University of British Columbia

CPSC 414 Computer Graphics

Midterm Review

Week 7, Wed 16 Oct 2003

- midterm review
- project 1 demos, hall of fame

News

- homework 1 due now
  – one day late if in handin box 18 by 9am Thu
  – two days late if in at class beginning Fri
  – no homeworks accepted after Fri 9am!
    - solutions out then

Midterm Exam

- Monday Oct 20 9am-9:50am
  - you may use one handwritten 8.5”x11” sheet
    - OK to use both sides of page
  - no other notes, no books
  - nonprogrammable calculators OK
  - arrive on time!!

What's Covered

- transformations
- viewing and projections
- coordinate systems of rendering pipeline
- picking
- lighting and shading
- scan conversion

- **not** sampling

Reading

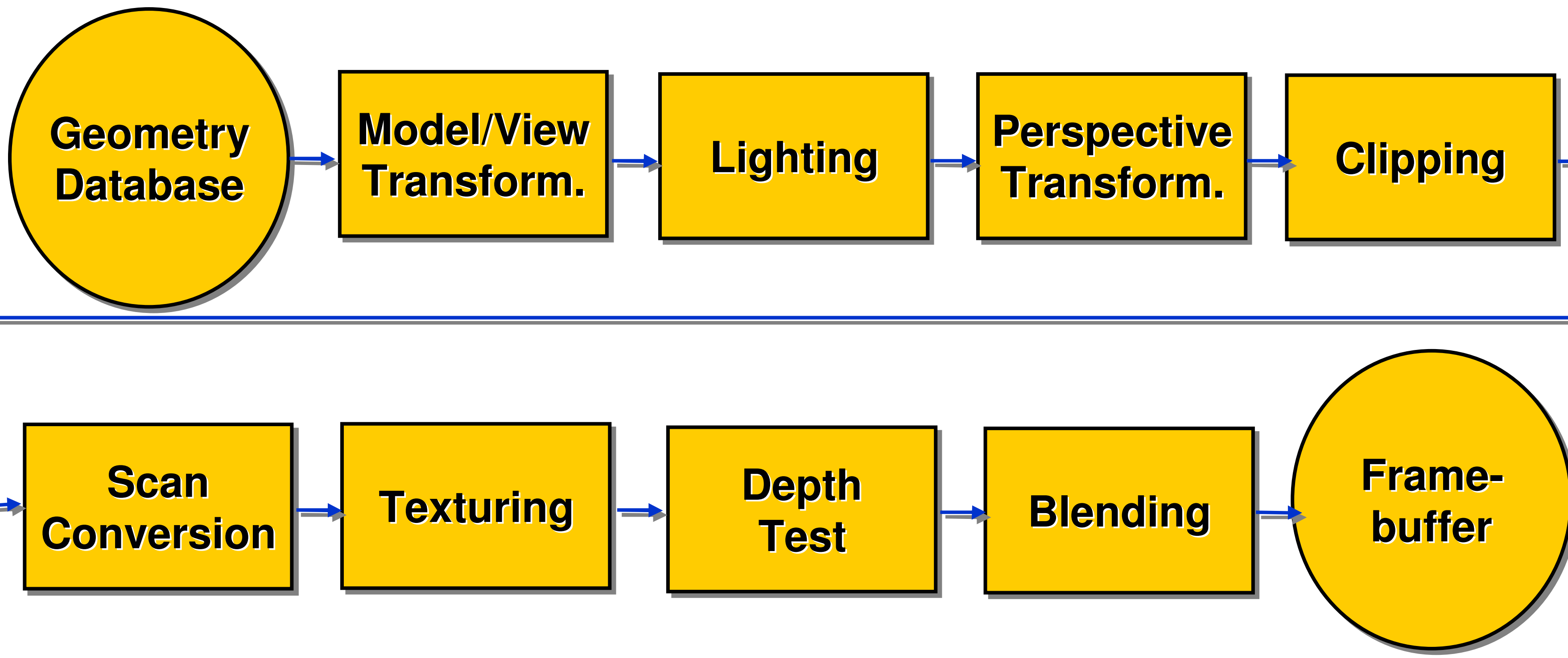
- Angel book
  - Chap 1, 2, 3, 4, 5, 6, 8.9-8.11, 9.1-9.6
  - you can be tested on material in book but not covered in lecture
  - you can be tested on material covered in lecture but not covered in book

Old Exams Posted

- see course web page

The Rendering Pipeline

- pros and cons of pipeline approach



Transformations

translate(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

scale(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\text{Rotate}(x, \theta)$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & \cos\theta & -\sin\theta & \\ & \sin\theta & \cos\theta & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\text{Rotate}\ (y, \theta)$

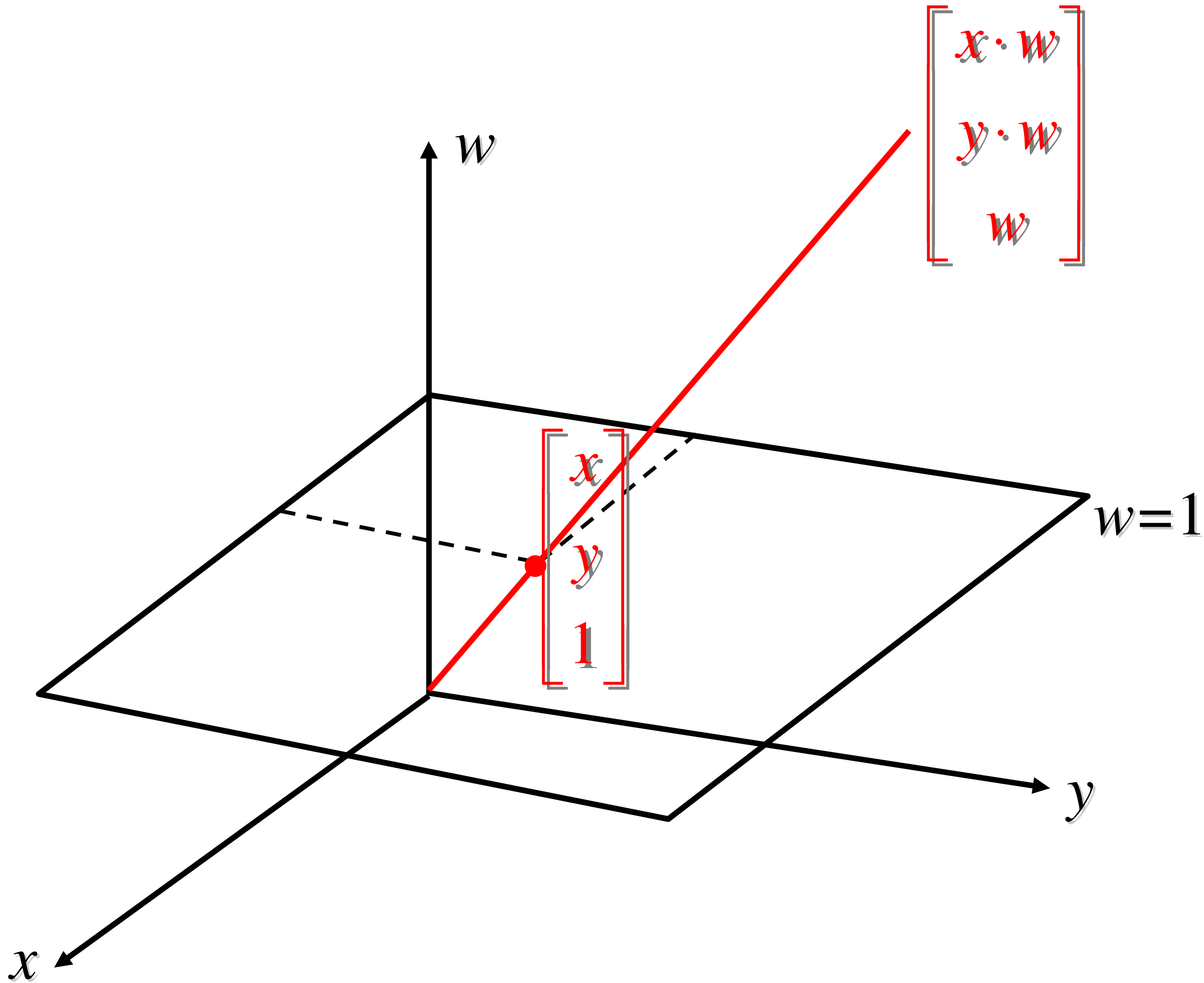
$$\begin{bmatrix} \cos\theta & & \sin\theta & \\ & 1 & & \\ -\sin\theta & & \cos\theta & \\ & & & 1 \end{bmatrix}$$

$\text{Rotate}\ (z, \theta)$

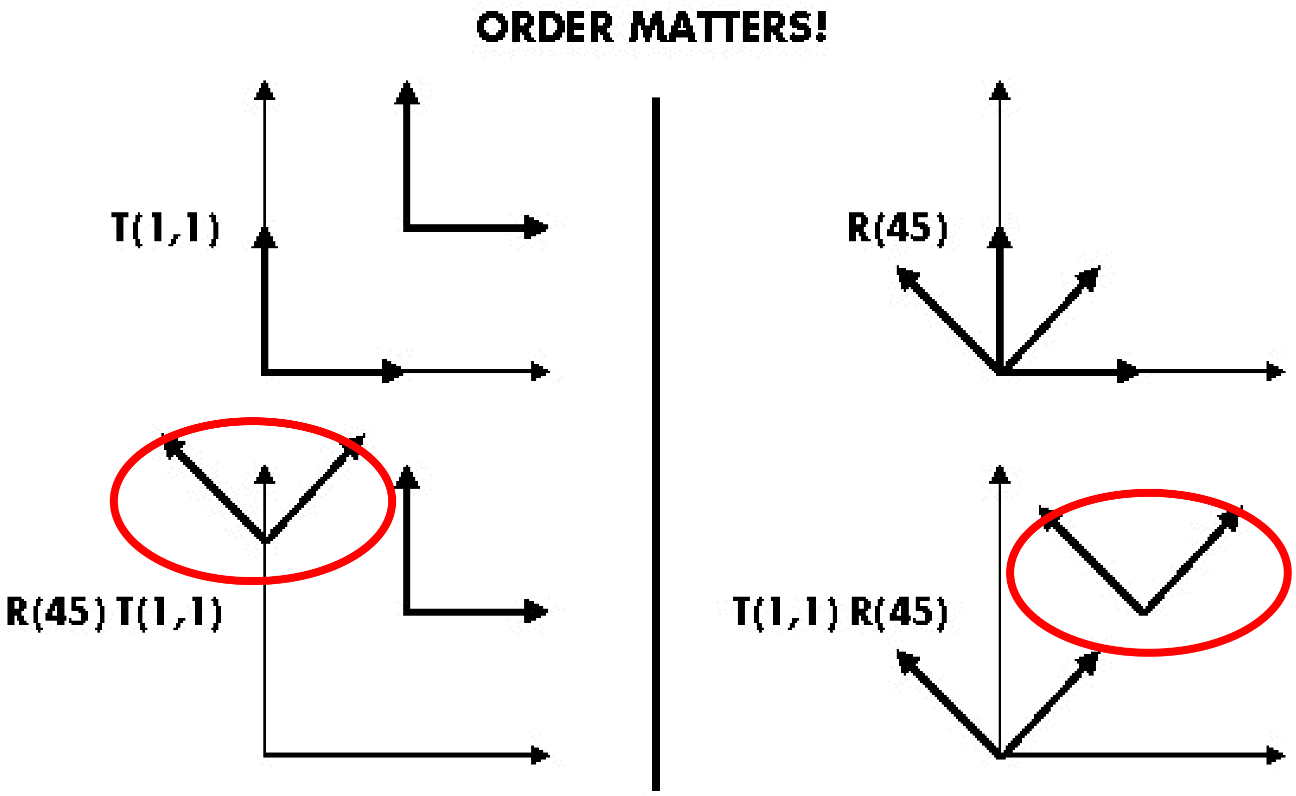
$$\begin{bmatrix} \cos\theta & -\sin\theta & & \\ \sin\theta & \cos\theta & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

Homogeneous Coordinates

•



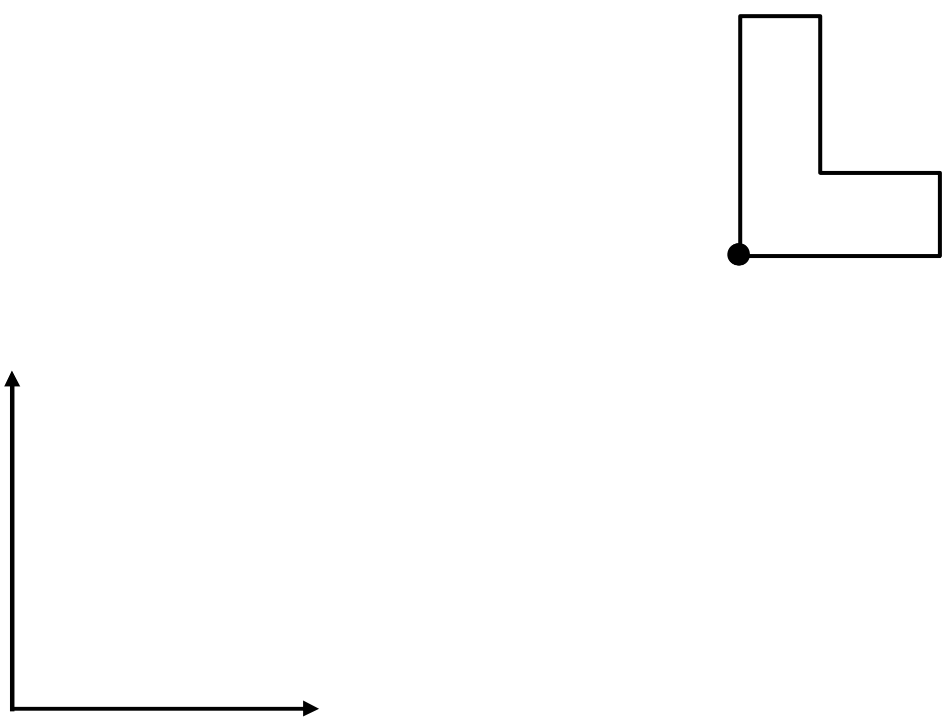
Composing Transformations



Ta Tb = Tb Ta, but Ra Rb != Rb Ra and Ta Rb != Rb Ta

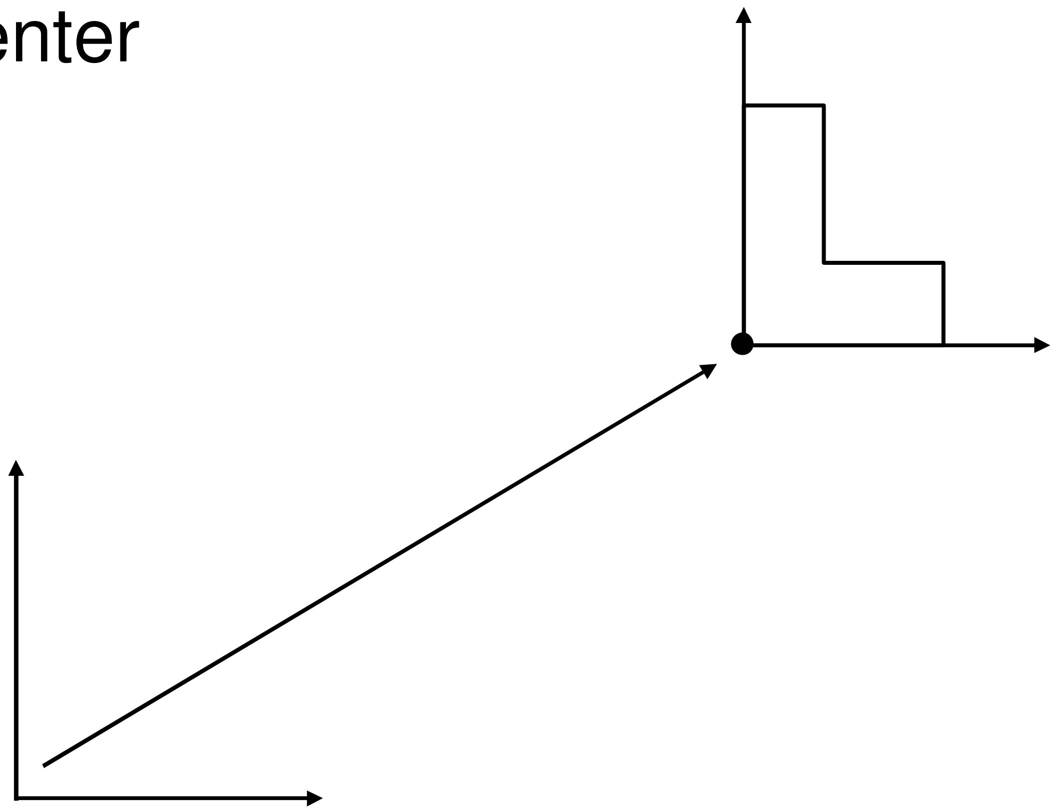
Composing Transformations

- example: rotation around arbitrary center



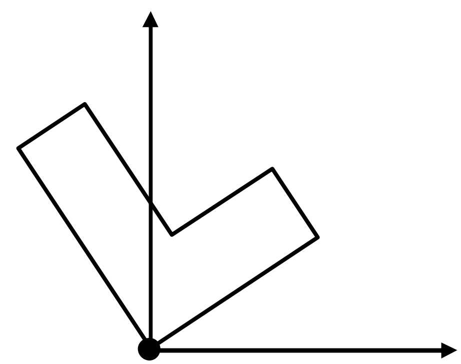
Composing Transformations

- example: rotation around arbitrary center
  - step 1: translate coordinate system to rotation center



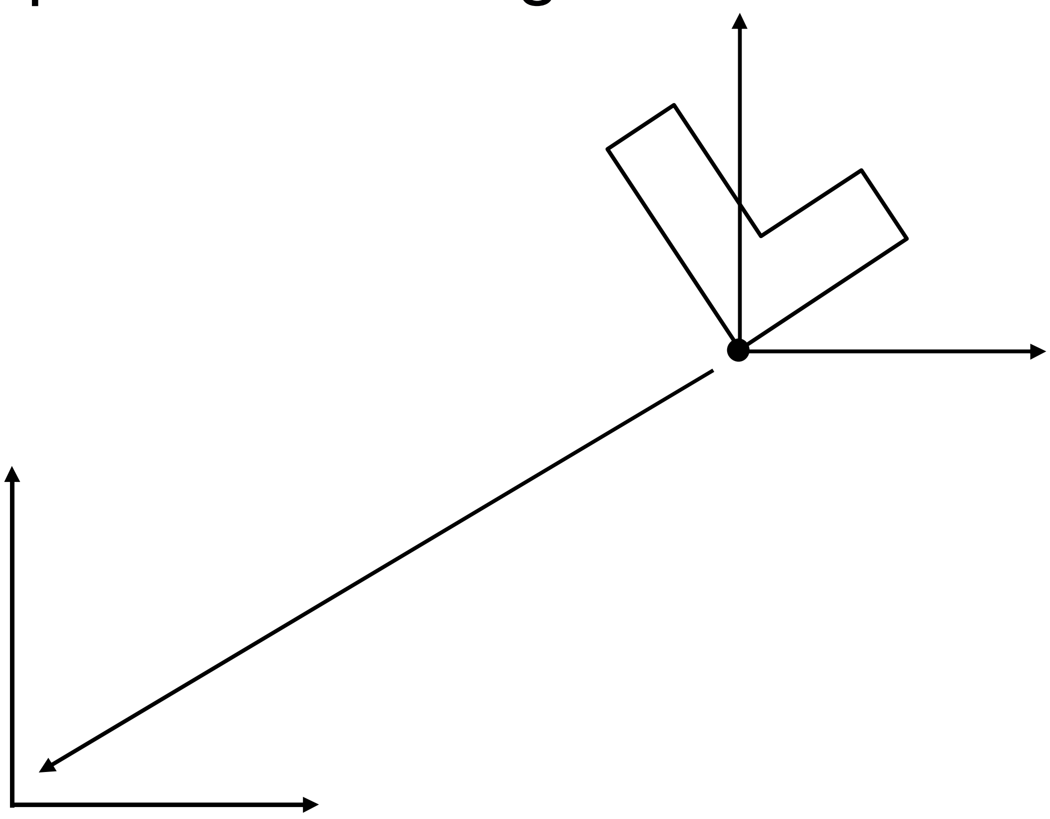
Composing Transformations

- example: rotation around arbitrary center
  - step 2: perform rotation



Composing Transformations

- example: rotation around arbitrary center
  – step 3: back to original coordinate system



Composing Transformations

- rotation about a fixed point

  $p' = TRT^{-1}p$
- rotation around an arbitrary axis

- considering frame vs. object

OpenGL:

D

C

B

A

draw p

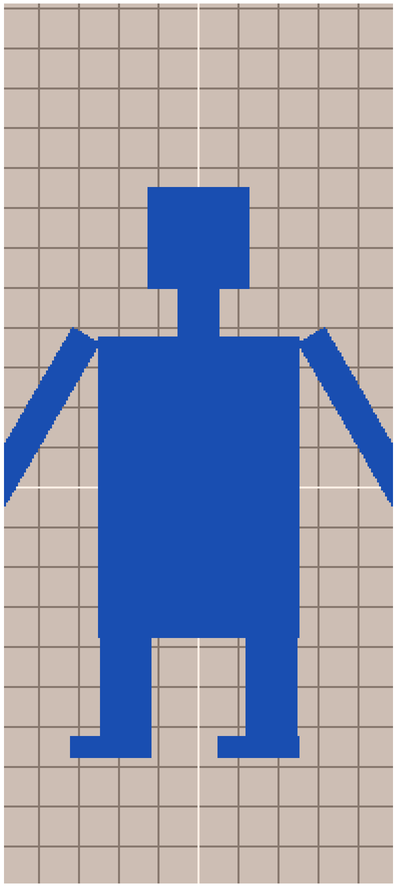
$$\xrightarrow{\text{frame}}$$

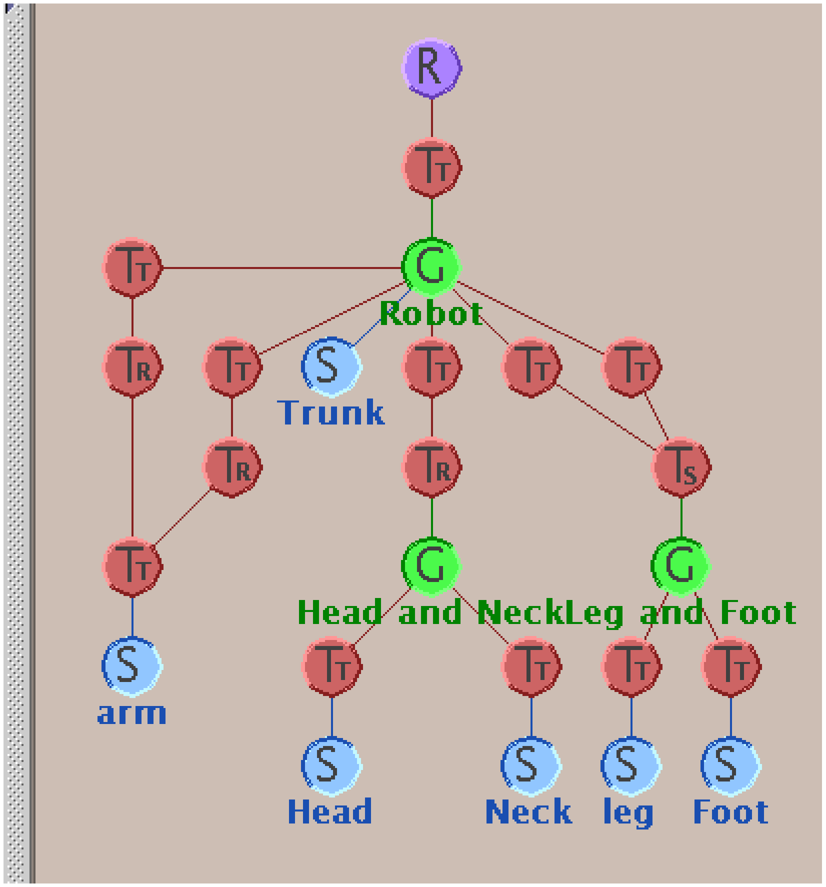
$$p' = DCBAp$$

$$\xleftarrow[\text{object}]{}$$

Transformation Hierarchies

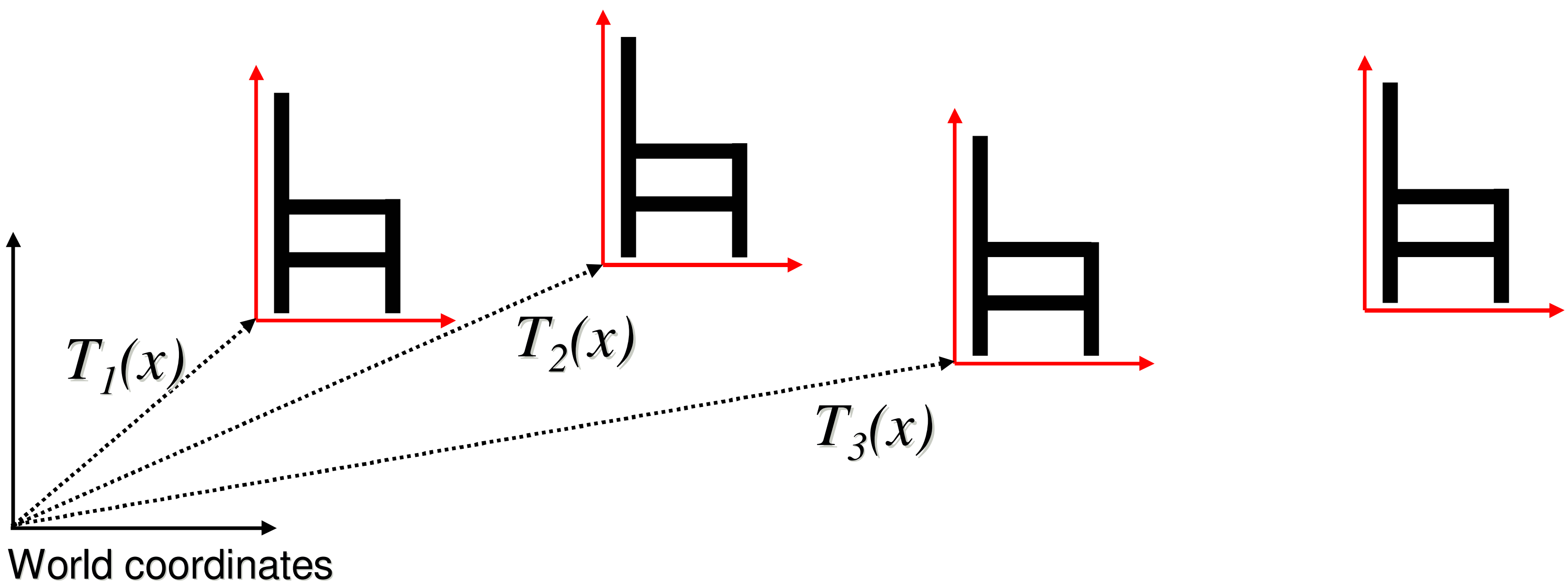
- hierarchies don't fall apart when changed
- transforms apply to graph nodes beneath





Matrix Stacks

- push and pop matrix stack
  - avoid computing inverses or incremental xforms
  - avoid numerical error

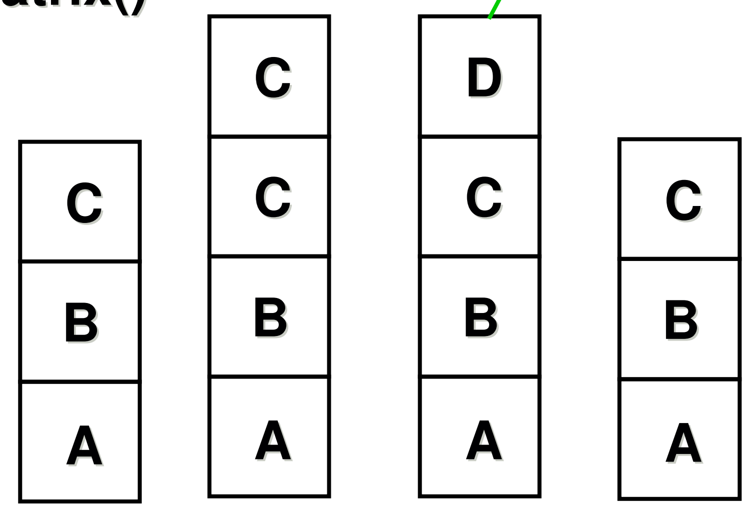


Matrix Stacks

D = C scale(2,2,2) trans(1,0,0)

glPushMatrix()

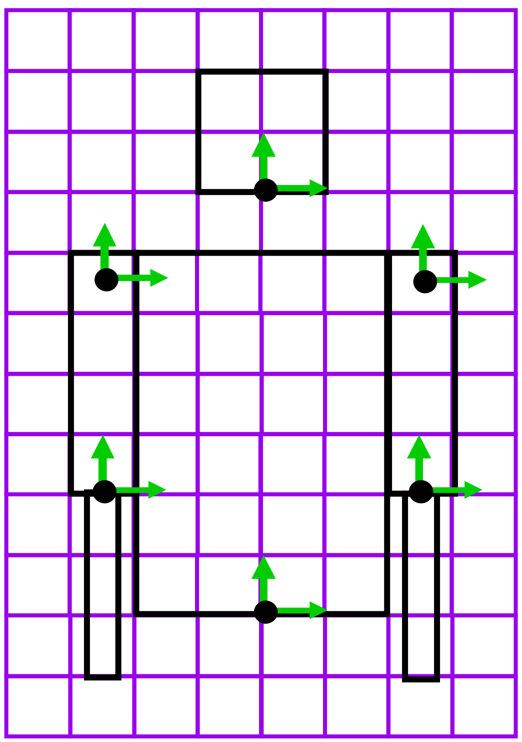
glPopMatrix()

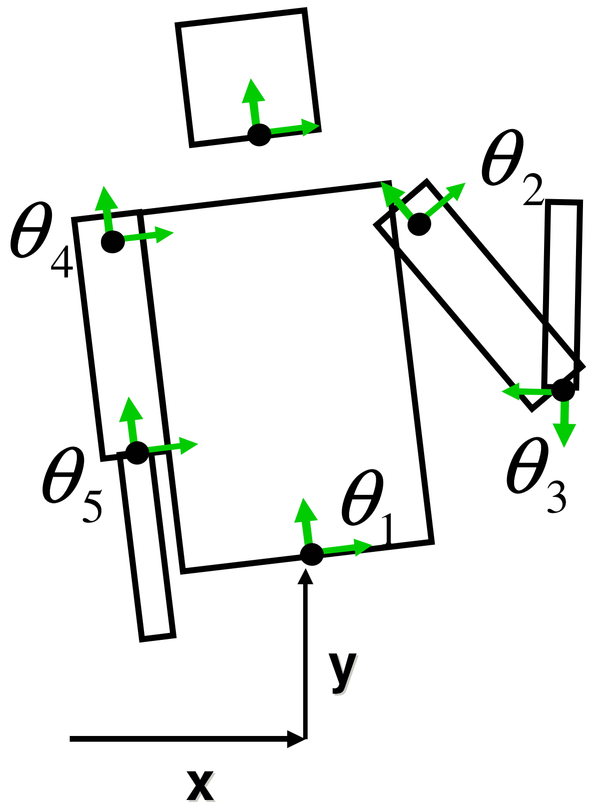


DrawSquare()
glPushMatrix()
glScale3f(2,2,2)
glTranslate3f(1,0,0)
DrawSquare()
glPopMatrix()

Transformation Hierarchies

- example






```
glTranslate3f(x,y,0);
glRotatef(θ1,0,0,1);
DrawBody();
glPushMatrix();
  glTranslate3f(0,7,0);
  DrawHead();
glPopMatrix();
glPushMatrix();
  glTranslate(2.5,5.5,0);
  glRotatef(θ2,0,0,1);
  DrawUArm();
  glTranslate(0,-3.5,0);
  glRotatef(θ3,0,0,1);
  DrawLArm();
glPopMatrix();
... (draw other arm)
```

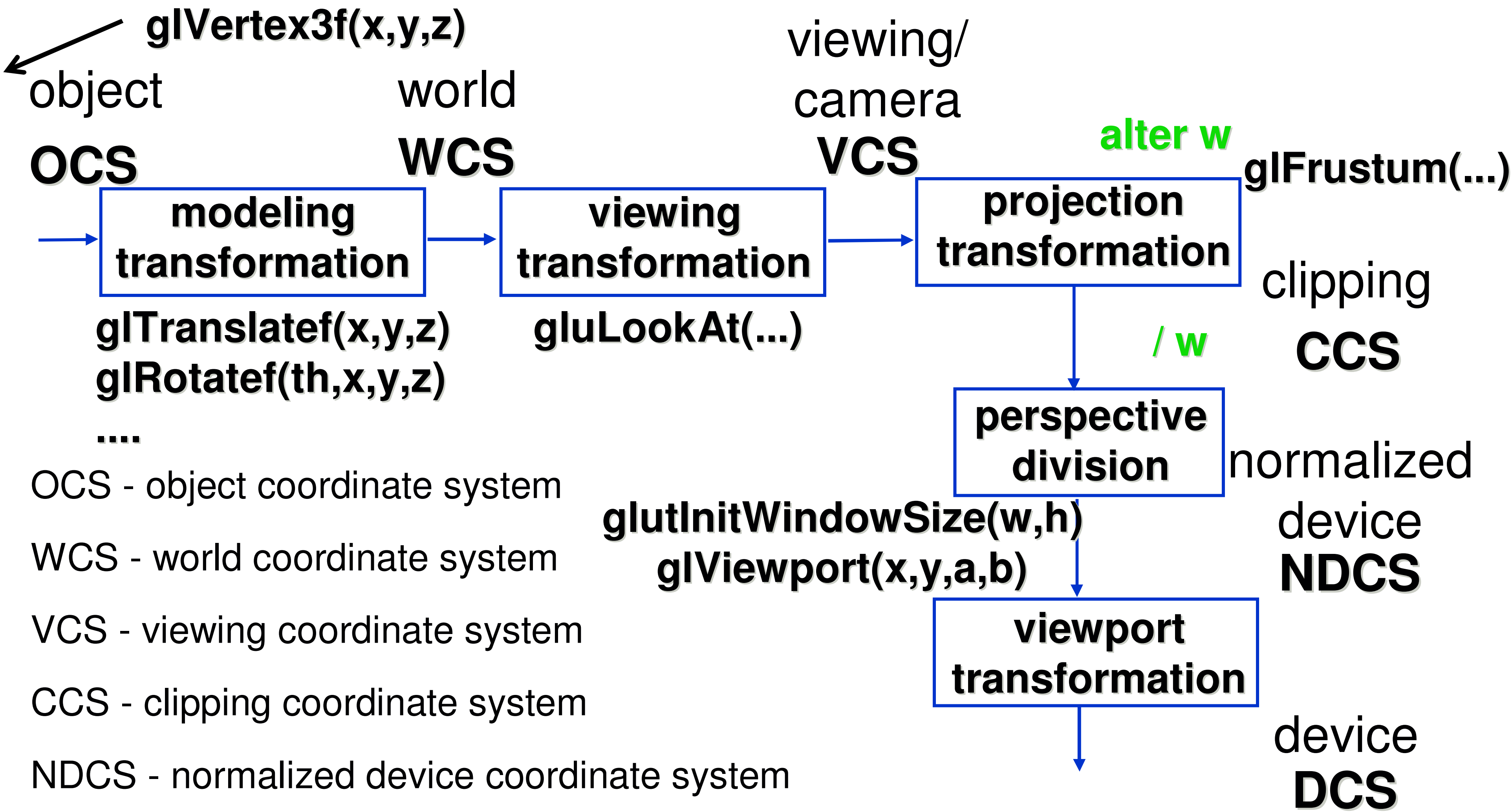
Display Lists

- reuse block of OpenGL code
- more efficient than immediate mode
  - code reuse, driver optimization
- good for static objects redrawn often
  - can’t change contents
  - not just for multiple instances
    - interactive graphics: objects redrawn every frame
- nest when possible for efficiency

Double Buffering

- two buffers, front and back
  - while front is on display, draw into back
  - when drawing finished, swap the two
- avoid flicker

Projective Rendering Pipeline



OCS - object coordinate system

WCS - world coordinate system

VCS - viewing coordinate system

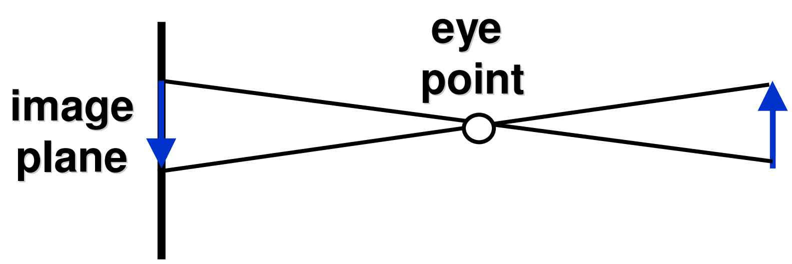
CCS - clipping coordinate system

NDCS - normalized device coordinate system

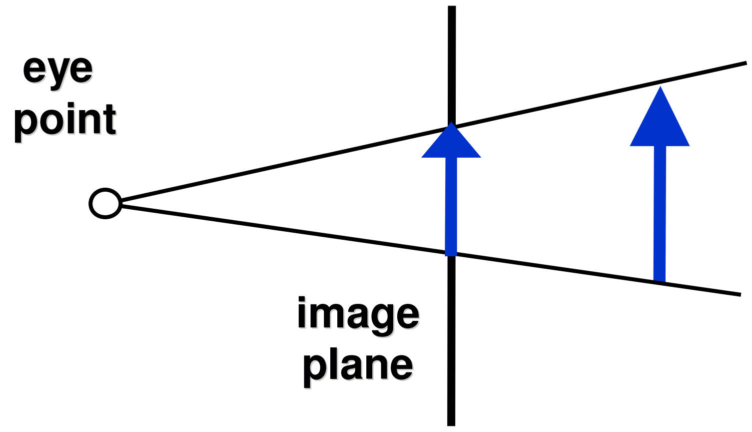
DCS - device coordinate system

Projection

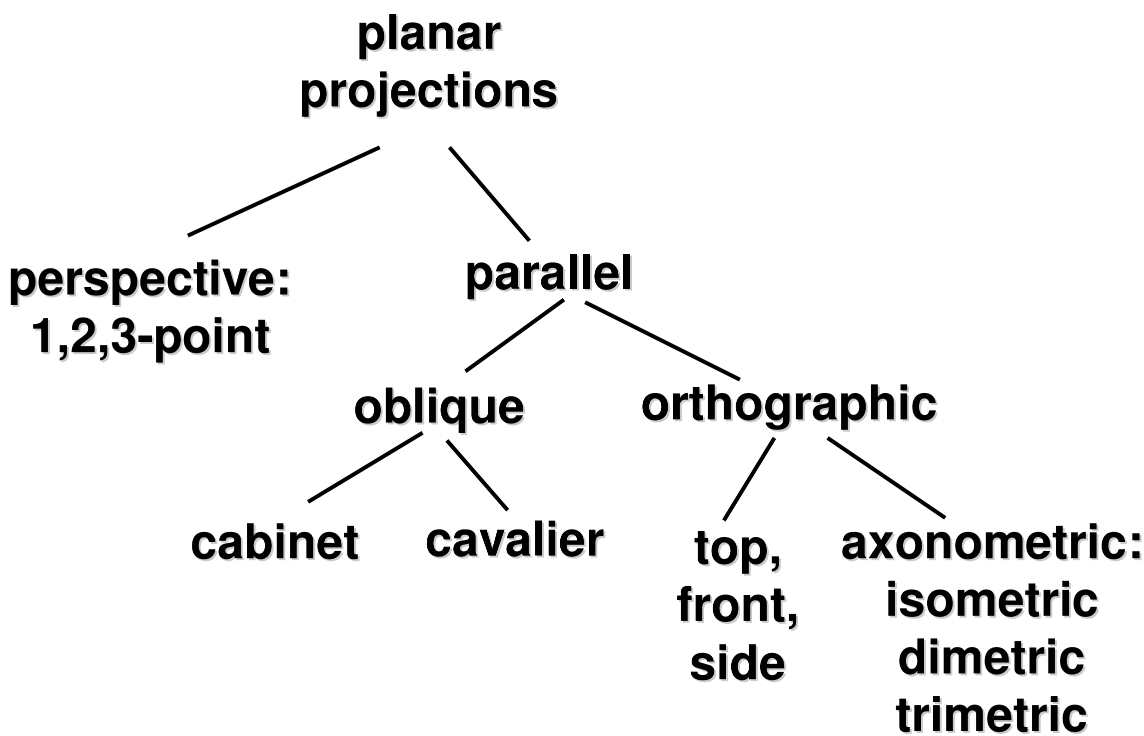
- theoretical pinhole camera



  - image inverted, more convenient equivalent

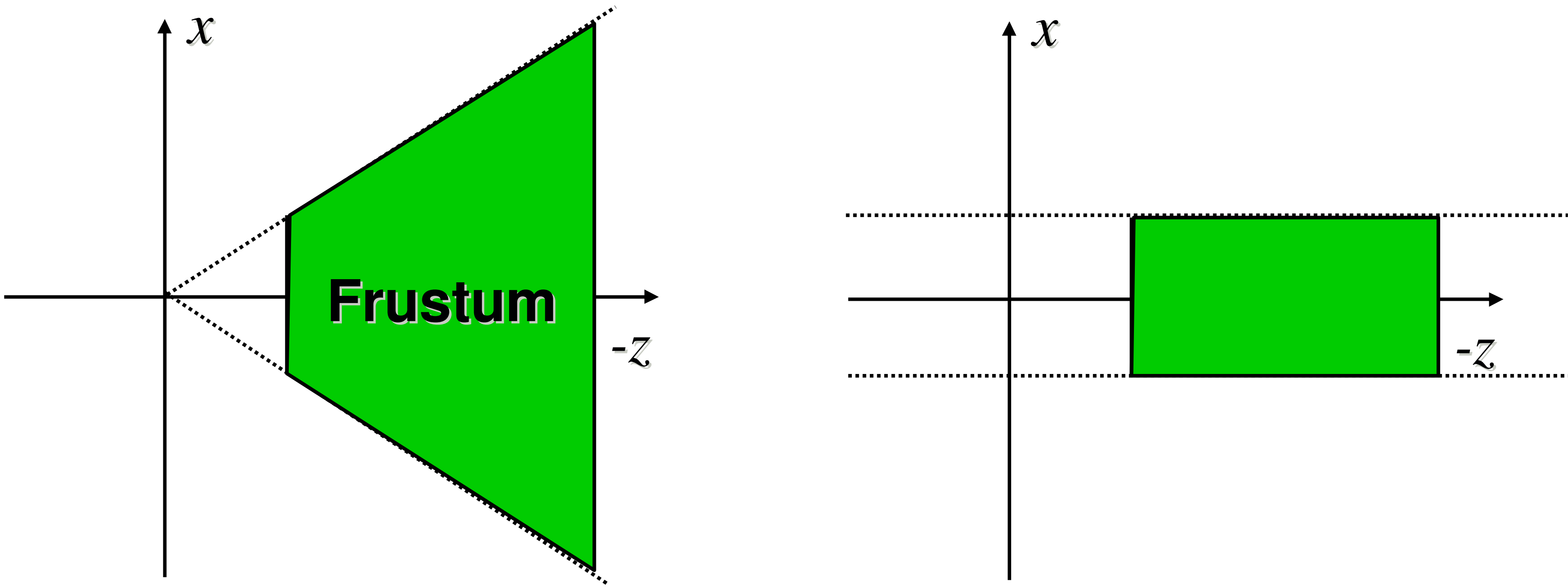


Projection Taxonomy



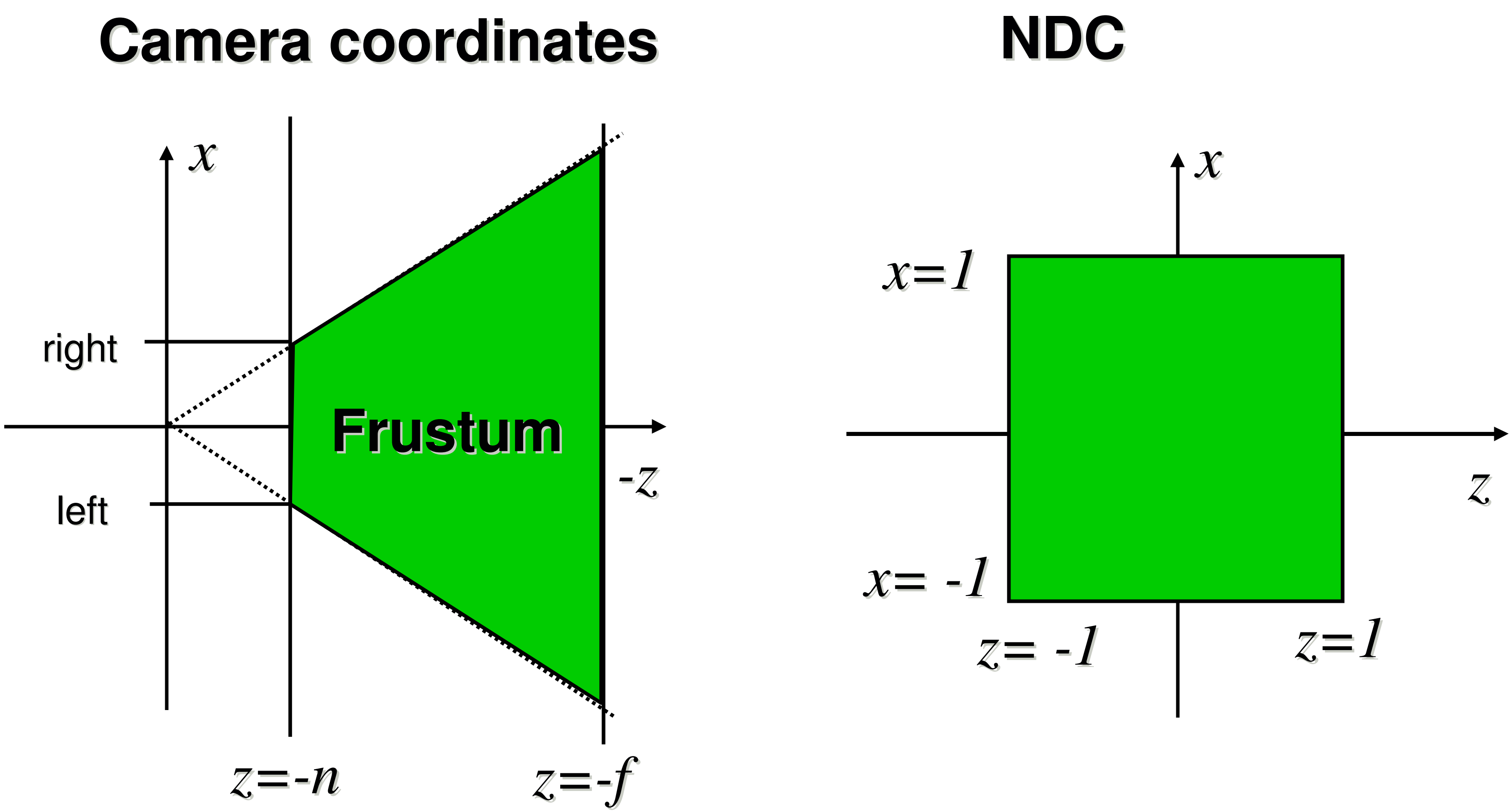
Projective Transformations

- transformation of space
  - center of projection moves to infinity
  - viewing frustum transformed into a parallelpiped



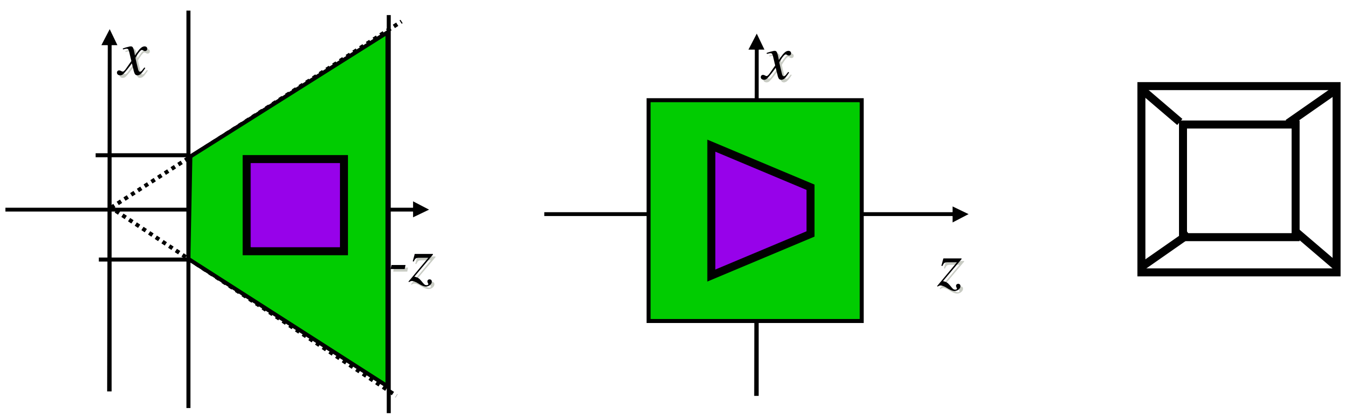
Normalized Device Coordinates

left/right x =+/- 1, top/bottom y =+/- 1, near/far z =+/- 1

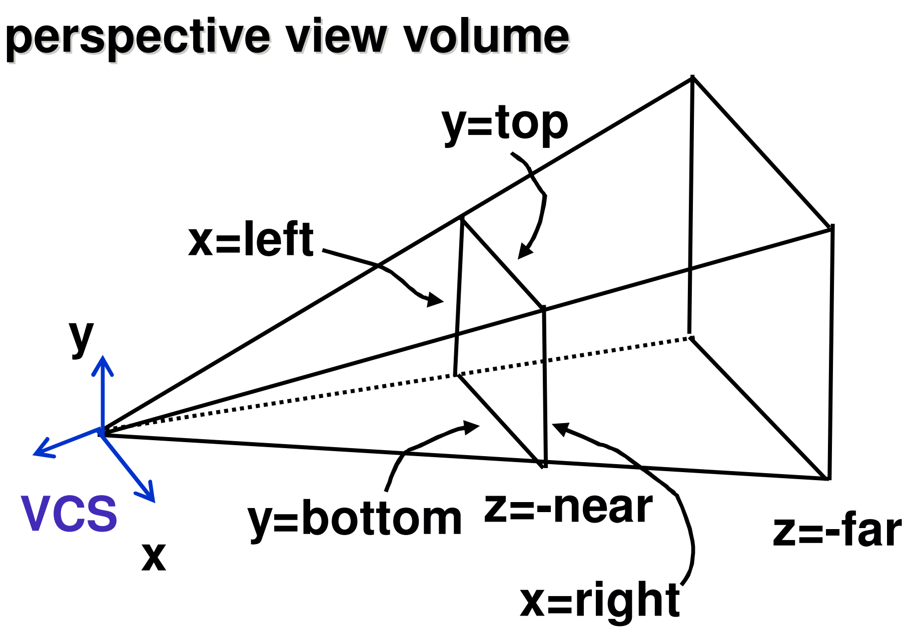


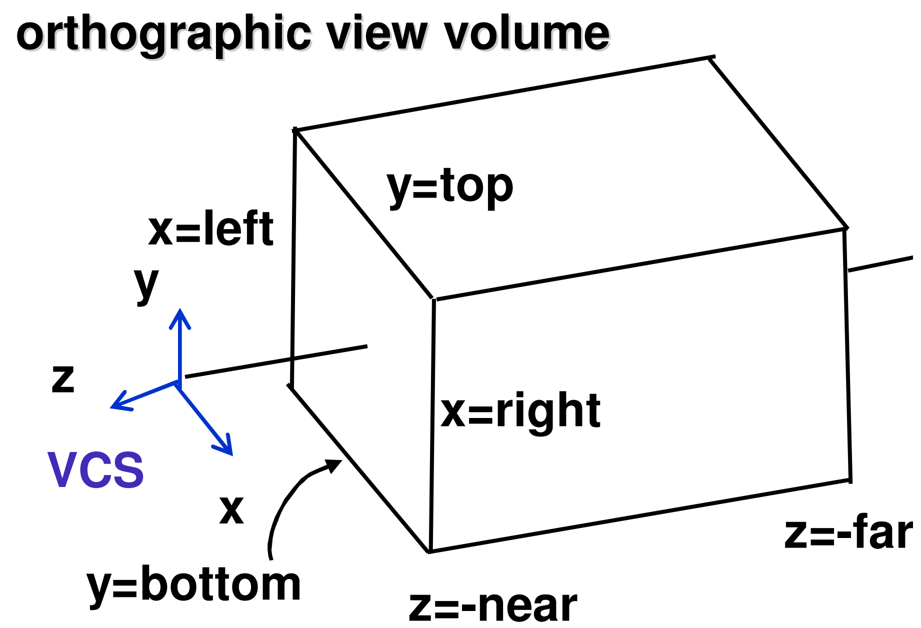
Projection Normalization

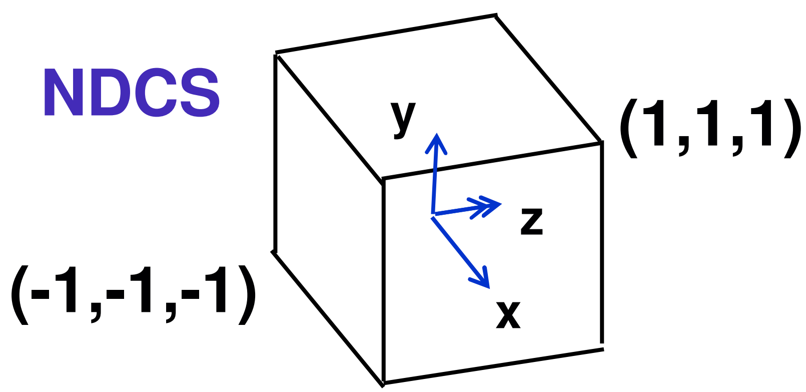
- distort such that orthographic projection of distorted objects is desired persp projection



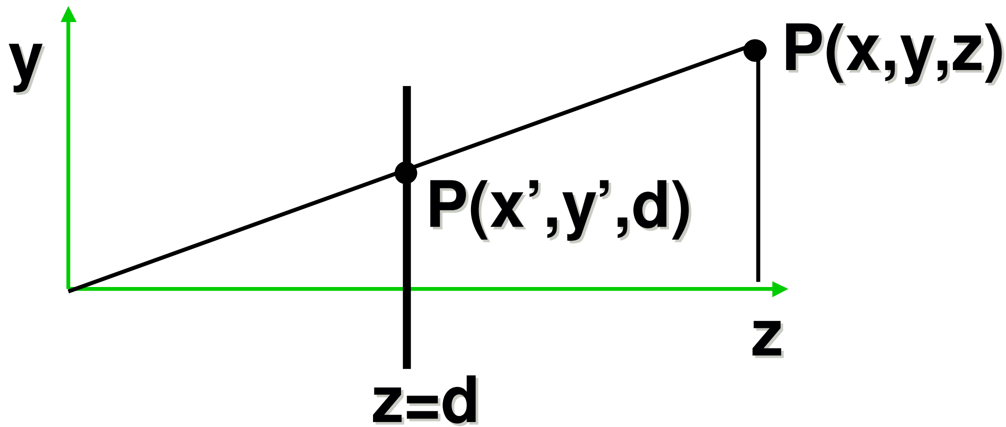
Transforming View Volumes







Basic Perspective Projection



similar triangles

$$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z}$$ **also** $$x' = \frac{x \cdot d}{z}$$ **but** $$z' = d$$

- nonuniform foreshortening
  - not affine

Basic Perspective Projection

- can express as homogenous 4x4 matrix!

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \xrightarrow{/w} \begin{bmatrix} x \cdot d/z \\ y \cdot d/z \\ d \end{bmatrix}$$

Projective Transformations

- determining the matrix representation
  - need to observe 5 points in general position, e.g.
    - $[left,0,0,1]^T \rightarrow [-1,0,0,1]^T$
    - $[0,top,0,1]^T \rightarrow [0,1,0,1]^T$
    - $[0,0,-f,1]^T \rightarrow [0,0,1,1]^T$
    - $[0,0,-n,1]^T \rightarrow [0,0,-1,1]^T$
    - $[left*f/n,top*f/n,-f,1]^T \rightarrow [-1,1,1,1]^T$
  - solve resulting equation system to obtain matrix

OpenGL Orthographic Matrix

- scale, translate, reflect for new coord sys
  - understand derivation!

$$\begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

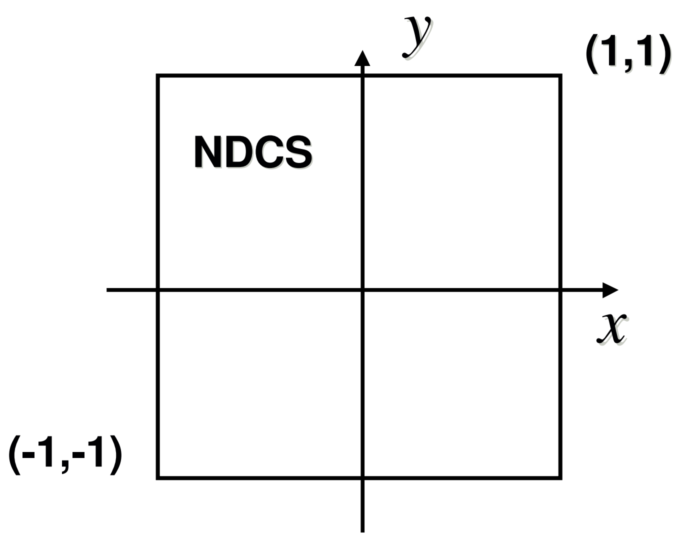
OpenGL Perspective Matrix

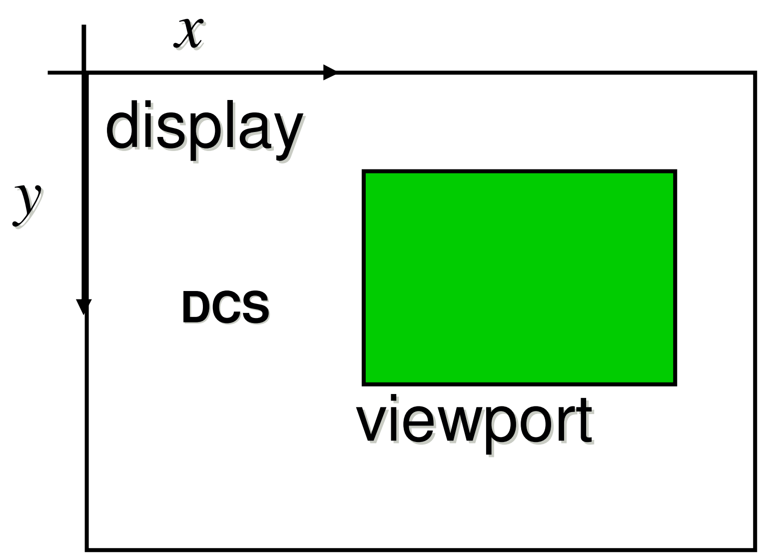
- shear, scale, reflect for new coord sys
  - understand derivation!

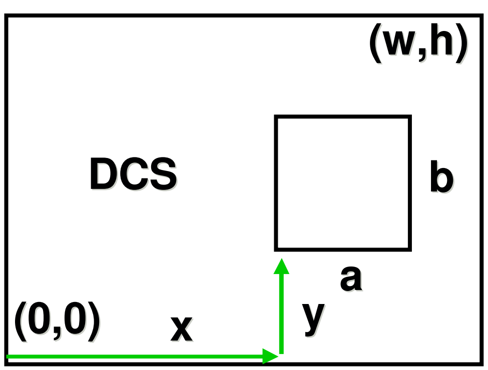
$$\begin{bmatrix} \frac{2 \cdot near}{right - left} & 0 & \frac{right + left}{right - left} & 0 \\ 0 & \frac{2 \cdot near}{top - bot} & \frac{top + bot}{top - bot} & 0 \\ 0 & 0 & \frac{-(far + near)}{far - near} & \frac{-2 \cdot far \cdot near}{far - near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

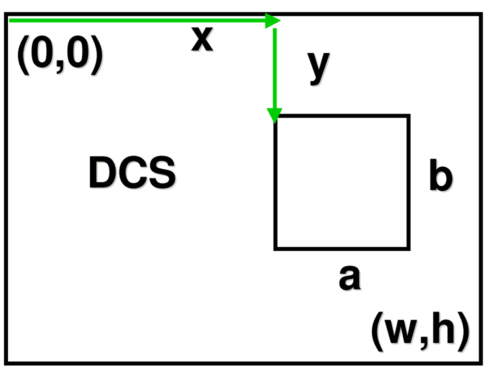
Viewport Transformation

onscreen pixels: map from [-1,1] to [0, displaywidth]









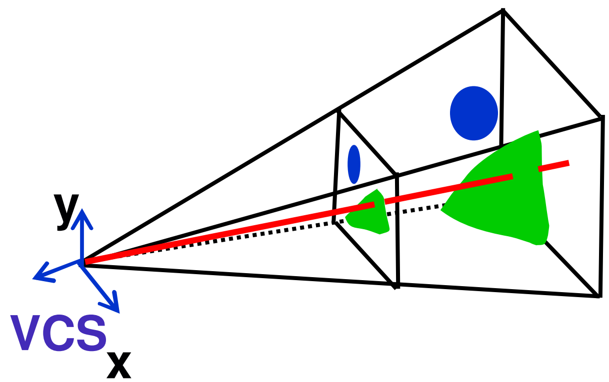
$$x_{DCS} = w\frac{(x_{NDCS}+1)}{2}$$

$$y_{DCS} = h\frac{(y_{NDCS}+1)}{2}$$

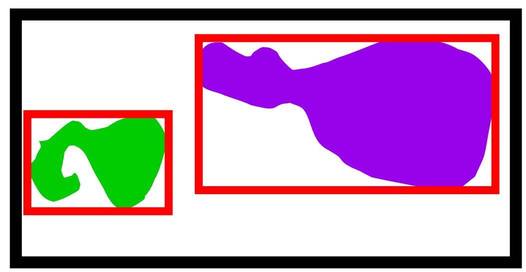
$$z_{DCS} = \frac{(z_{NDCS}+1)}{2}$$

3 Simple Picking Approaches

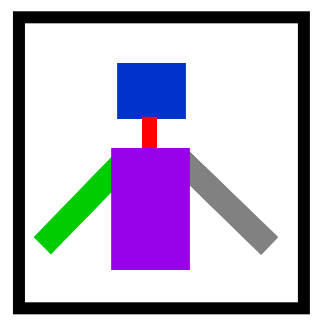
- manual ray intersection



- bounding extents

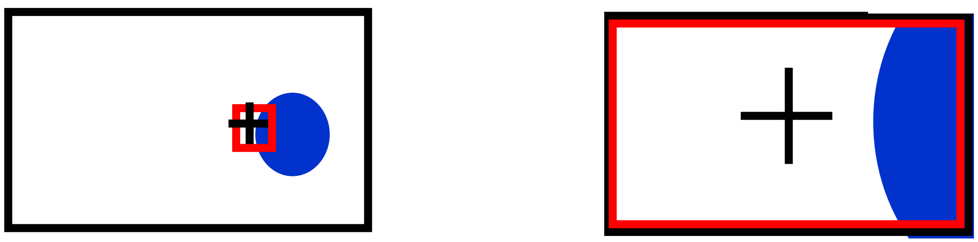


- backbuffer coloring



Picking Select/Hit

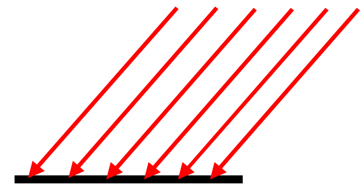
- assign (hierarchical) integer key/name(s)
- small region around cursor as new viewport

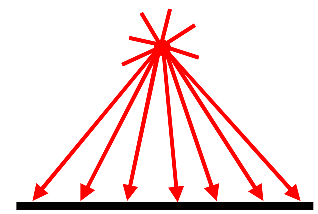


- redraw in selection mode
  - equivalent to casting pick "tube"
  - store keys, depth for drawn objects in hit list
- examine hit list
  - usually use frontmost, but up to application

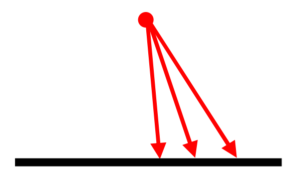
Light Sources

- directional/parallel lights
  - point at infinity: $(x,y,z,0)^T$

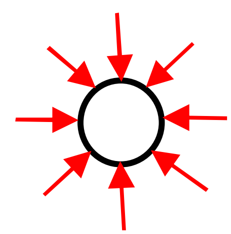




- point lights
  - finite position: $(x,y,z,1)^T$



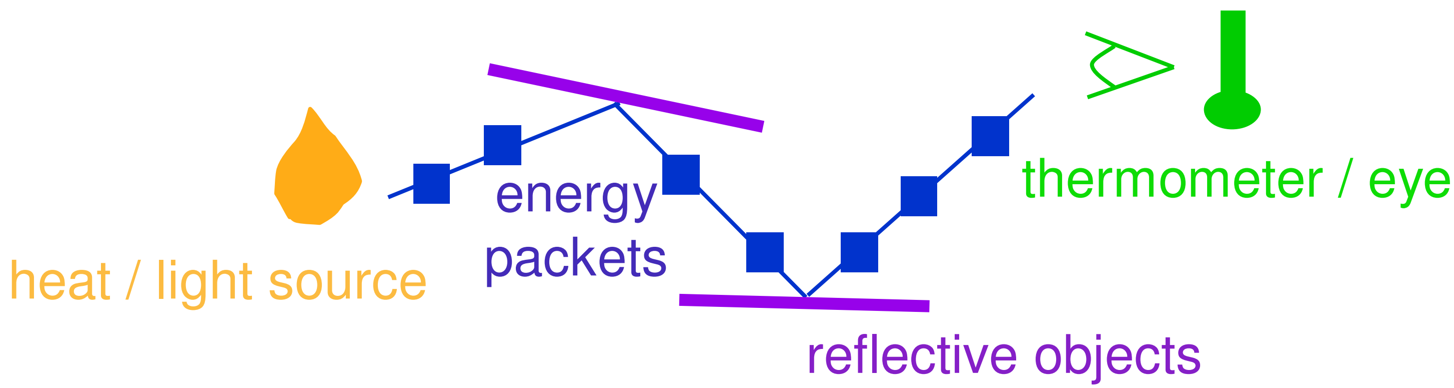
- spotlights
  - position, direction, angle



- ambient lights

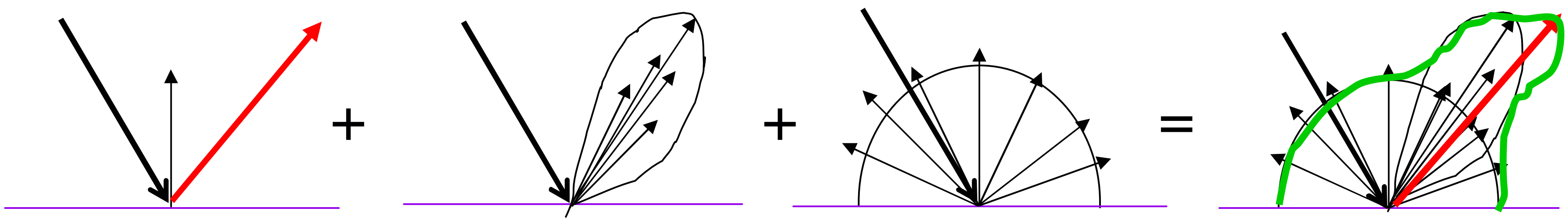
Illumination as Radiative Transfer

– model light transport as packet flow
  - particles not waves



Reflectance

- *specular*: perfect mirror with no scattering
- *gloss*: mixed, partial specularity
- *diffuse*: all directions with equal energy

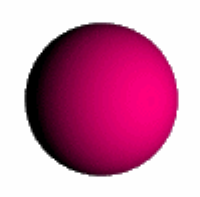


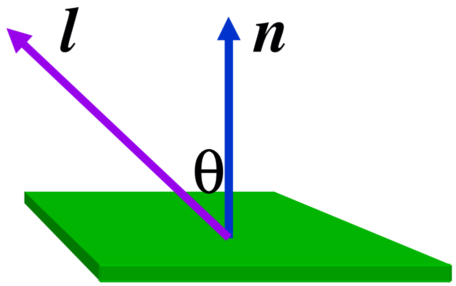
specular + glossy + diffuse =

reflectance distribution

Reflection Equations

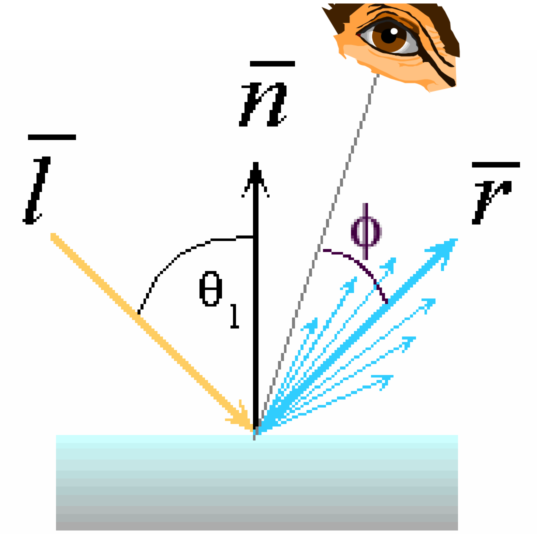
$$I_{diffuse} = k_d \, I_{light} \, (\mathbf{n} \bullet \mathbf{l})$$





$$I_{specular} = k_s I_{light} \, (\mathbf{v} \bullet \mathbf{r})^{n_{shiny}}$$

$$2\,(\mathbf{N}\,(\mathbf{N} \cdot \mathbf{L})) - \mathbf{L} = \mathbf{R}$$

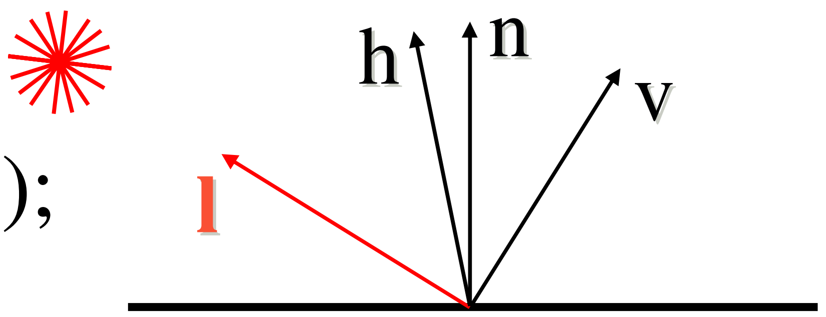


Reflection Equations

- Blinn improvement

$$I_{out}(\mathbf{x}) = k_s \cdot (\mathbf{h} \cdot \mathbf{n})^{n_{shiny}} \cdot I_{in}(\mathbf{x});$$

$$\mathbf{h} = (\mathbf{l} + \mathbf{v}) / 2$$



- full Phong lighting model
  - combine ambient, diffuse, specular components

$$I_{total} = k_a I_{ambient} + \sum_{i=1}^{\#lights} I_i \left(k_d \left(\mathbf{n} \times \mathbf{l_i} \right) + k_s \left(\mathbf{v} \times \mathbf{r_i} \right)^{n_{shiny}} \right)$$

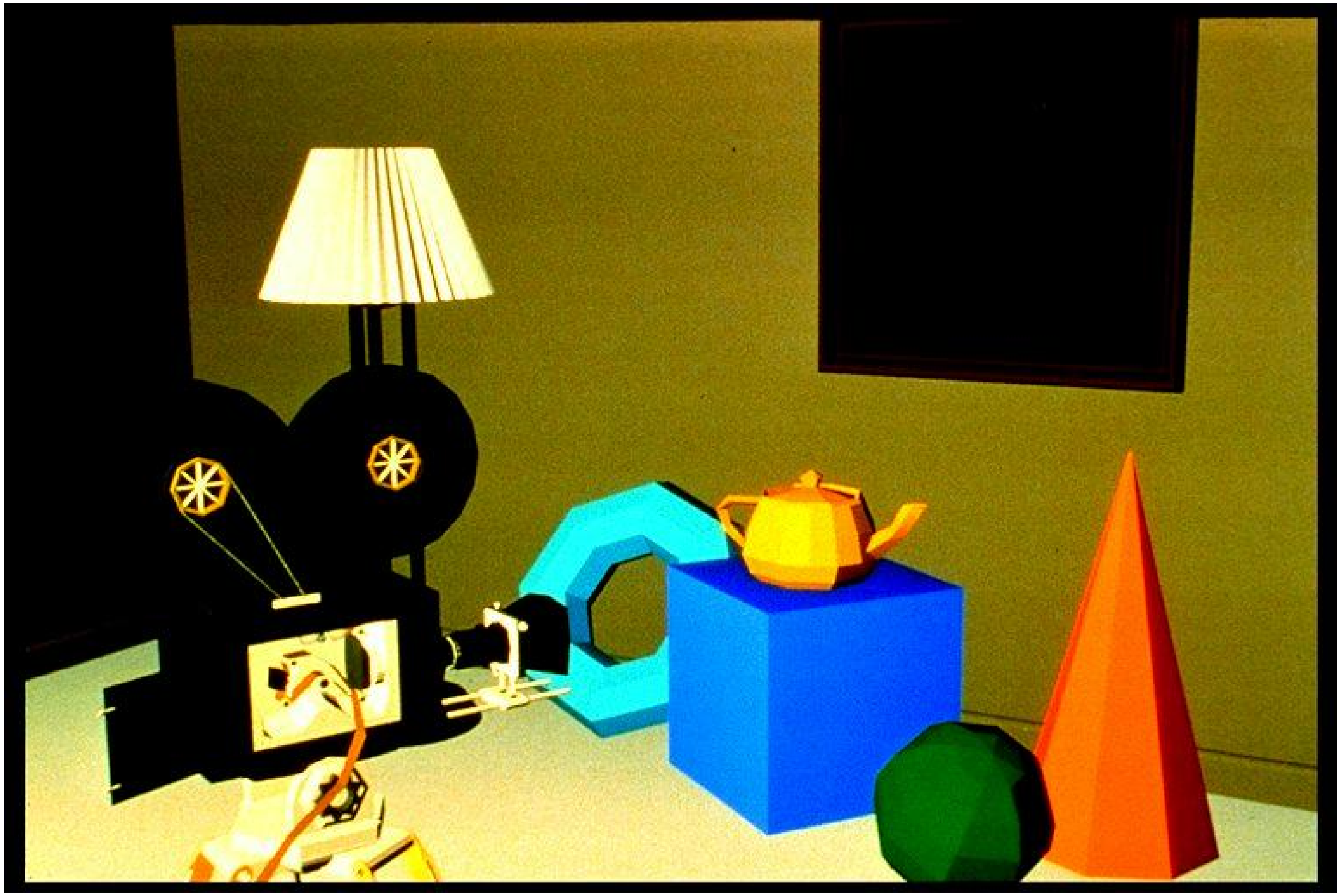
Lighting vs. Shading

- **lighting**
  - simulating the interaction of light with surface
- **shading**
  - deciding pixel color
  - continuum of realism: when do we do lighting calculation?

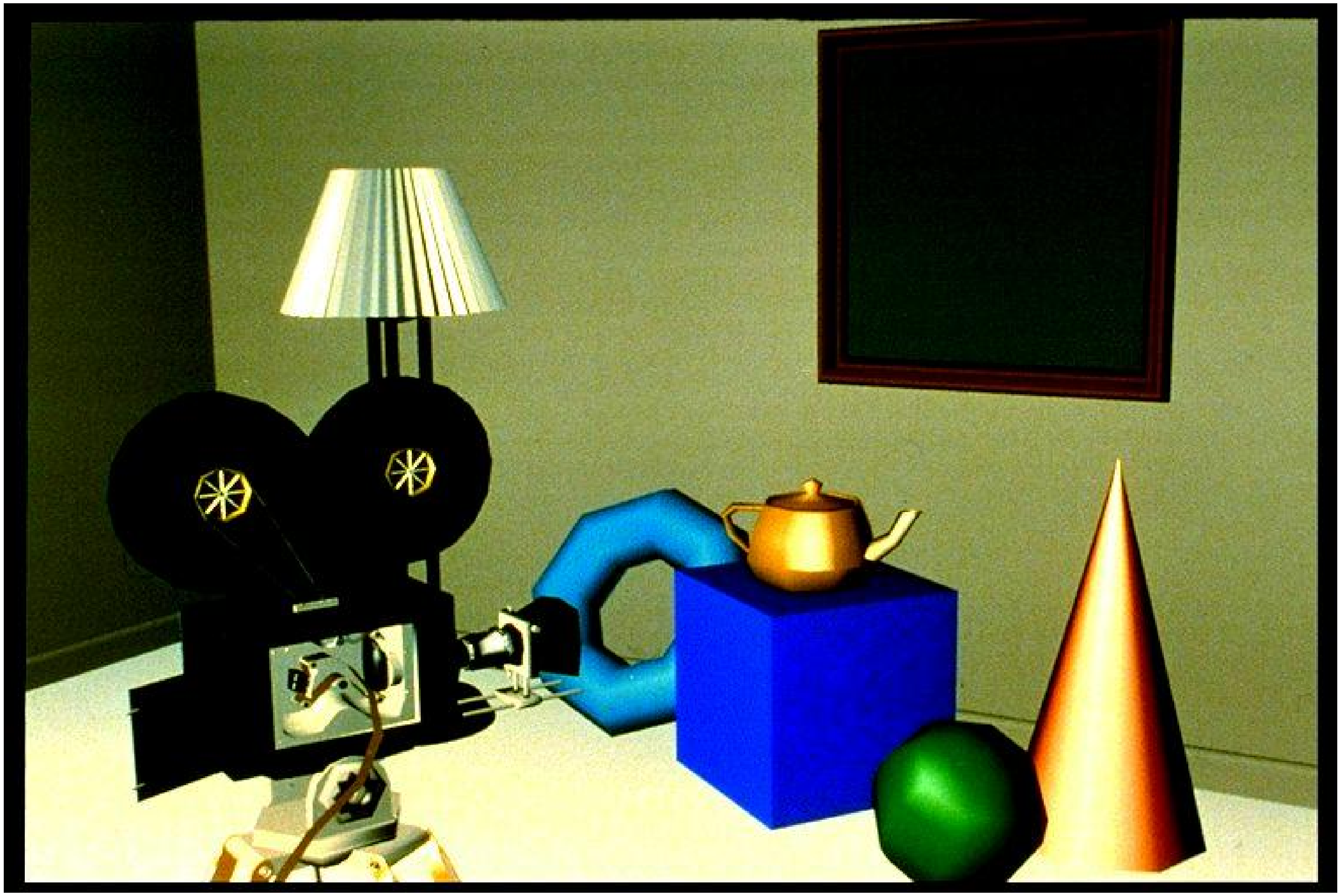
Shading Models

- flat shading
  - compute Phong lighting once for entire polygon
- Gouraud shading
  - compute Phong lighting at the vertices and interpolate lighting values across polygon
- Phong shading
  - compute averaged vertex normals
  - interpolate normals across polygon and perform Phong lighting across polygon

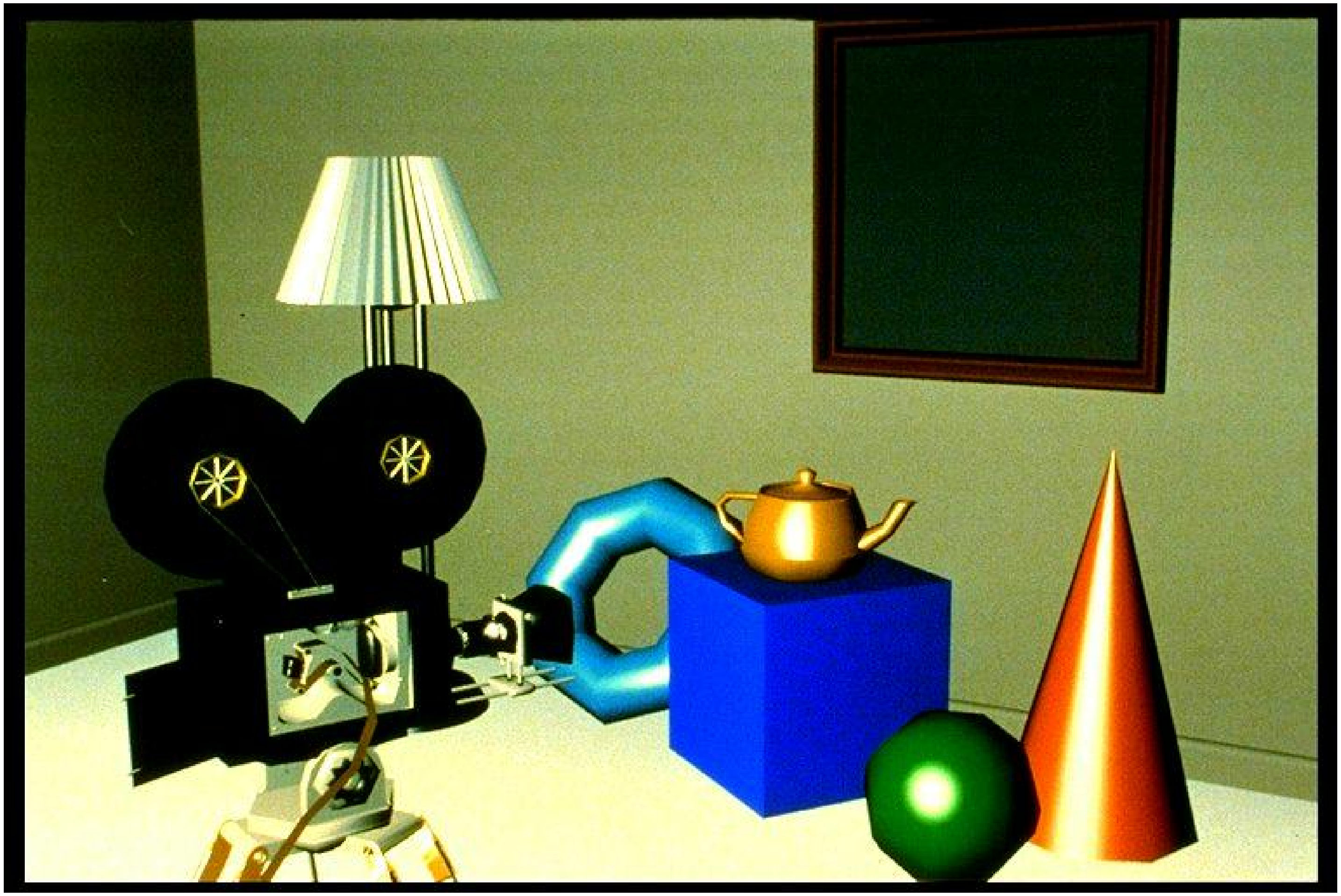
Shutterbug: Flat Shading



Shutterbug: Gouraud Shading



Shutterbug: Phong Shading

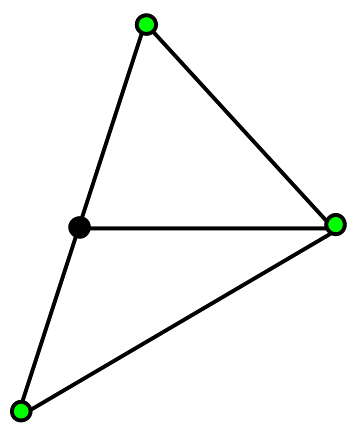


Scanline Algorithms

- given vertices, fill in the pixels

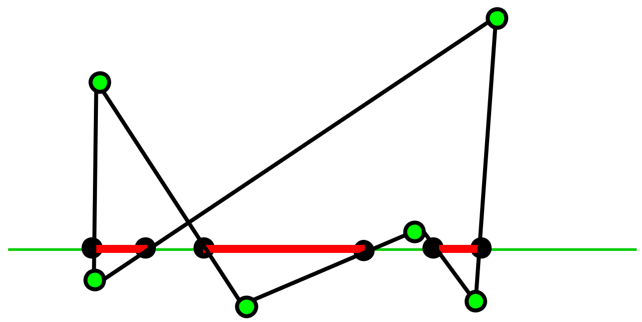
triangles

- split into two regions
- fill in between edges



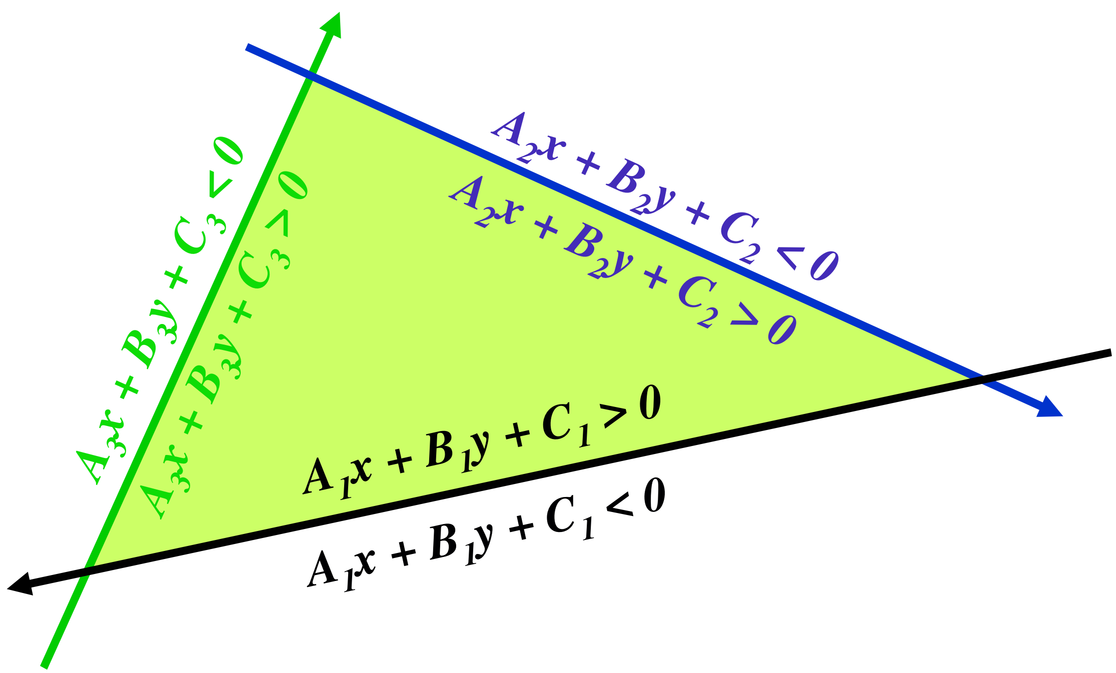
arbitrary polygons (non-simple, non-convex)

- build edge table
- for each scanline
  - obtain list of intersections, i.e., AEL
  - use parity test to determine in/out and fill in the pixels



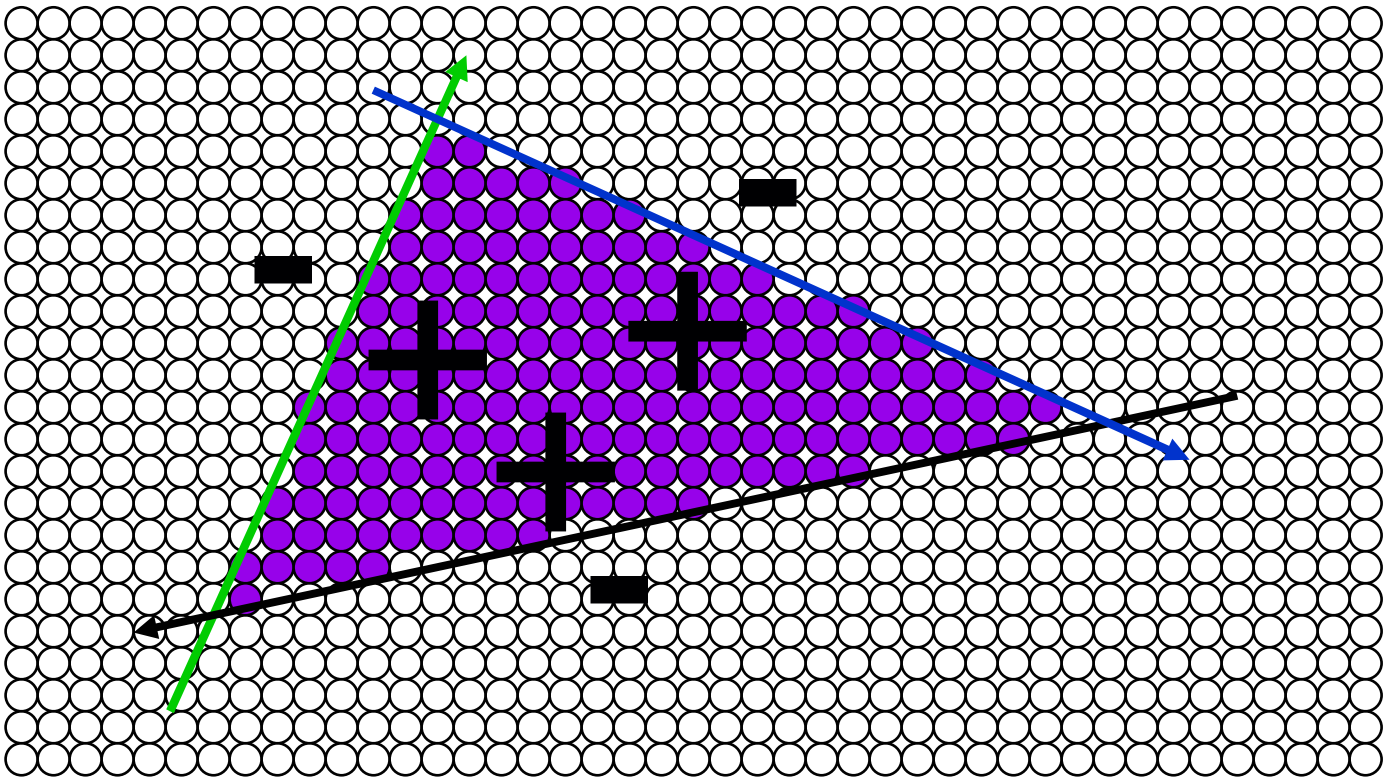
Edge Equations

- define triangle as intersection of three positive half-spaces:



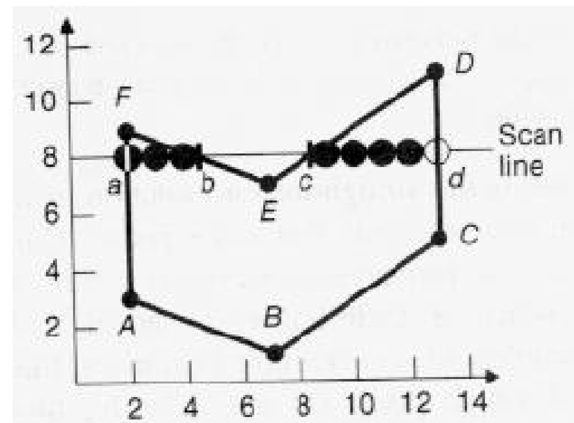
Edge Equations

- So…simply turn on those pixels for which all edge equations evaluate to > 0:



Parity for General Case

- use parity for interior test
  - draw pixel if edgecount odd
  - horizontal lines: count
  - vertical max: count
  - vertical min: don't count



Edge Tables

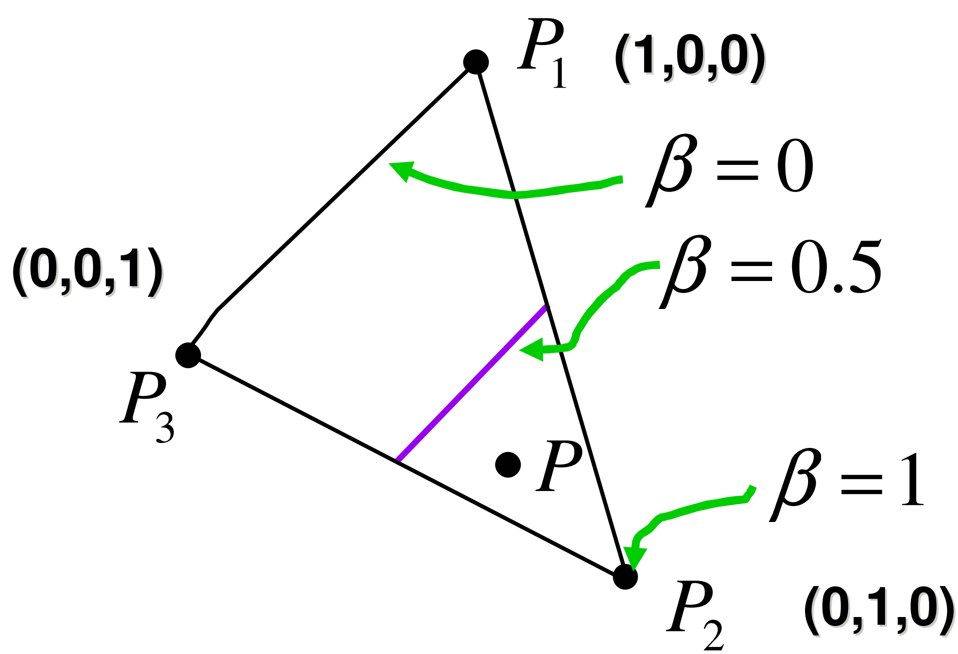
- edge table (ET)
  – store edges sorted by y in linked list
    - at ymin, store ymax, xmin, slope
- active edge table (AET)
  – active: currently used for computation
  – store active edges sorted by x
    - update each scanline, store ET values + current_x
  – for each scanline (from bottom to top)
    - do EAT bookkeeping
    - traverse EAT (from leftmost x to rightmost x)
      – draw pixels if parity odd

Barycentric Coordinates

- weighted combination of vertices
  - understand derivation!

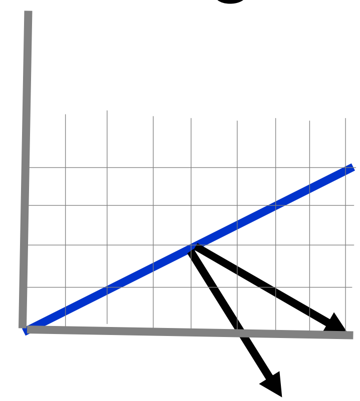
$$\begin{cases} P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3 \\ \alpha + \beta + \gamma = 1 \\ 0 \le \alpha, \beta, \gamma \le 1 \end{cases}$$

"convex combination of points"



Transforming Normals

- apply nonuniform scale: stretch along x by 2
  - can't transform normal by modelling matrix



- solution:

$$P \qquad\qquad P' = MP$$
$$N \longrightarrow N' = QN$$

$$\boxed{Q = \left(M^{-1}\right)^T}$$

normal to any surface transformed by inverse transpose of modelling transformation