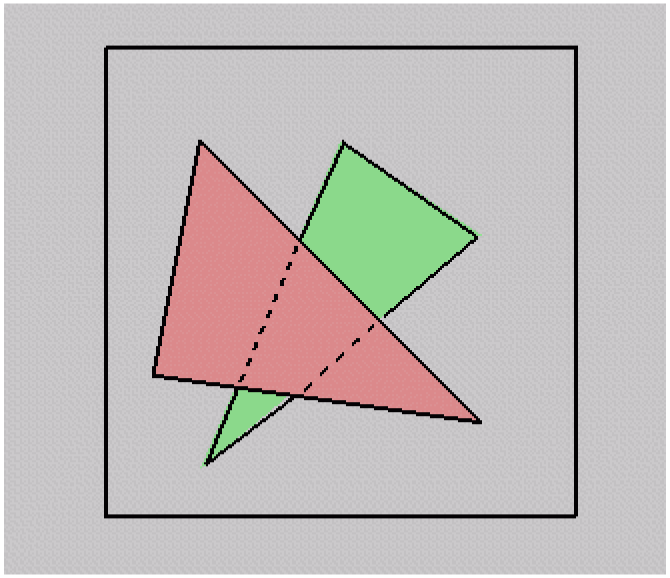
The Z-Buffer Algorithm

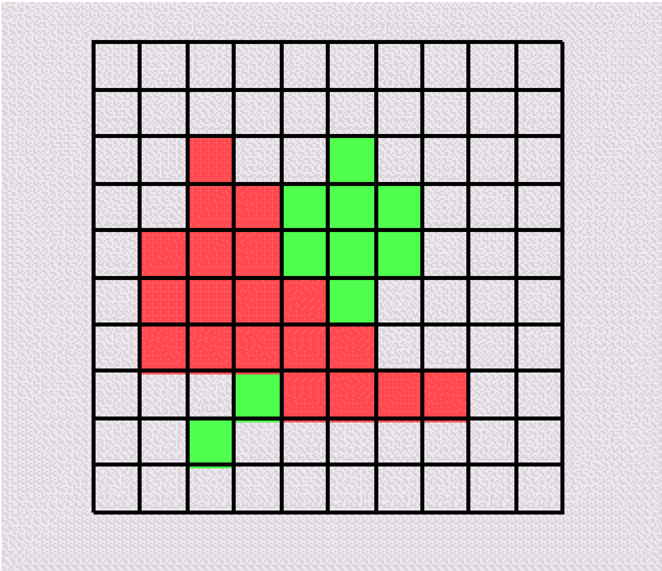
- we know how to rasterize polygons into an image discretized into pixels:



The Z-Buffer Algorithm

- *what happens if multiple primitives occupy the same pixel on the screen? Which is allowed to paint the pixel?*





The Z-Buffer Algorithm

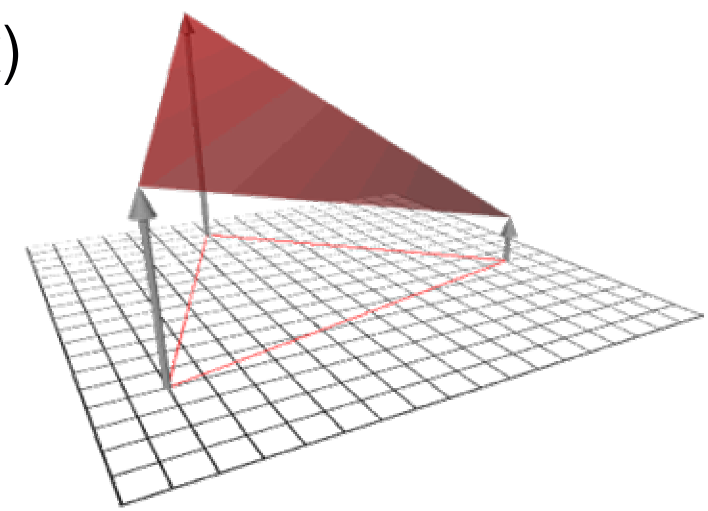
- idea: retain depth (Z in eye coordinates) through projection transform
  - use canonical viewing volumes
  - each vertex has z coordinate (relative to eye point) intact

The Z-Buffer Algorithm

- augment color framebuffer with Z-buffer or *depth buffer* which stores Z value at each pixel
  - at frame beginning, initialize all pixel depths to ∞
  - when rasterizing, interpolate depth (Z) across polygon and store in pixel of Z-buffer
  - suppress writing to a pixel if its Z value is more distant than the Z value already stored there

Interpolating Z

- edge equations: Z just another planar parameter:
    - $z = (-D - Ax - By) / C$
    - if walking across scanline by (Dx) $znew = zold - (A/C)(Dx)$
  - total cost:
    - 1 more parameter to increment in inner loop
    - 3x3 matrix multiply for setup



- edge walking: just interpolate Z along edges and across spans

Z-buffer

- store (r,g,b,z) for each pixel
  - typically 8+8+8+24 bits, can be more

```
for all i,j {
 Depth[i,j] = MAX_DEPTH
 Image[i,j] = BACKGROUND_COLOUR
}
for all polygons P {
  for all pixels in P {
    if (Z_pixel < Depth[i,j]) {
      Image[i,j] = C_pixel
      Depth[i,j] = Z_pixel
    }
  }
}
```

Depth Test Precision

– reminder: projective transformation maps eye-space *z* to generic *z*-range (NDC)

– simple example:

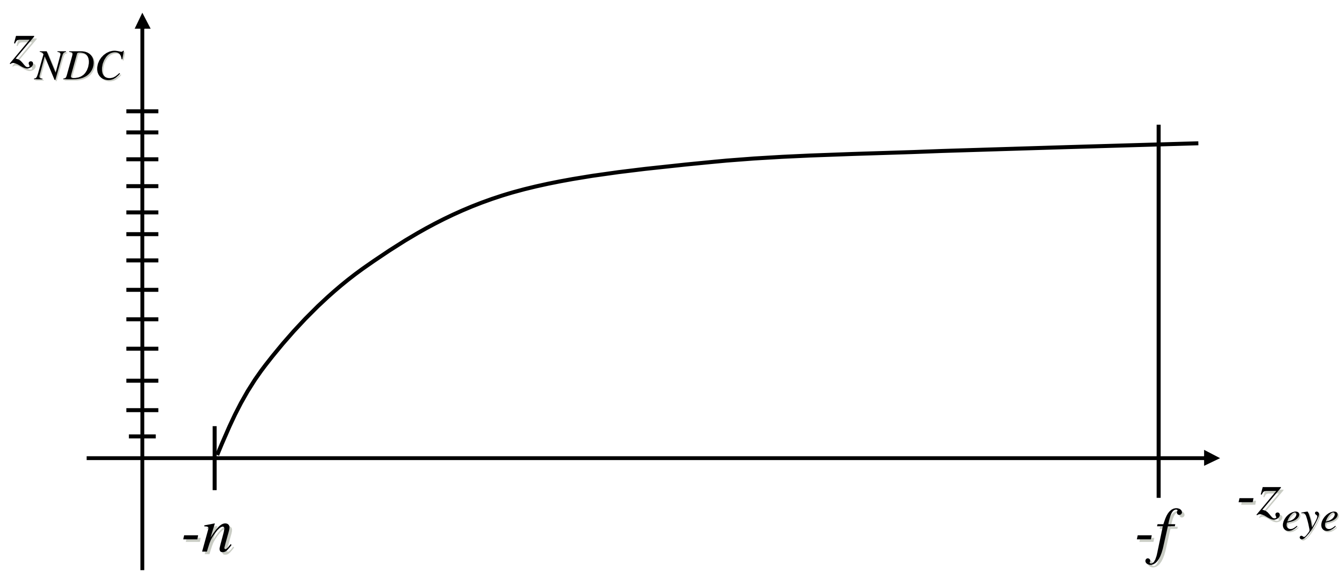
$$T\left(\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

– thus:

$$z_{NDC} = \frac{a \cdot z_{eye} + b}{z_{eye}} = a + \frac{b}{z_{eye}}$$

Depth Test Precision

- therefore, depth-buffer essentially stores $1/z$, rather than z!
- this yields precision problems with integer depth buffers:

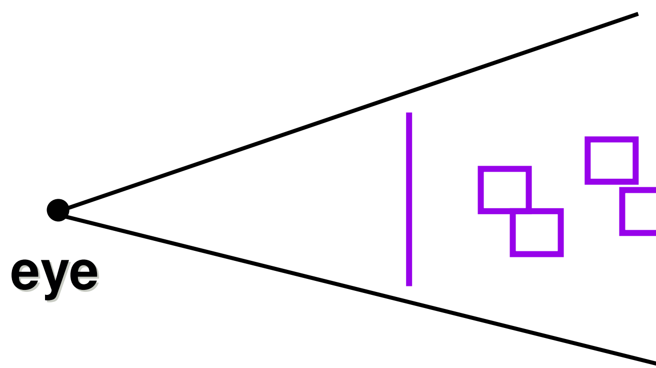


Depth Test Precision

- precision of depth buffer is bad for far objects
- depth fighting: two different depths in eye space get mapped to same depth in framebuffer
  - which object “wins” depends on drawing order and scan-conversion
- gets worse for larger ratios $f{:}n$
  - *rule of thumb:* $\mathrm{f{:}n} < 1000$ *for 24 bit depth buffer*

Z-buffer

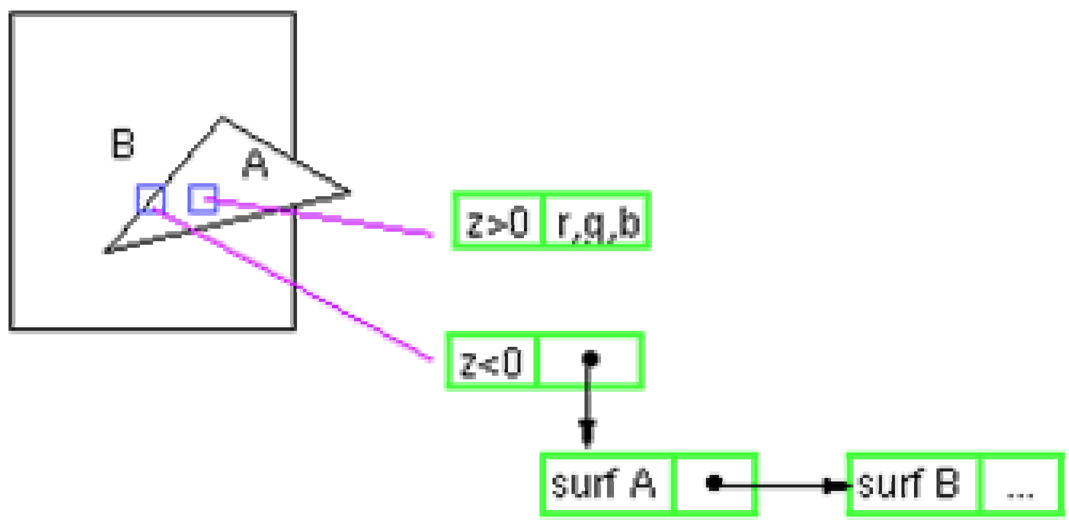
– hardware support in graphics cards
– poor for high-depth-complexity scenes
    • need to render all polygons, even if most are invisible



– “jaggies”: pixel staircase along edges

The A-Buffer

- antialiased, area-averaged accumulation buffer
  - z-buffer: one visible surface per pixel
  - A-buffer: linked list of surfaces



- data for each surface includes
  - *RGB, Z, area-coverage percentage, ...*

The Z-Buffer Algorithm

- how much memory does the Z-buffer use?
- does the image rendered depend on the drawing order?
- does the time to render the image depend on the drawing order?
- how does Z-buffer load scale with visible polygons? with framebuffer resolution?

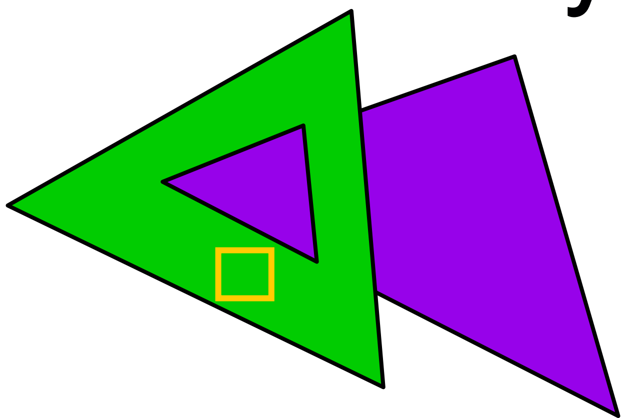
Z-Buffer Pros

- simple!!!
- easy to implement in hardware
- polygons can be processed in arbitrary order
- easily handles polygon interpenetration
- enables *deferred shading*
  – rasterize shading parameters (e.g., surface normal) and only shade final visible fragments

Z-Buffer Cons

- lots of memory (e.g. 1280x1024x32 bits)
  - with 16 bits cannot discern millimeter differences in objects at 1 km distance
- Read-Modify-Write in inner loop requires fast memory
- hard to do analytic antialiasing
  - we don't know which polygon to map pixel back to
- shared edges are handled inconsistently
  - ***ordering dependent***
- hard to simulate translucent polygons
  - we throw away color of polygons behind closest one

Visibility



– object space algorithms
  • explicitly compute visible portions of polygons
  • painter's algorithm: depth-sorting, BSP trees
– image space algorithms
  • operate on pixels or scan-lines
  • visibility resolved to the precision of the display
  • Z-buffer

Hidden Surface Removal

- 2 classes of methods
  - image-space algorithms
    - perform visibility test for very pixel independently
    - limited to resolution of display
    - performed late in rendering pipeline
  - object-space algorithms
    - determine visibility on a polygon level in camera coordinates
    - resolution independent
    - early in rendering pipeline (after clipping)
    - expensive