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CPSC 414 Computer Graphics

Scan Conversion

Week 6, Wed 8 Oct 2003

- recap: polygon scan conversion
- interpolation
- barycentric coords
- transforming normals
- sampling

News

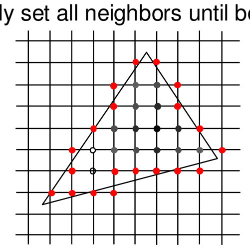
- Homework 1
  - problem 18
    - **x and y transposed** in bottom layer
  - problem 4
    - rotate 30 deg **around x axis** with fixed point of (3,5,12,1)
  - new correct PDF posted
  - reminder: no late work after Fri 17 Oct 9am
    - handin box 18 CICSR basement
- Project 1
  - solution, hall of fame on Friday

News

- Office hours reminder: FSC 2618
  - Mondays 10:30-11:30 or by appointment
  - exceptions: Oct 20, Nov 10
- Readings
  - Chap 8.9-8.11, Fri 10/3 slide notes

Simple Polygon Scanconversion

- flood fill: simple, slow
  - start with *seed point*
  - recursively set all neighbors until boundary is hit

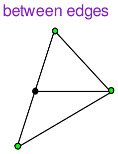


Scanline Algorithms

- given vertices, fill in the pixels

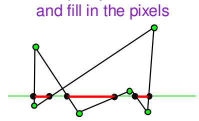
triangles

- split into two regions
- fill in between edges



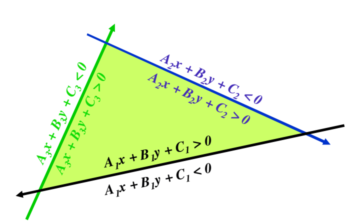
arbitrary polygons (non-simple, non-convex)

- build edge table
- for each scanline
  - obtain list of intersections, i.e., AEL
  - use parity test to determine in/out and fill in the pixels



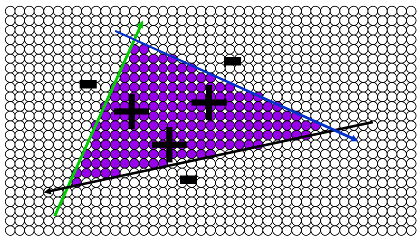
Edge Equations

- define triangle as intersection of three positive half-spaces:



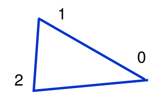
Edge Equations

- So…simply turn on those pixels for which all edge equations evaluate to > 0:



Edge Equation Consistency

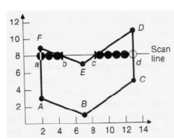
- how to get same +/- state for all equations?
  - consistent counterclockwise vertex traversal
    - good: [0,1], [1,2], [2,0] or [0,2], [2,1], [1,0]
    - bad: [0,1], [2,1], [2,0]



- how to ensure interior is positive?
  - explicit area test: if negative, flip all param signs
    - if (area < 0) { A = -A; B = -B; C = -C; }

Parity for General Case

- use parity for interior test
  - draw pixel if edgecount odd
  - horizontal lines: don't count
  - vertical max: don't count
  - vertical min: count



Edge Tables

- edge table (ET)
  - store edges sorted by y in linked list
    - at ymin, store ymax, xmin, slope
- active edge table (AET)
  - active: currently used for computation
  - store active edges sorted by x
    - update each scanline, store ET values + current_x
  - for each scanline (from bottom to top)
    - do EAT bookkeeping
    - traverse EAT (from leftmost x to rightmost x)
      - draw pixels if parity odd

Edge Table Bookkeeping

- setup: sorting in y
  - bucket sort, one bucket per pixel
  - add: simple check of ET[current_y]
  - delete edges if edge.ymax > current_y
- main loop: sorting in x
  - for polygons that do not self-intersect, order of edges does not change between two scanlines
  - so insertion sort while adding new edges suffices

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Interpolation

Scan Conversion

- done:
  - how to determine pixels covered by a primitive
- next:
  - how to assign pixel colors
    - interpolation of colors across triangles
    - interpolation of other properties

Interpolation During Scanconvert

- interpolate values between vertices
  - z values
  - r,g,b colour components
  - u,v texture coordinates
  - N_x, N_y, N_z surface normals
- three equivalent methods (for triangles)
  1. bilinear interpolation
  2. plane equation
  3. barycentric coordinates

1. Bilinear Interpolation

- interpolate quantity along left-hand and right-hand edges, as a function of y
  - then interpolate quantity as a function of x
- only triangles guarantee orientation-independent interpolation
- compute efficiently by using known values at previous scanline, previous pixel

2. Plane Equation

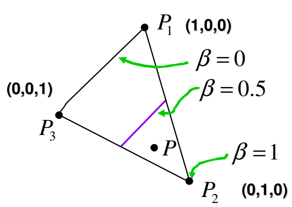
- implicit plane equation
  - z = f(x,y)
- parametric plane equation
  - $a(x-x_0)+ b(y-y_0)+c(z-z_0) = 0$
- explicit plane equation
  - $Plane = A\cdot x + B\cdot y + C\cdot z + D$

3. Barycentric Coordinates

- weighted combination of vertices

$$\begin{cases} P = \alpha\cdot P_1 + \beta\cdot P_2 + \gamma\cdot P_3 \\ \alpha+\beta+\gamma = 1 \\ 0 \le \alpha,\beta,\gamma \le 1 \end{cases}$$

"convex combination of points"



Barycentric Coordinates

- how to compute α,β,γ ?
  - use bilinear interpolation or plane equations

interpolate α,β,γ just like we did for z

$$\alpha = a\cdot x + b\cdot y + c\cdot z + d$$
$$\beta = \ldots$$

  - once computed, use to interpolate any # of parameters from their vertex values

$$z = \alpha\cdot z_1 + \beta\cdot z_2 + \gamma\cdot z_3$$
$$r = \alpha\cdot r_1 + \beta\cdot r_2 + \gamma\cdot r_3$$
$$g = \alpha\cdot g_1 + \beta\cdot g_2 + \gamma\cdot g_3$$

etc.

Interpolatation: Gouraud Shading

- need linear function over triangle that yields original vertex colors at vertices
- use barycentric coordinates for this
  - every pixel in interior gets colors resulting from mixing colors of vertices with weights corresponding to barycentric coordinates
  - color at pixels is affine combination of colors at vertices

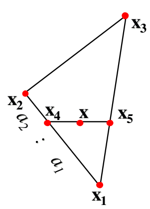
$$Color(\alpha \cdot \mathbf{x}_1 + \beta \cdot \mathbf{x}_2 + \gamma \cdot \mathbf{x}_3) :=$$
$$\alpha \cdot Color(\mathbf{x}_1) + \beta \cdot Color(\mathbf{x}_2) + \gamma \cdot Color(\mathbf{x}_3)$$

Interpolatation: Gouraud Shading

- we know
  - affine combinatons are invariant under affine transformations
- thus
  - does not matter whether colors are interpolated *before* or after *affine* transformations!
  - colors do not shift around on the surface with affine transformations, but stay attached to every surface point

Computing Barycentric Coords

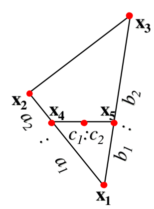
- how do we find barycentric coordinates for every pixel efficiently?
  - look at a point **x** on a scanline:



$$\mathbf{x}_4 = \mathbf{x}_1 + \frac{a_1}{a_1 + a_2}(\mathbf{x}_2 - \mathbf{x}_1)$$
$$= (1 - \frac{a_1}{a_1 + a_2}) \cdot \mathbf{x}_1 + \frac{a_1}{a_1 + a_2} \cdot \mathbf{x}_2$$
$$= \frac{a_2}{a_1 + a_2} \cdot \mathbf{x}_1 + \frac{a_1}{a_1 + a_2} \cdot \mathbf{x}_2$$

Computing Barycentric Coords

- similarly:



$$\mathbf{x}_5 = \frac{b_2}{b_1 + b_2} \cdot \mathbf{x}_1 + \frac{b_1}{b_1 + b_2} \cdot \mathbf{x}_3$$

$$\mathbf{x} = \frac{c_2}{c_1 + c_2} \cdot \mathbf{x}_4 + \frac{c_1}{c_1 + c_2} \cdot \mathbf{x}_5$$

Computing Barycentric Coords

combining

$$\mathbf{x} = \frac{c_2}{c_1 + c_2} \cdot \mathbf{x}_4 + \frac{c_1}{c_1 + c_2} \cdot \mathbf{x}_5 \qquad \mathbf{x}_4 = \frac{a_2}{a_1 + a_2} \cdot \mathbf{x}_1 + \frac{a_1}{a_1 + a_2} \cdot \mathbf{x}_2$$

$$\mathbf{x}_5 = \frac{b_2}{b_1 + b_2} \cdot \mathbf{x}_1 + \frac{b_1}{b_1 + b_2} \cdot \mathbf{x}_3$$

gives

$$\mathbf{x} = \frac{c_2}{c_1 + c_2} \cdot \left(\frac{a_2}{a_1 + a_2} \cdot \mathbf{x}_1 + \frac{a_1}{a_1 + a_2} \cdot \mathbf{x}_2 \right) + \frac{c_1}{c_1 + c_2} \cdot \left(\frac{b_2}{b_1 + b_2} \cdot \mathbf{x}_1 + \frac{b_1}{b_1 + b_2} \cdot \mathbf{x}_3 \right)$$

Computing Barycentric Coords

thus

$$\mathbf{x} = \alpha \cdot \mathbf{x}_1 + \beta \cdot \mathbf{x}_2 + \gamma \cdot \mathbf{x}_3$$

$$\text{with } \alpha = \frac{c_2}{c_1 + c_2} \cdot \frac{a_2}{a_1 + a_2} + \frac{c_1}{c_1 + c_2} \cdot \frac{b_2}{b_1 + b_2},$$

$$\beta = \frac{c_2}{c_1 + c_2} \cdot \frac{a_1}{a_1 + a_2}$$

$$\gamma = \frac{c_1}{c_1 + c_2} \cdot \frac{b_1}{b_1 + b_2}$$

Computing Barycentric Coords

- can prove correct by verifying barycentric properties
  - $\alpha + \beta + \gamma = 1$
  - $0 \leq \alpha, \beta, \gamma \leq 1$

Gouraud Shading with Bary Coords

- algorithm
  - modify scanline algorithm for polygon scan-conversion as follows:
    - linearly interpolate colors along edges of triangle to obtain colors for endpoints of span of pixels
    - linearly interpolate colors from these endpoints within the scanline

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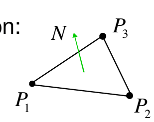
Transforming Normals

Interpolation During Scanconvert

- interpolate values between vertices
  - z values
  - r,g,b colour components
  - u,v texture coordinates
  - N_x, N_y, N_z surface normals

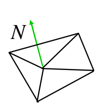
Computing Normals

- polygon:



$$N = (P_2 - P_1) \times (P_3 - P_1)$$

- assume vertices ordered CCW when viewed from visible side of polygon
- normal for a vertex
  - used for lighting
  - supplied by model (i.e., sphere), or computed from neighboring polygons



Transforming Normals

- what is a normal?
  - a **direction**
  - earlier: vector as direction

$$\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

  - so if points transformed by modelview vector M, can we just transform vector by M too?

$$\begin{bmatrix} Nx' \\ Ny' \\ Nz' \\ 0 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & T_x \\ m_{21} & m_{22} & m_{23} & T_y \\ m_{31} & m_{32} & m_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Nx \\ Ny \\ Nz \\ 0 \end{bmatrix}$$

Transforming Normals

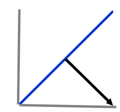
$$\begin{bmatrix} x' \\ y' \\ z' \\ 0 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & T_x \\ m_{21} & m_{22} & m_{23} & T_y \\ m_{31} & m_{32} & m_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

- translations OK: w=0 means unaffected
- rotations OK
- uniform scaling OK

- these all maintain direction

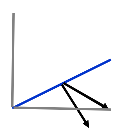
Transforming Normals

- nonuniform scaling does not work
- x-y=0 plane
  - line x=y
  - normal: [1,-1,0]
    - ignore normalization for now



Transforming Normals

- apply nonuniform scale: stretch along x by 2
  - new plane x = 2y
- transformed normal



$$\begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

  - x = -2y or x+2y=0
  - not perpendicular!
  - should be 2x = -y

Planes and Normals

- plane is all points such that $\mathbf{N} \bullet \mathbf{P} = 0$
  or $\mathbf{N}^{\mathrm{T}} \mathbf{P} = 0$ (must transpose for matrix mult!)

$$\mathbf{N} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \mathbf{P} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- explicit form

$$Plane = A \cdot x + B \cdot y + C \cdot z + D$$

Finding Correct Normal Transform

- transform a plane

$$\begin{matrix} P \\ N \end{matrix} \longrightarrow \begin{matrix} P' = MP \\ N' = QN \end{matrix}$$

if we know M, what should Q be?

$N'^{T} P' = 0$ **stay perpendicular**

$(QN)^T (MP) = 0$ **substitute from above**

$N^T \underbrace{Q^T M}P = 0$ $\quad (\mathbf{AB})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}} \mathbf{A}^{\mathrm{T}}$

$Q^T M = I$ $\quad N^T P = 0$

$\boxed{Q = \left(M^{-1}\right)^T}$

thus the normal to any surface has to be transformed by the inverse transpose of the modelling transformation

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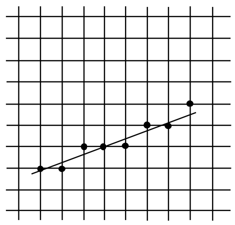
Sampling and Antialiasing

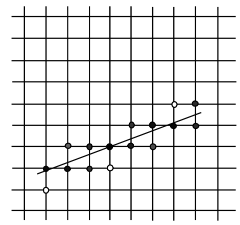
Jaggy Lines

- rasterized lines were not smooth
  - “stair-stepping”, “jaggies”



Smoother Lines

- solution
  – set 2 points with intensity proportional to distance from line



Smoothing Bresenham

- can modify Bresenham alg to do this
  – for every column of pixels, set the two pixels between which the line intersects the column
  – means that decision variable has to be shifted down one pixel $d = F(x+1, y+1)$
  – increments for E and NE can be determined as before (but results slightly different)
  – d can directly be used to multiply pixel intensities
    - fully integer implementation possible

General Problem

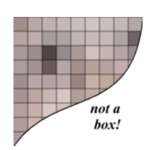
- “jaggies”: undesirable artifact
  – name for general problem is “aliasing”
- solving the general problem
  – “antialiasing”
- theoretical framework
  – sampling, signal processing

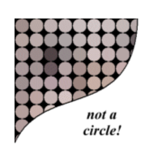
Samples

- most things in the real world are continuous
- everything in a computer is discrete
- the process of mapping a continuous function to a discrete one is called sampling
- the process of mapping a discrete function to a continuous one is called reconstruction
- the process of mapping a continuous variable to a discrete one is called quantization
- rendering an image requires sampling and quantization
- displaying an image involves reconstruction

What is a pixel?

- a pixel is not…
  – a box
  – a disk
  – a teeny tiny little light
- a pixel is a point
  – it has no dimension
  – it occupies no area
  – it cannot be seen
  – it can have a coordinate

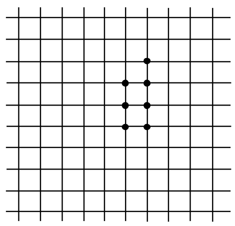




a pixel is *more* than a point, it is a *sample*

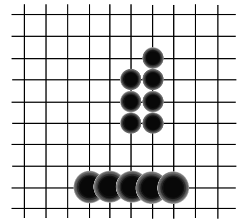
Pixels

- point samples
  – pixels have no extent!



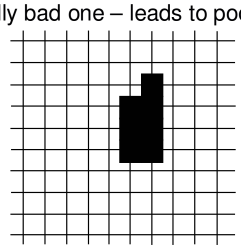
Pixel Display

- reconstruction yields continuous function
  – displays constructed to create reconstruction
  – footprints can overlap! (e.g. Gaussians)

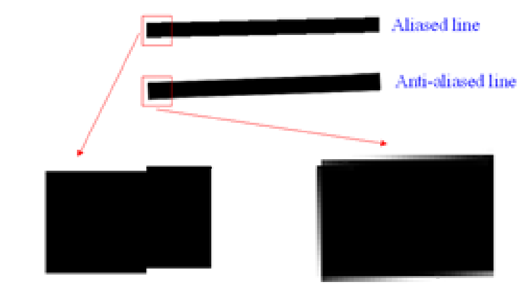


Pixels

- square pixel model
  – Just ONE possible reconstruction function
  – and a really bad one – leads to poor quality



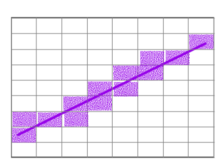
Anti-Aliasing: Example



Samples

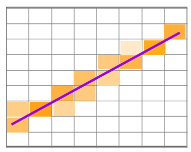
Line Segments

- we tried to sample a line segment so it would map to a 2D raster display
- we quantized the pixel values to 0 or 1
- we saw stair steps, or jaggies



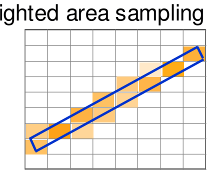
Line Segments

- instead, quantize to many shades
- but what sampling algorithm is used?



Area Sampling

- shade pixels according to the area covered by thickened line
- this is unweighted area sampling



- a rough approximation formulated by dividing each pixel into a finer grid of pixels

Unweighted Area Sampling

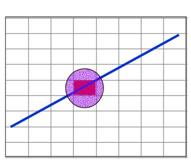
- primitive cannot affect intensity of pixel if it does not intersect the pixel
- equal areas cause equal intensity, regardless of distance from pixel center to area

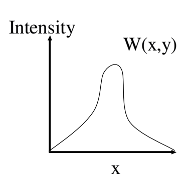
Weighted Area Sampling

- unweighted sampling colors two pixels identically when the primitive cuts the same area through the two pixels
- intuitively, pixel cut through the center should be more heavily weighted than one cut along corner

Weighted Area Sampling

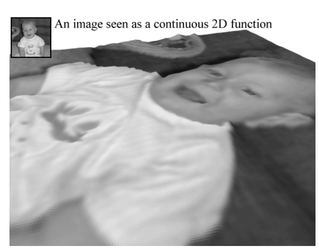
- weighting function, W(x,y)
  - specifies the contribution of primitive passing through the point (x, y) from pixel center





Images

- an image is a 2D function I(x, y) that specifies intensity for each point (x, y)

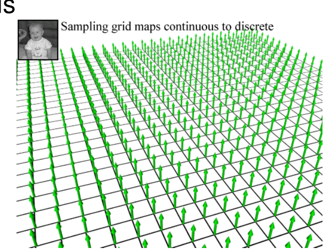


Sampling and Image

- our goal is to convert the continuous image to a discrete set of samples
- the graphics system's display hardware will attempt to reconvert the samples into a continuous image: *reconstruction*

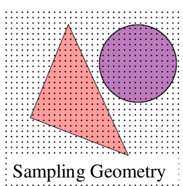
Point Sampling an Image

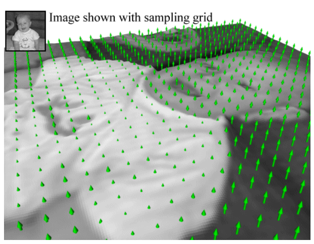
- simplest sampling is on a grid
- sample depends solely on value at grid points



Point Sampling

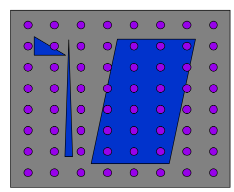
- multiply sample grid by image intensity to obtain a discrete set of points, or samples.





Sampling Errors

- some objects missed entirely, others poorly sampled



Fixing Sampling Errors

- *supersampling*
  - take more than one sample for each pixel and combine them
    - how many samples is enough?
    - how do we know no features are lost?