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Geometric Transformations

- ***recap***
- ***composition of transformations***
- ***rendering pipeline overview***

News

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- newsgroup is working
- labs start today, CICSR 011
  - *waitlisted: pick a lab, attend to hear TA talk*
  - *accounts: newsgroup answer on activate/reenable*
- readings
  - *Chap 1: graphics overview*
  - *Chap 2: graphics programming*
  - *Chap 4: transformations*

Transformations recap

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Affine transformations

- linear transformation + translations
- can be expressed as a 3x3 matrix + 3 vector

$$\mathbf{x}' = \mathbf{M} \cdot \mathbf{x} + \mathbf{t}$$

Homogeneous coordinates

- Unified representation as 4-vector (in 3D) for
  - *Points*
  - *Vectors / directions*
- Affine transformations become 4x4 matrices
  - *Composing multiple affine transformations involves simply multiplying the matrices*

Homogeneous Coordinates

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Homogeneous representation of points:

- Add an additional component $w=1$ to all *points*
- All multiples of this vector are considered to represent the same 3D point
- why bother? need unified representation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \equiv \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} x \cdot w \\ y \cdot w \\ z \cdot w \\ w \end{bmatrix} \forall w \neq 0$$

Geometrically In 2D

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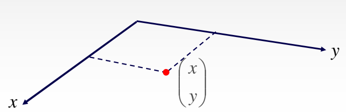
Cartesian Coordinates:



Geometrically In 2D

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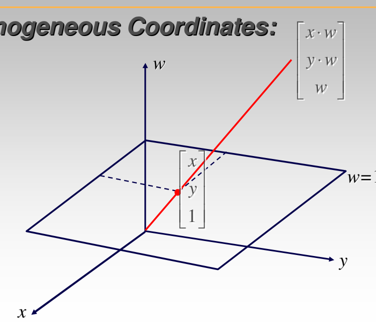
Homogeneous Coordinates:



Homogeneous Matrices

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Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & 0 \\ m_{2,1} & m_{2,2} & m_{2,3} & 0 \\ m_{3,1} & m_{3,2} & m_{3,3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & 0 \\ m_{2,1} & m_{2,2} & m_{2,3} & 0 \\ m_{3,1} & m_{3,2} & m_{3,3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & t_x \\ 0 & 0 & 0 & t_y \\ 0 & 0 & 0 & t_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Homogeneous Matrices

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Combining the two matrices into one:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & 0 \\ m_{2,1} & m_{2,2} & m_{2,3} & 0 \\ m_{3,1} & m_{3,2} & m_{3,3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & t_x \\ 0 & 0 & 0 & t_y \\ 0 & 0 & 0 & t_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & t_x \\ m_{2,1} & m_{2,2} & m_{2,3} & t_y \\ m_{3,1} & m_{3,2} & m_{3,3} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Homogeneous Matrices

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Note:

- Multiplication of the matrix with a constant does not change the transformation!

$$\tilde{T}\left(\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\right) = \begin{bmatrix} m_{1,1}\cdot k & m_{1,2}\cdot k & m_{1,3}\cdot k & t_x\cdot k \\ m_{2,1}\cdot k & m_{2,2}\cdot k & m_{2,3}\cdot k & t_y\cdot k \\ m_{3,1}\cdot k & m_{3,2}\cdot k & m_{3,3}\cdot k & t_z\cdot k \\ 0 & 0 & 0 & k \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x'\cdot k \\ y'\cdot k \\ z'\cdot k \\ k \end{bmatrix}$$

$$\equiv \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = T\left(\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\right)$$

normalize: divide by w

Homogeneous Vectors

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point: w >= 1

- What about vectors (directions)?
- What is the affine transformation of a vector?
  - *Rotation*
  - *Scaling*
  - *Translation*

Vectors are invariant under translation!

Homogeneous Vectors

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Representing vectors/directions in homogeneous coordinates

- Need representation that is only affected by linear transformations, but not by translations
- Answer: $w=0$

$$T\left(\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}\right) = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & t_x \\ m_{2,1} & m_{2,2} & m_{2,3} & t_y \\ m_{3,1} & m_{3,2} & m_{3,3} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 0 \end{bmatrix}$$

Transformations

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translate(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

scale(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\mathrm{Rotate}(x,\theta)$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & \cos\theta & -\sin\theta & \\ & \sin\theta & \cos\theta & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\mathrm{Rotate}\ (y,\theta)$

$$\begin{bmatrix} \cos\theta & & \sin\theta & \\ & 1 & & \\ -\sin\theta & & \cos\theta & \\ & & & 1 \end{bmatrix}$$

$\mathrm{Rotate}\ (z,\theta)$

$$\begin{bmatrix} \cos\theta & -\sin\theta & & \\ \sin\theta & \cos\theta & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

Transformations

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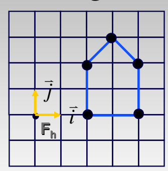
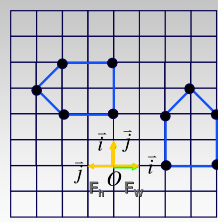
Arriving at a transformation...



object defined in local coords



place object in world

Transformations

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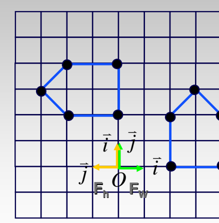
Arriving at a transformation...



$Rotate(z,90)$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & & \\ \sin\theta & \cos\theta & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}_{\mathbf{w}} = \begin{bmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{\mathbf{h}}$$

$$P = O_h + x_h \vec{i}_h + y_h \vec{j}_h$$

Transformations

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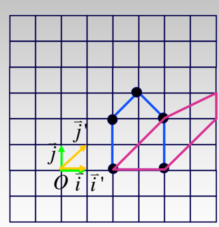
How about the following?



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & & \\ 0 & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

shear

Composing Transformations

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translation

$$T1 = T(dx_1, dy_1) = \begin{bmatrix} 1 & & dx_1 & a \\ & 1 & dy_1 & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \qquad T2 = T(dx_2, dy_2) = \begin{bmatrix} 1 & & dx_2 & a \\ & 1 & dy_2 & b \\ & & 1 & c \\ & & & 1 \end{bmatrix}$$

$$P'' = T2 \bullet P' = T2 \bullet [T1 \bullet P] = [T2 \bullet T1] \bullet P, \textit{ where}$$

$$T2 \bullet T1 = \begin{bmatrix} 1 & & dx_1 + dx_2 & a \\ & 1 & dy_1 + dy_2 & b \\ & & 1 & c \\ & & & 1 \end{bmatrix}$$

so translations add

Composing Transformations

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scaling

$$S2 \bullet S1 = \begin{bmatrix} sx_1 * dx_2 & & & \\ & sy_1 * sy_2 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

so scales multiply

rotation

$$R2 \bullet R1 = \begin{bmatrix} \cos(\theta 1 + \theta 2) & -\sin(\theta 1 + \theta 2) & & \\ \sin(\theta 1 + \theta 2) & \cos(\theta 1 + \theta 2) & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

so rotations add

Composing Transformations

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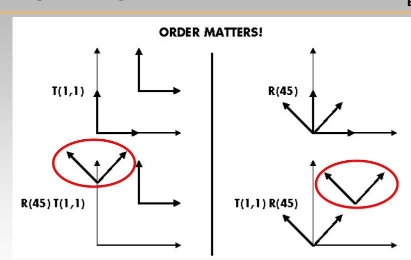

Ta Tb = Tb Ta, but Ra Rb != Rb Ra and Ta Rb != Rb Ta

Composing Transformations

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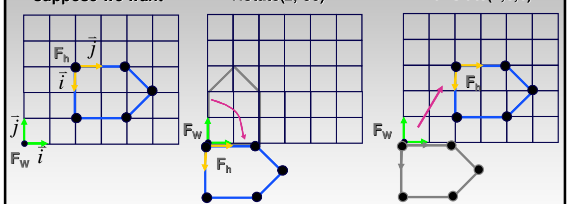

$P_A = \boxed{Rot(z,-90)\, P_h}$ $\quad P_W = Trans(2,3,0)\, P_A$

$P_W = Trans(2,3,0)\, Rot(z,-90)\, P_h$

Composing Transformations

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$P_W = Trans(2,3,0)\, Rot(z,-90)\, P_h$

Which direction to read?

- R-to-L: interpret operations wrt fixed coords [object]
- L-to-R: interpret operations wrt local coords [coord sys]
- OpenGL (L-to-R, local coords)

glTranslatef(2,3,0); $\quad M = M \cdot Trans(2,3,0)$

glRotatef(-90,0,0,1); $\quad M = M \cdot Rot(z,-90)$

updates current transformation matrix by postmultiplying

Composing Transformations

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Undoing Transformations: inverses

$$Trans(x,y,z)^{-1} = Trans(-x,-y,-z)$$
$$Trans(x,y,z)\; Trans(-x,-y,-z) = I$$

$$Rot(z,\theta)^{-1} = Rot(z,-\theta) = Rot^T(z,\theta)$$ **(R is orthogonal)**

$$Rot(z,\theta)\; Rot(z,-\theta) = I$$

$$Scale(sx,sy,sz)^{-1} = Scale(\frac{1}{sx},\frac{1}{sy},\frac{1}{sz})$$

Rotation about a point

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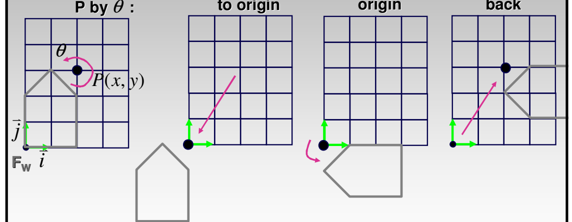

$$Trans(x,y,z)\, Rot(z,\theta)\, Trans(-x,-y,-z)$$

Composing of Linear Transformations

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In general:

- Transformation of geometry into coordinate system where operation becomes simpler
- Perform operation
- Transform geometry back to original coordinate system

Also works for affine transformations

Composing of Affine Transformations

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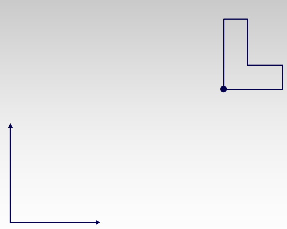
Example: Rotation around arbitrary center



Composing of Affine Transformations

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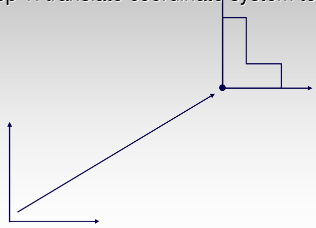
Example: Rotation around arbitrary center

- Step 1: translate coordinate system to rotation center



Composing of Affine Transformations

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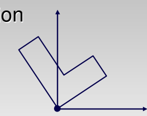
Example: Rotation around arbitrary center

- Step 2: perform rotation



Composing of Affine Transformations

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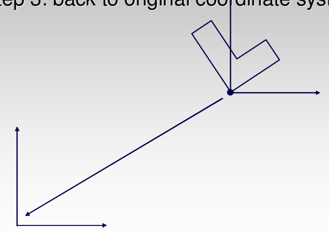
Example: Rotation around arbitrary center

- Step 3: back to original coordinate system



Rotation about an arbitrary axis

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- axis defined by two points
- translate point to the origin
- rotate to align axis with z-axis (or x or y)
- perform rotation
- undo aligning rotations
- undo translation

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The Rendering Pipeline – An Overview

3D Graphics

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Modeling:

- Representing object properties
  - *Geometry: polygons, smooth surfaces etc.*
  - *Materials: reflection models etc.*

Rendering:

- Generation of images from models
  - *Interactive rendering*
  - *Ray-tracing*

Animation:

- Making geometric models move and deform

Rendering

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Goal:
- Transform computer models into images
- May or may not be photo-realistic

Interactive rendering:
- Fast, but until recently low quality
- Roughly follows a fixed patterns of operations
  - ➢ Rendering Pipeline

Offline rendering:
- Ray-tracing
- Global illumination

Rendering

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Tasks that need to be performed (in no particular order):
- Project all 3D geometry onto the image plane
  - *Geometric transformations*
- Determine which primitives or parts of primitives are visible
  - *Hidden surface removal*
- Determine which pixels a geometric primitive covers
  - *Scan conversion*
- Compute the color of every visible surface point
  - *Lighting, shading, texture mapping*

The Rendering Pipeline

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What is it? All of this:
- Abstract model for sequence of operations to transform a geometric model into a digital image
- An abstraction of the way graphics hardware works
- The underlying model for application programming interfaces (APIs) that allow the programming of graphics hardware
  - *OpenGL*
  - *Direct 3D*

Actual implementations of the rendering pipeline will vary in the details

The Rendering Pipeline

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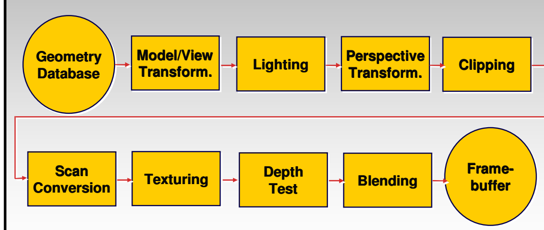

The Rendering Pipeline
Geometry Database

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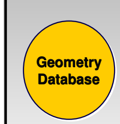

Geometry database:
- Application-specific data structure for holding geometric information
- Depends on specific needs of application
  - – *Independent triangles, connectivity information etc.*

The Rendering Pipeline
Model/View Transformation

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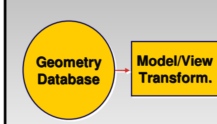

Modeling transformation:
- Map all geometric objects from a local coordinate system into a world coordinate system

Viewing transformation:
- Map all geometry from world coordinates into camera coordinates

The Rendering Pipeline
Lighting

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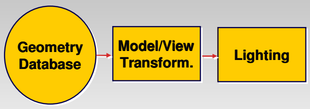

Lighting:

- Compute the brightness of every point based on its material properties (e.g. Lambertian diffuse) and the light position(s)
- Computation is performed *per-vertex*

The Rendering Pipeline
Perspective Transformation

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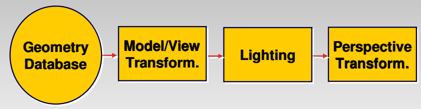

Perspective transformation

- Projecting the geometry onto the image plane
- Projective transformations and model/view transformations can all be expressed with 4x4 matrix operations

The Rendering Pipeline
Clipping

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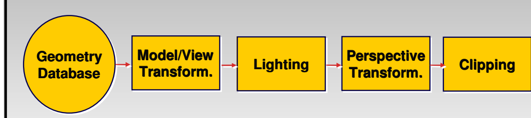

Clipping

- Removal of parts of the geometry that fall outside the visible screen or window region
- May require *re-tessellation* of geometry

The Rendering Pipeline
Scan Conversion

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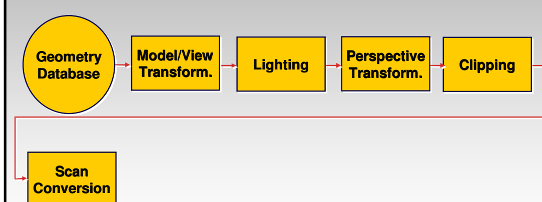

The Rendering Pipeline
Scan Conversion

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Scan conversion

- Turning 2D drawing primitives (lines, polygons etc.) into individual pixels (*discretizing/sampling*)
- Interpolation of colors across the geometric primitive
- This yields a *fragment* (pixel data associated with a particular location in the final image and color values, depth, and some additional information)

The Rendering Pipeline
Texture Mapping

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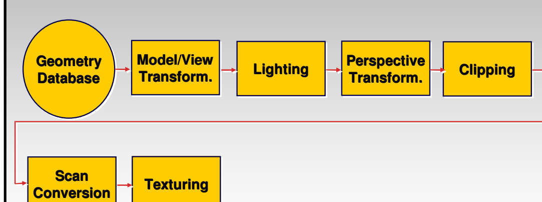

The Rendering Pipeline
Texture Mapping

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Texture mapping

- “gluing images onto geometry”
- The color of every fragment is altered by looking up a new color value from an image

The Rendering Pipeline
Depth Test

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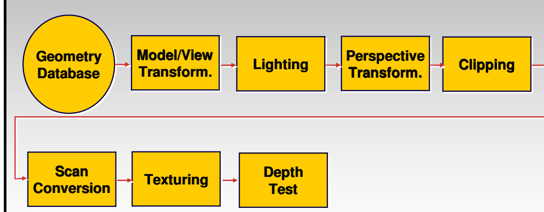

The Rendering Pipeline
Depth Test

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Depth test:

- Removes parts of the geometry that are hidden behind other geometry
- Test is performed on every individual fragment
  - *we will also discuss other approaches later*

The Rendering Pipeline
Blending

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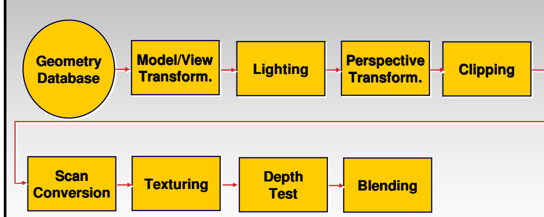

The Rendering Pipeline
Blending

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Blending:

- Fragments are written to pixels in the final image
- Rather than simply replacing the previous color value, the new and the old value can be combined with some arithmetic operations (blending)
- The video memory on the graphics board that holds the resulting image and is used to display it is called the *framebuffer*

The Rendering Pipeline

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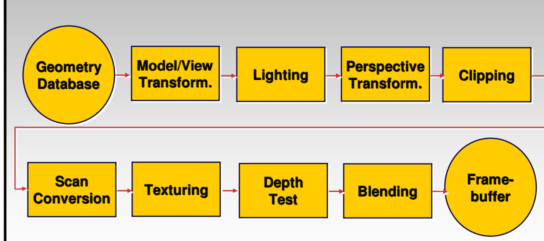

Discussion

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Advantages of a pipeline structure

- Logical separation of the different components, modularity
- Easy to parallelize:
  - *Earlier stages can already work on new data while later stages still work with previous data*
  - *Similar to pipelining in modern CPUs*
  - *But much more aggressive parallelization possible (special purpose hardware!)*
  - *Important for hardware implementations!*
- Only local knowledge of the scene is necessary

Discussion

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Disadvantages:

- Limited flexibility
- Some algorithms would require different ordering of pipeline stages
  - *Hard to achieve while still preserving compatibility*
- Only local knowledge of scene is available
  - *Shadows*
  - *Global illumination*