



University of British Columbia
CPSC 414 Computer Graphics

Midterm Review

Week 7, Wed 16 Oct 2003

- midterm review
- project 1 demos, hall of fame

News

- homework 1 due now
 - one day late if in handin box 18 by 9am Thu
 - two days late if in at class beginning Fri
 - no homeworks accepted after Fri 9am!
 - solutions out then

Midterm Exam

- Monday Oct 20 9am-9:50am
 - you may use one handwritten 8.5"x11" sheet
 - OK to use both sides of page
 - no other notes, no books
 - nonprogrammable calculators OK
 - arrive on time!!

What's Covered

- transformations
- viewing and projections
- coordinate systems of rendering pipeline
- picking
- lighting and shading
- scan conversion

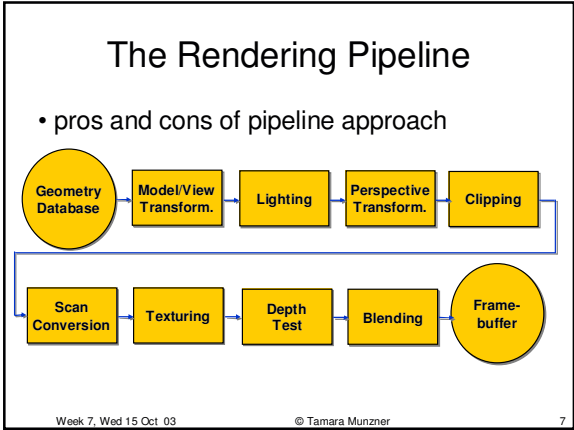
- **not** sampling

Reading

- Angel book
 - Chap 1, 2, 3, 4, 5, 6, 8.9-8.11, 9.1-9.6
 - you can be tested on material in book but not covered in lecture
 - you can be tested on material covered in lecture but not covered in book

Old Exams Posted

- see course web page



Transformations

translate(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & a & 0 & x \\ 0 & 1 & b & y \\ 0 & 0 & 1 & c & z \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

scale(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & x \\ 0 & b & 0 & y \\ 0 & 0 & c & z \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Rotate(x, θ)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & \cos \theta & -\sin \theta & y \\ 0 & \sin \theta & \cos \theta & z \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

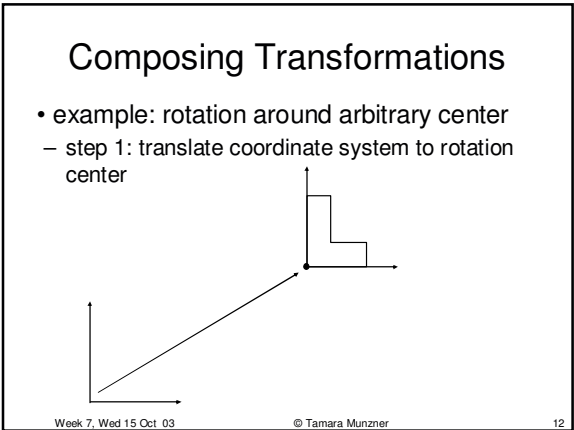
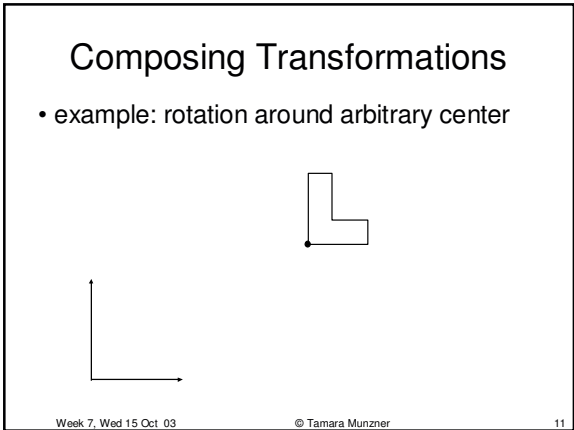
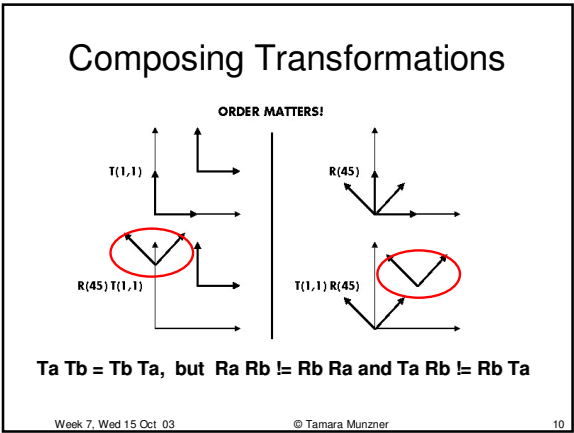
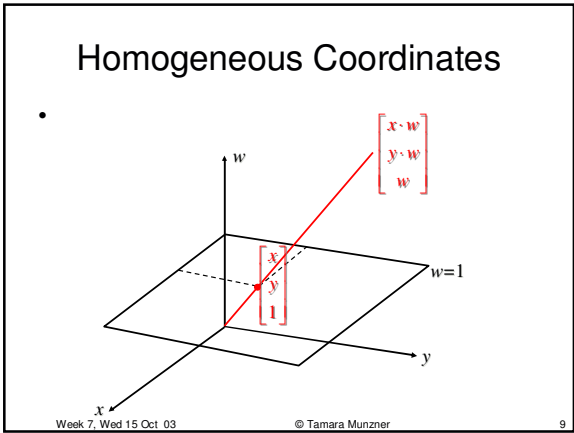
Rotate(y, θ)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & 0 & x \\ 0 & 1 & 0 & y \\ -\sin \theta & 0 & 0 & z \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Rotate(z, θ)

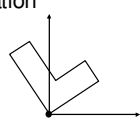
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & x \\ \sin \theta & \cos \theta & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

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Composing Transformations

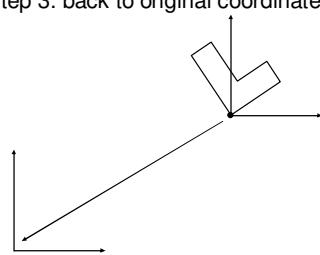
- example: rotation around arbitrary center
- step 2: perform rotation



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Composing Transformations

- example: rotation around arbitrary center
- step 3: back to original coordinate system



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Composing Transformations

- rotation about a fixed point
 $p' = TRT^{-1}p$
- rotation around an arbitrary axis
- considering frame vs. object

frame

→

$p' = DCBAp$

←

object

OpenGL:

D

C

B

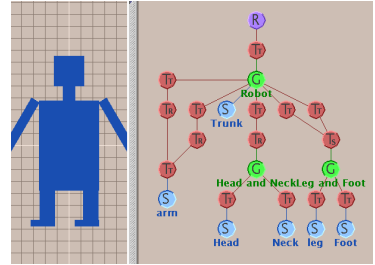
A

draw p

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Transformation Hierarchies

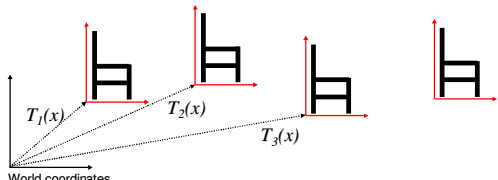
- hierarchies don't fall apart when changed
- transforms apply to graph nodes beneath



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Matrix Stacks

- push and pop matrix stack
- avoid computing inverses or incremental xforms
- avoid numerical error



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Matrix Stacks

`glPushMatrix()`

`glPopMatrix()`

$D = C \text{ scale}(2,2,2) \text{ trans}(1,0,0)$

C
C
B
A

C
C
B
A

D
C
B
A

C
B
A

`DrawSquare()`

`glPushMatrix()`

`glScale3f(2,2,2)`

`glTranslate3f(1,0,0)`

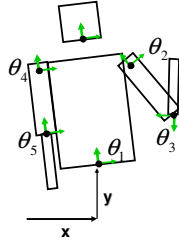
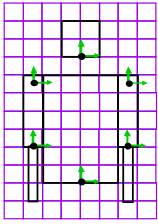
`DrawSquare()`

`glPopMatrix()`

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Transformation Hierarchies

- example



```

glTranslatef(x,y,0);
glRotatef(theta_1,0,0,1);
DrawBody();
glPushMatrix();
glTranslatef(0,7,0);
DrawHead();
glPopMatrix();
glPushMatrix();
glTranslatef(2.5,5.5,0);
glRotatef(theta_2,0,0,1);
DrawUArm();
glTranslatef(0,-3.5,0);
glRotatef(theta_3,0,0,1);
DrawLArm();
glPopMatrix();
... (draw other arm)
    
```

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Display Lists

- reuse block of OpenGL code
- more efficient than immediate mode
 - code reuse, driver optimization
- good for static objects redrawn often
 - can't change contents
 - not just for multiple instances
 - interactive graphics: objects redrawn every frame
- nest when possible for efficiency

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Double Buffering

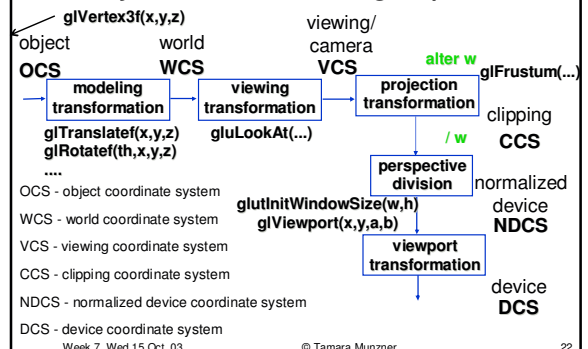
- two buffers, front and back
 - while front is on display, draw into back
 - when drawing finished, swap the two
- avoid flicker

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Projective Rendering Pipeline



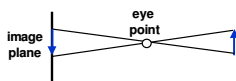
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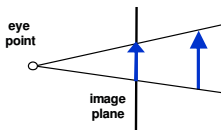
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Projection

- theoretical pinhole camera



- image inverted, more convenient equivalent

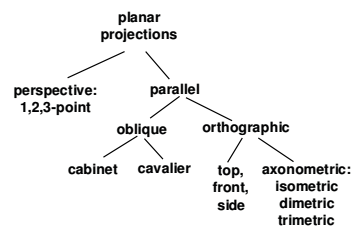


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Projection Taxonomy



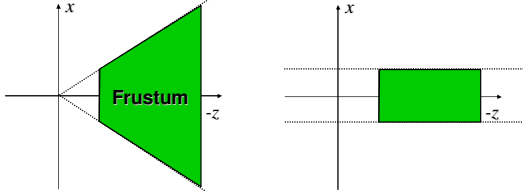
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Projective Transformations

- transformation of space
 - center of projection moves to infinity
 - viewing frustum transformed into a parallelepiped



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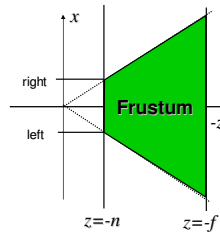
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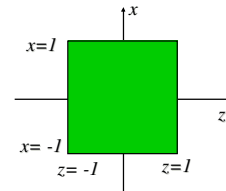
Normalized Device Coordinates

left/right $x = +/- 1$, top/bottom $y = +/- 1$, near/far $z = +/- 1$

Camera coordinates



NDC



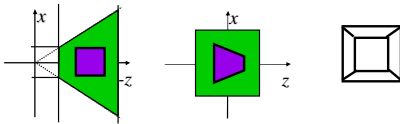
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Projection Normalization

- distort such that orthographic projection of distorted objects is desired persp projection



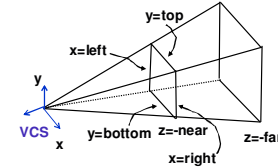
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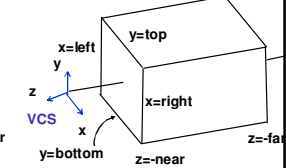
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Transforming View Volumes

perspective view volume



orthographic view volume



NDCS

(-1,-1,-1)

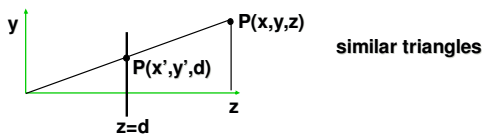
(1,1,1)

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Basic Perspective Projection



$$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z} \quad \text{also} \quad x' = \frac{x \cdot d}{z} \quad \text{but} \quad z' = d$$

- nonuniform foreshortening
 - not affine

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Basic Perspective Projection

- can express as homogenous 4x4 matrix!

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \xrightarrow{f_w} \begin{bmatrix} x \cdot d / z \\ y \cdot d / z \\ d \\ d \end{bmatrix}$$

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Projective Transformations

- determining the matrix representation
 - need to observe 5 points in general position, e.g.
 - $[left, 0, 0, 1]^T \rightarrow [-1, 0, 0, 1]^T$
 - $[0, top, 0, 1]^T \rightarrow [0, 1, 0, 1]^T$
 - $[0, 0, -f, 1]^T \rightarrow [0, 0, 1, 1]^T$
 - $[0, 0, -n, 1]^T \rightarrow [0, 0, -1, 1]^T$
 - $[left * f/n, top * f/n, -f, 1]^T \rightarrow [-1, 1, 1, 1]^T$
- solve resulting equation system to obtain matrix

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OpenGL Orthographic Matrix

- scale, translate, reflect for new coord sys
 - understand derivation!

$$\begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bot} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \frac{-2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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OpenGL Perspective Matrix

- shear, scale, reflect for new coord sys
 - understand derivation!

$$\begin{bmatrix} \frac{2 \cdot near}{right - left} & 0 & \frac{right + left}{right - left} & 0 \\ 0 & \frac{2 \cdot near}{top - bot} & \frac{top + bot}{top - bot} & 0 \\ 0 & 0 & \frac{-(far + near)}{far - near} & \frac{-2 \cdot far \cdot near}{far - near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

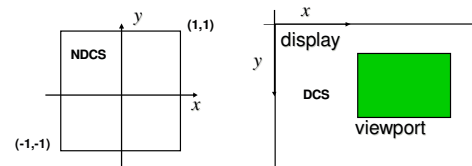
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Viewport Transformation

onscreen pixels: map from $[-1, 1]$ to $[0, displaywidth]$



$$\begin{aligned} x_{DCS} &= w \frac{(x_{NDCS} + 1)}{2} \\ y_{DCS} &= h \frac{(y_{NDCS} + 1)}{2} \\ z_{DCS} &= \frac{(z_{NDCS} + 1)}{2} \end{aligned}$$

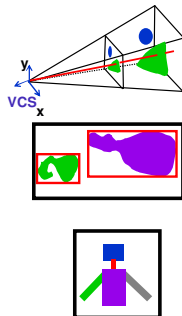
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3 Simple Picking Approaches

- manual ray intersection
 - bounding extents
 - backbuffer coloring



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Picking Select/Hit

- assign (hierarchical) integer key/name(s)
- small region around cursor as new viewport



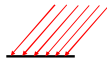



- redraw in selection mode
 - equivalent to casting pick "tube"
 - store keys, depth for drawn objects in hit list
- examine hit list
 - usually use frontmost, but up to application

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Light Sources

- **directional/parallel lights**
 - point at infinity: $(x,y,z,0)^T$
- **point lights**
 - finite position: $(x,y,z,1)^T$
- **spotlights**
 - position, direction, angle
- **ambient lights**


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Illumination as Radiative Transfer

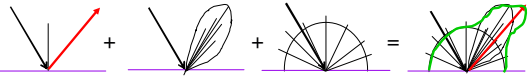
– model light transport as packet flow

- particles not waves

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Reflectance

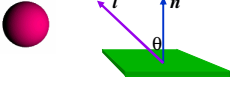
- *specular*: perfect mirror with no scattering
- *gloss*: mixed, partial specularity
- *diffuse*: all directions with equal energy



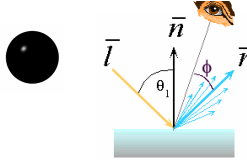
specular + glossy + diffuse =
reflectance distribution

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Reflection Equations

$$I_{\text{diffuse}} = k_d I_{\text{light}} (\mathbf{n} \cdot \mathbf{l})$$


$$I_{\text{specular}} = k_s I_{\text{light}} (\mathbf{v} \cdot \mathbf{r})^{n_{\text{shiny}}}$$

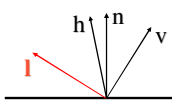
$$2(\mathbf{N} \cdot \mathbf{L}) - \mathbf{L} = \mathbf{R}$$


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Reflection Equations

- Blinn improvement

$$I_{\text{out}}(\mathbf{x}) = k_s \cdot (\mathbf{h} \cdot \mathbf{n})^{n_{\text{shiny}}} \cdot I_{\text{in}}(\mathbf{x});$$

$$\mathbf{h} = (\mathbf{l} + \mathbf{v}) / 2$$

- full Phong lighting model
 - combine ambient, diffuse, specular components
$$I_{\text{total}} = k_a I_{\text{ambient}} + \sum_{i=1}^{\#lights} I_i (k_d (\mathbf{n} \cdot \mathbf{l}_i) + k_s (\mathbf{v} \cdot \mathbf{r}_i)^{n_{\text{shiny}}})$$

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Lighting vs. Shading

- **lighting**
 - simulating the interaction of light with surface
- **shading**
 - deciding pixel color
 - continuum of realism: when do we do lighting calculation?

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Shading Models

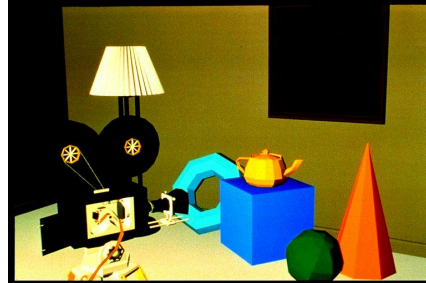
- flat shading
 - compute Phong lighting once for entire polygon
- Gouraud shading
 - compute Phong lighting at the vertices and interpolate lighting values across polygon
- Phong shading
 - compute averaged vertex normals
 - interpolate normals across polygon and perform Phong lighting across polygon

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Shutterbug: Flat Shading

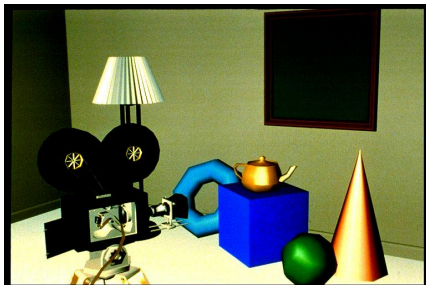


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Shutterbug: Gouraud Shading



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Shutterbug: Phong Shading



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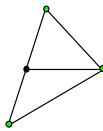
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Scanline Algorithms

- given vertices, fill in the pixels

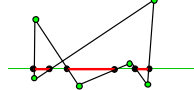
triangles

- split into two regions
- fill in between edges



arbitrary polygons (non-simple, non-convex)

- build edge table
- for each scanline
 - obtain list of intersections, i.e., AEL
 - use parity test to determine in/out and fill in the pixels



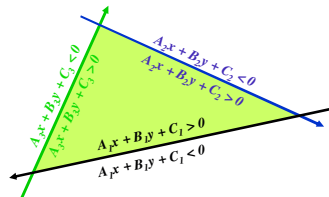
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Edge Equations

- define triangle as intersection of three positive half-spaces:



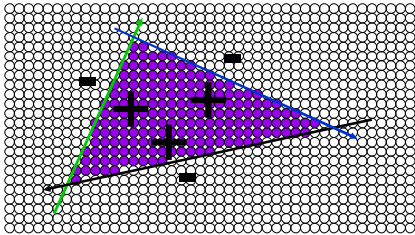
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Edge Equations

- So...simply turn on those pixels for which all edge equations evaluate to > 0 :



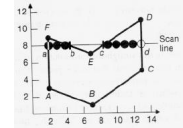
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Parity for General Case

- use parity for interior test
 - draw pixel if edgecount odd
 - horizontal lines: count
 - vertical max: count
 - vertical min: don't count



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Edge Tables

- edge table (ET)
 - store edges sorted by y in linked list
 - at ymin, store ymax, xmin, slope
- active edge table (AET)
 - active: currently used for computation
 - store active edges sorted by x
 - update each scanline, store ET values + current_x
 - for each scanline (from bottom to top)
 - do EAT bookkeeping
 - traverse EAT (from leftmost x to rightmost x)
 - draw pixels if parity odd

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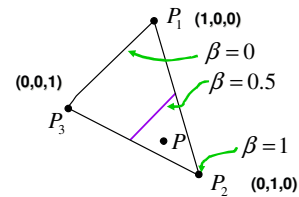
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Barycentric Coordinates

- weighted combination of vertices
 - understand derivation!

$$\begin{cases} P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3 \\ \alpha + \beta + \gamma = 1 \\ 0 \leq \alpha, \beta, \gamma \leq 1 \end{cases}$$

"convex combination of points"



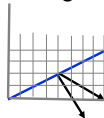
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Transforming Normals

- apply nonuniform scale: stretch along x by 2
 - can't transform normal by modelling matrix



- solution:

$$\begin{matrix} P \\ N \end{matrix} \longrightarrow \begin{matrix} P' = MP \\ N' = QN \end{matrix}$$

$$Q = (M^{-1})^T$$

normal to any surface transformed by inverse transpose of modelling transformation

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