University of British Columbia

CPSC 414 Computer Graphics

Procedural Approaches
Curves

Week 12, Fri 21 Nov 2003

News

- midterm solutions out
- late hw2: handin box 18, CICSR basement
- project 3 out

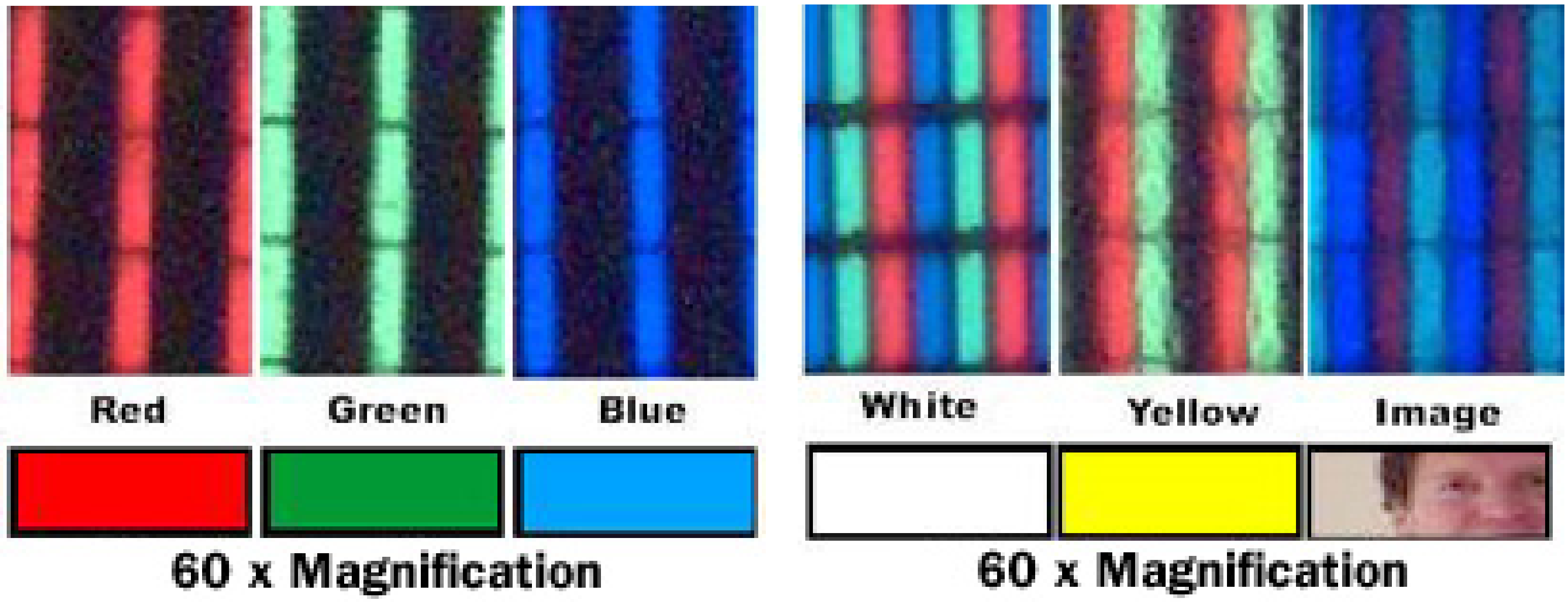
- final location announced
  - LSK 200
  - noon Tue Dec 9
  - must have photo ID
    - student ID best

Display Technologies recap

- mobile display with laser to retina
- stereo glasses/display
- 3D scanners
  - laser stripe + camera
  - laser time-of-flight
  - cameras only, depth from stereo
- Shape Tape
- haptics (Phantom)
- 3D printers

Color LCD Answer

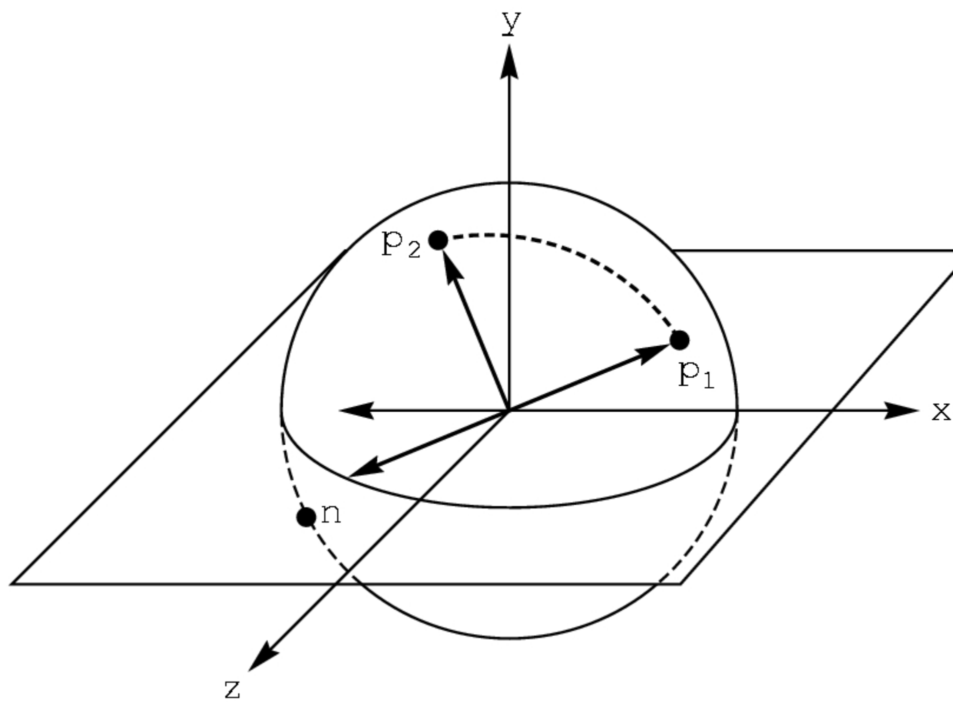
- three subpixels for each pixel
  - color filter on each



http://electronics.howstuffworks.com/lcd5.htm

Virtual Trackball recap

- Rotation about axis
  $\mathbf{n} = \mathbf{p_1} \times \mathbf{p_2}$
- Angle of rotation:
  $\mathbf{p_1} \bullet \mathbf{p_2} = |\mathbf{p_1}|\ |\mathbf{p_2}| \cos\theta$
- Fixed point is origin if use [-1, 1] cube



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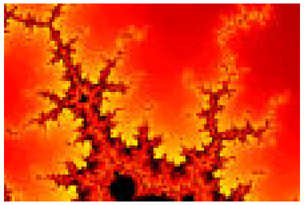
Procedural Approaches

Procedural Modeling

- textures, geometry
  - explicitly stored in memory
- procedural approach
  - compute something on the fly
  - often less memory cost
  - visual richness
- fractals, particle systems, noise

Fractal Landscapes

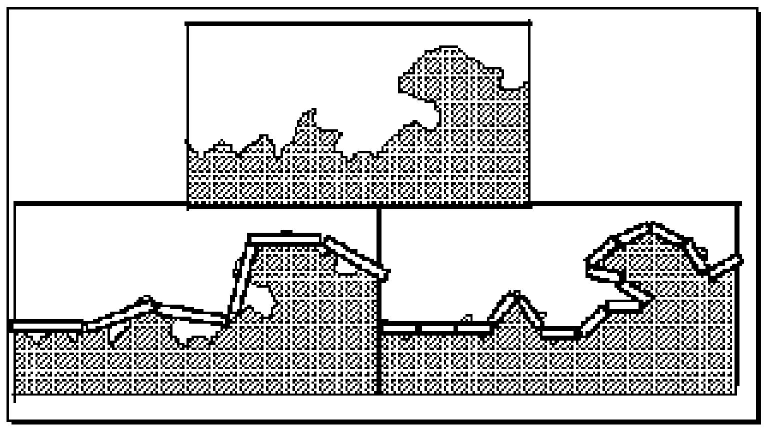


- fractals: not just for “showing math”
  - triangle subdivision
  - vertex displacement
  - recursive until termination condition

http://www.fractal-landscapes.co.uk/images.html

Self-Similarity

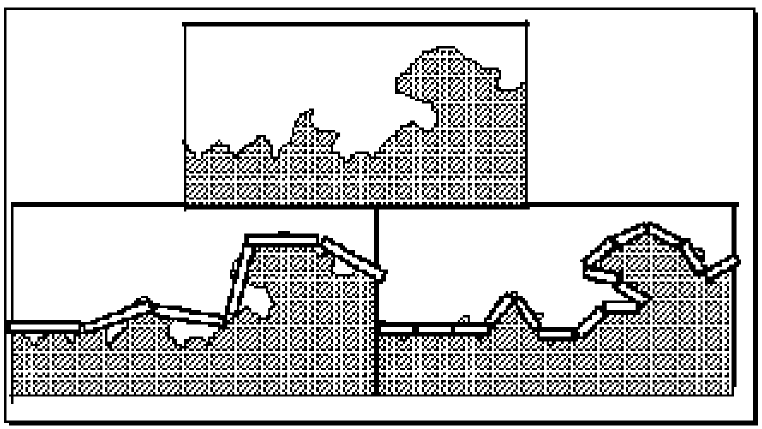
- infinite nesting of structure on all scales



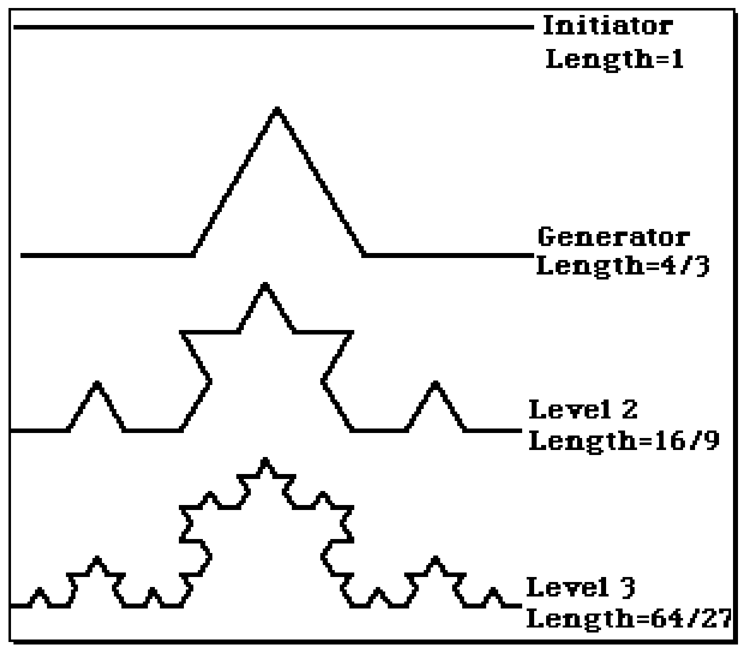
Fractal Dimension

- D = log(N)/log(r)
  N = measure, r = subdivision scale
  - Hausdorff dimension: noninteger

coastline of Britain



Koch snowflake

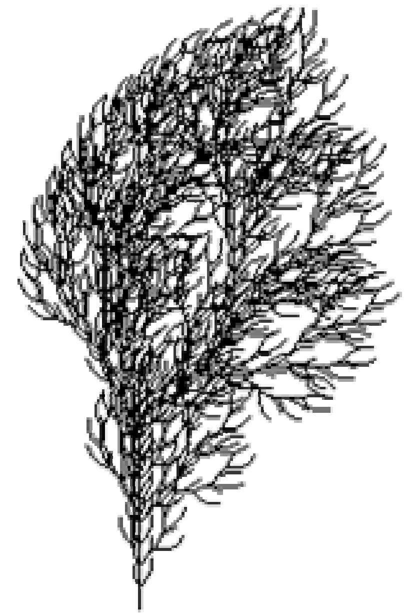


D = log(N)/log(r) D = log(4)/log(3) = 1.26

http://www.vanderbilt.edu/AnS/psychology/cogsci/chaos/workshop/Fractals.html

Language-Based Generation

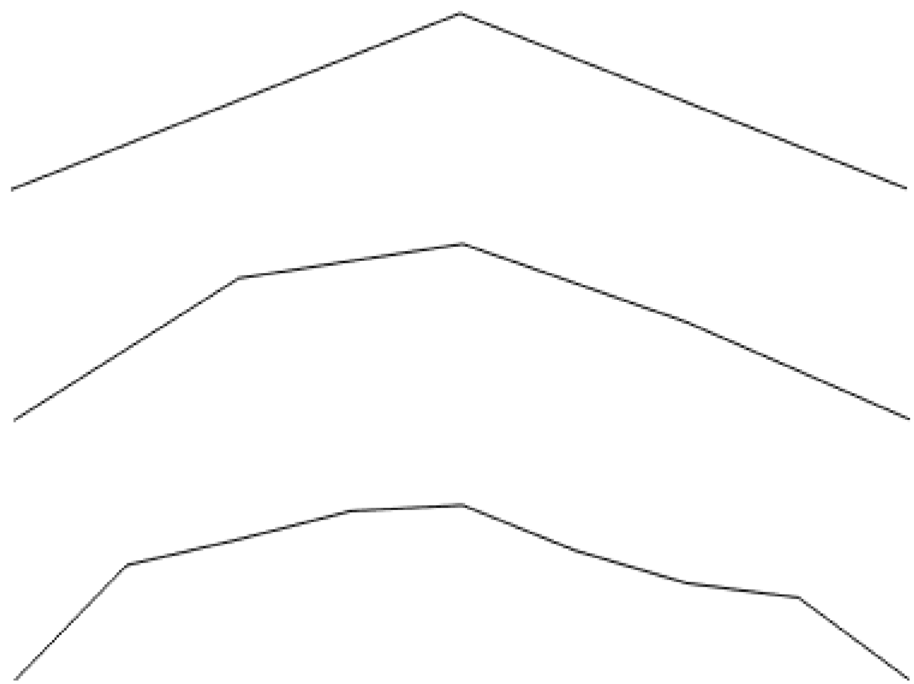
- L-Systems: after Lindenmayer
  - Koch snowflake: F :- FLFRRFLF
    - F: forward, R: right, L: left
  - Mariano's Bush:
    F=FF-[-F+F+F]+[+F-F-F] }
    - angle 16



http://spanky.triumf.ca/www/fractint/lsys/plants.html

1D: Midpoint Displacement

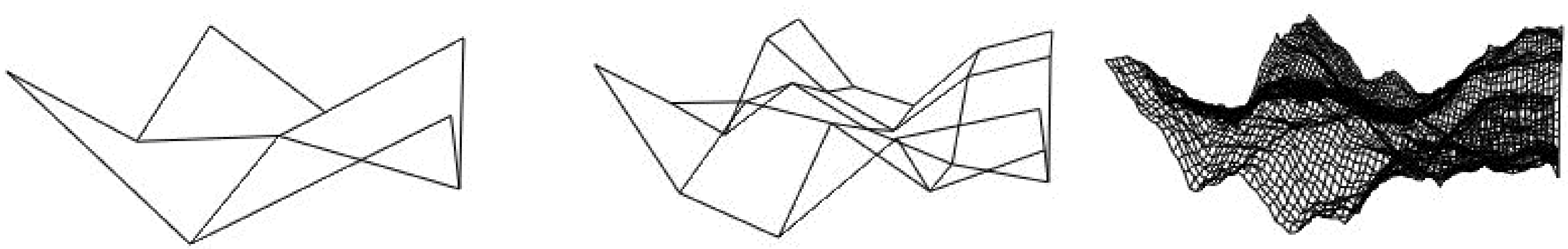
- divide in half
- randomly displace
- scale variance by half



http://www.gameprogrammer.com/fractal.html

2D: Diamond-Square

- diamond step
  - generate a new value at square midpoint
    - average corner values + random amount
    - gives diamonds when have multiple squares in grid
- square step
  - generate new value at diamond midpoint
    - average corner values + random amount
    - gives squares again in grid

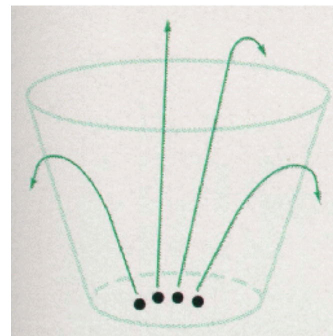


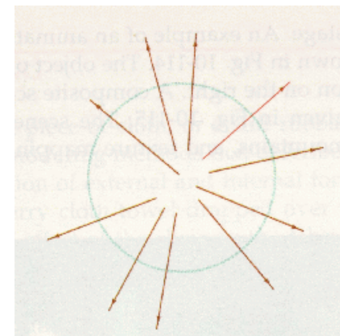
Particle Systems

- loosely defined
  - modeling, or rendering, or animation
- key criteria
  - collection of particles
  - random element controls attributes
    - position, velocity (speed and direction), color, lifetime, age, shape, size, transparency
    - predefined stochastic limits: bounds, variance, type of distribution

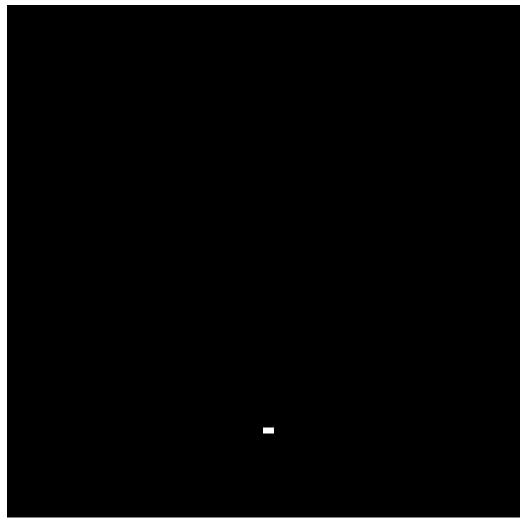
Particle System Examples

- objects changing fluidly over time
  - fire, steam, smoke, water
- objects fluid in form
  - grass, hair, dust
- physical processes
  - waterfalls, fireworks, explosions
- group dynamics: behavioral
  - birds/bats flock, fish school,
    human crowd, dinosaur/elephant stampede





Explosions Animation



http://www.cs.wpi.edu/%7Ematt/courses/cs563/talks/psys.html

Boid Animation

- bird-like objects
- http://www.red3d.com/cwr/boids/

Particle Life Cycle

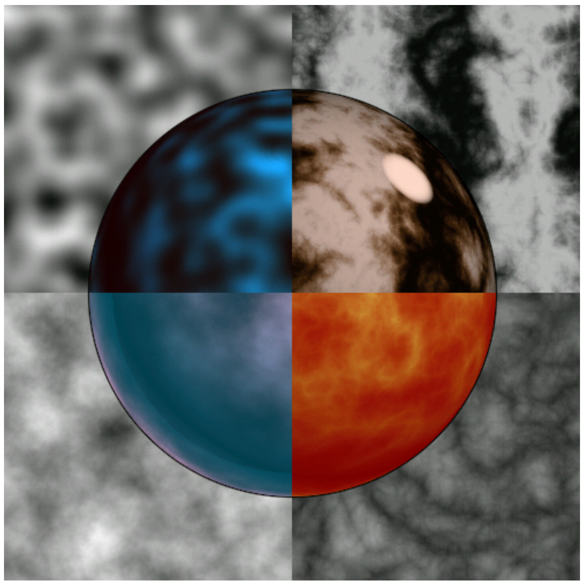
- generation
  – randomly within “fuzzy” location
  – initial attribute values: random or fixed
- dynamics
  – attributes of each particle may vary over time
    - color darker as particle cools off after explosion
  – can also depend on other attributes
    - position: previous particle position + velocity + time
- death
  – age and lifetime for each particle (in frames)
  – or if out of bounds, too dark to see, etc

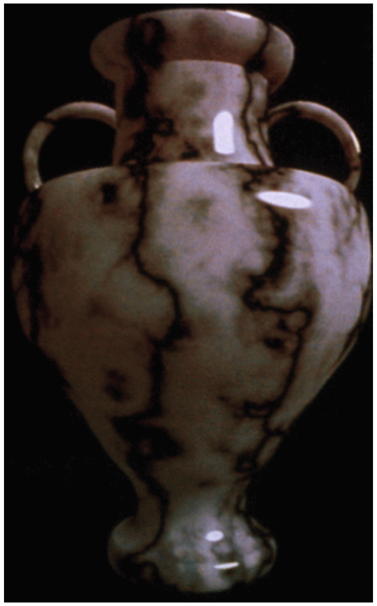
Particle System Rendering

- expensive to render thousands of particles
- simplify: avoid hidden surface calculations
  - each particle has small graphical primitive (blob)
  - pixel color: sum of all particles mapping to it
- some effects easy
  - temporal anti-aliasing (motion blur)
    - normally expensive: supersampling over time
    - position, velocity known for each particle
    - just render as streak

Perlin Noise

- excellent tutorial explanation
  http://www.kenperlin.com/talk1





http://mrl.nyu.edu/~perlin/planet/

Procedural Approaches Summary

- fractals
- L-systems
- particle systems
- Perlin noise

- not at all complete list!
  – big subject: entire classes on this alone

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Curves

Parametric Curves

- parametric form for a line:

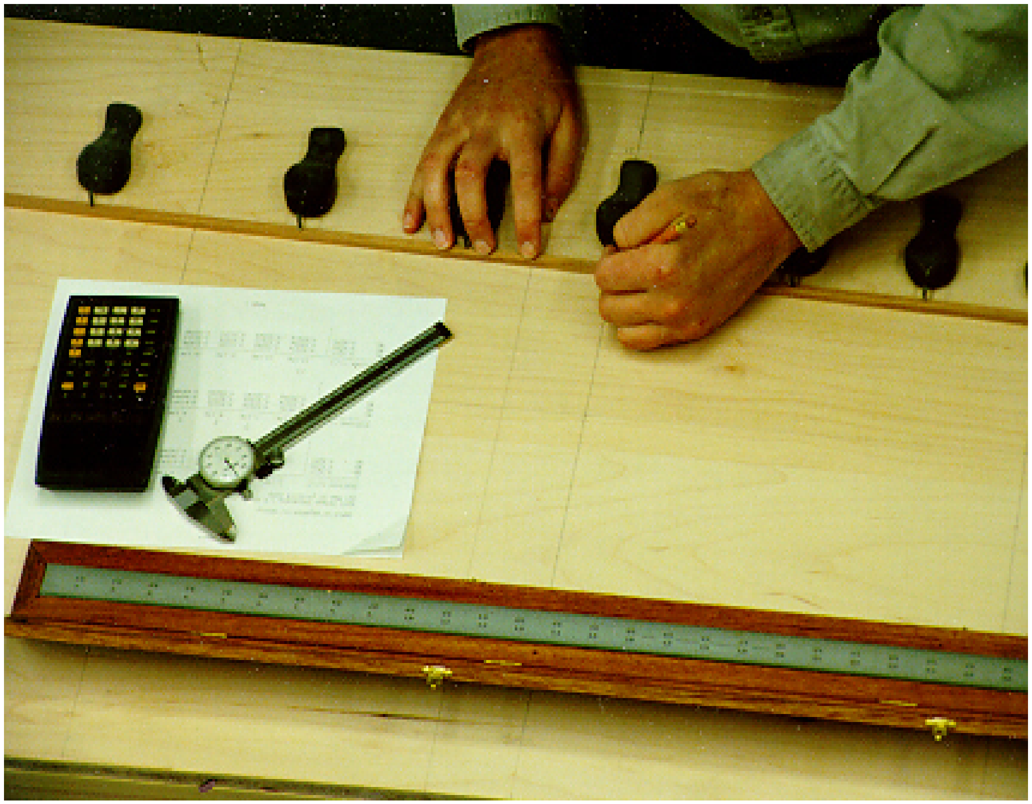
$$x = x_0 t + (1-t)x_1$$
$$y = y_0 t + (1-t)y_1$$
$$z = z_0 t + (1-t)z_1$$

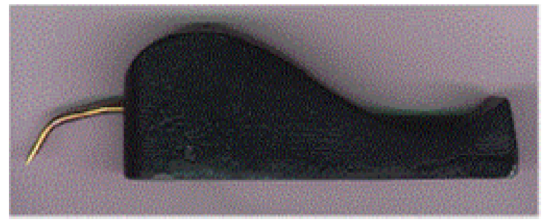
- x, y and z are each given by an equation that involves:
  - parameter t
  - some user specified control points, x_0 and x_1
- this is an example of a parametric curve

Splines

- a *spline* is a parametric curve defined by *control points*
  - term “spline” dates from engineering drawing, where a spline was a piece of flexible wood used to draw smooth curves
  - control points are *adjusted by the user* to control shape of curve

Splines – Old School

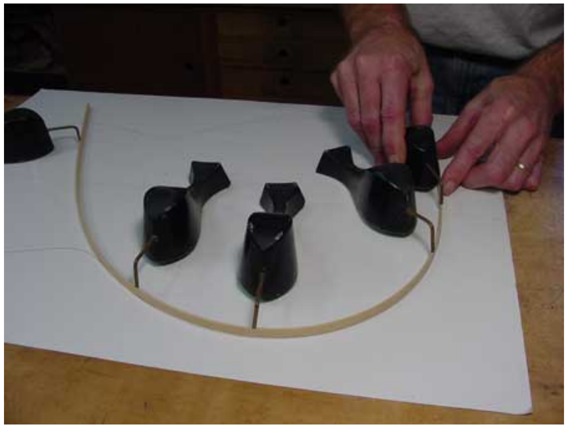




Splines - History

- Draftsman use 'ducks' and strips of wood (splines) to draw curves
- Wood splines have second-order continuity
- And pass through the control points

A Duck (weight)



Ducks trace out curve

Hermite Spline

- *Hermite spline* is curve for which user provides:
  - endpoints of the curve
  - parametric derivatives of the curve at the endpoints
    - parametric derivatives are *dx/dt, dy/dt, dz/dt*
  - more derivatives would be required for higher order curves

Hermite Cubic Splines

- example of knot and continuity constraints

Hermite Specification

Hermite Spline (2)

- say user provides x_0, x_1, x'_0, x'_1
- cubic spline has degree 3, is of the form:

$$x = at^3 + bt^2 + ct + d$$

  - for some constants a, b, c and d derived from the control points, but how?
- we have constraints:
  - curve must pass through x_0 when $t=0$
  - derivative must be x'_0 when $t=0$
  - curve must pass through x_1 when $t=1$
  - derivative must be x'_1 when $t=1$

Hermite Spline (3)

- solving for the unknowns gives

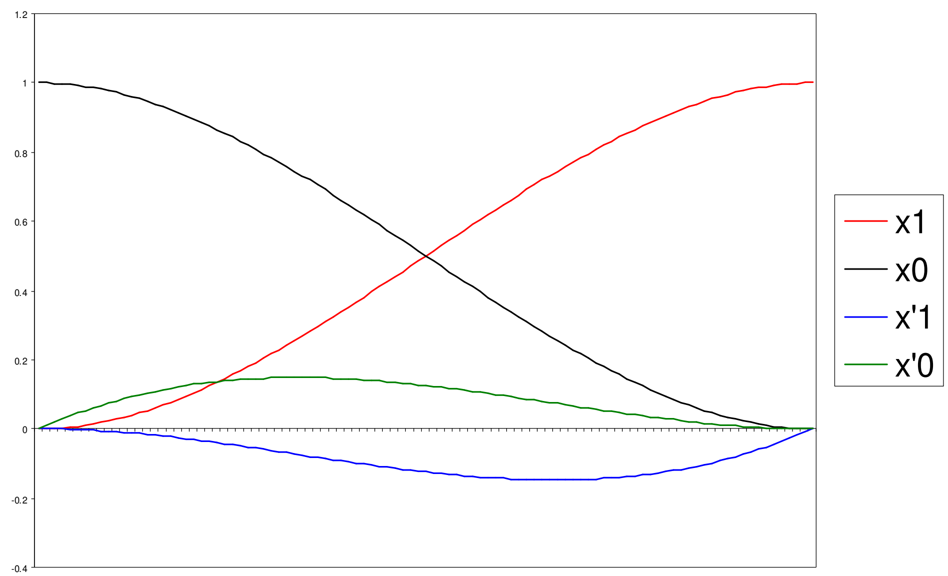
$$
\begin{aligned}
a &= -2x_1 + 2x_0 + x_1' + x_0' \\
b &= 3x_1 - 3x_0 - x_1' - 2x_0' \\
c &= x_0' \\
d &= x_0
\end{aligned}
$$

- rearranging gives

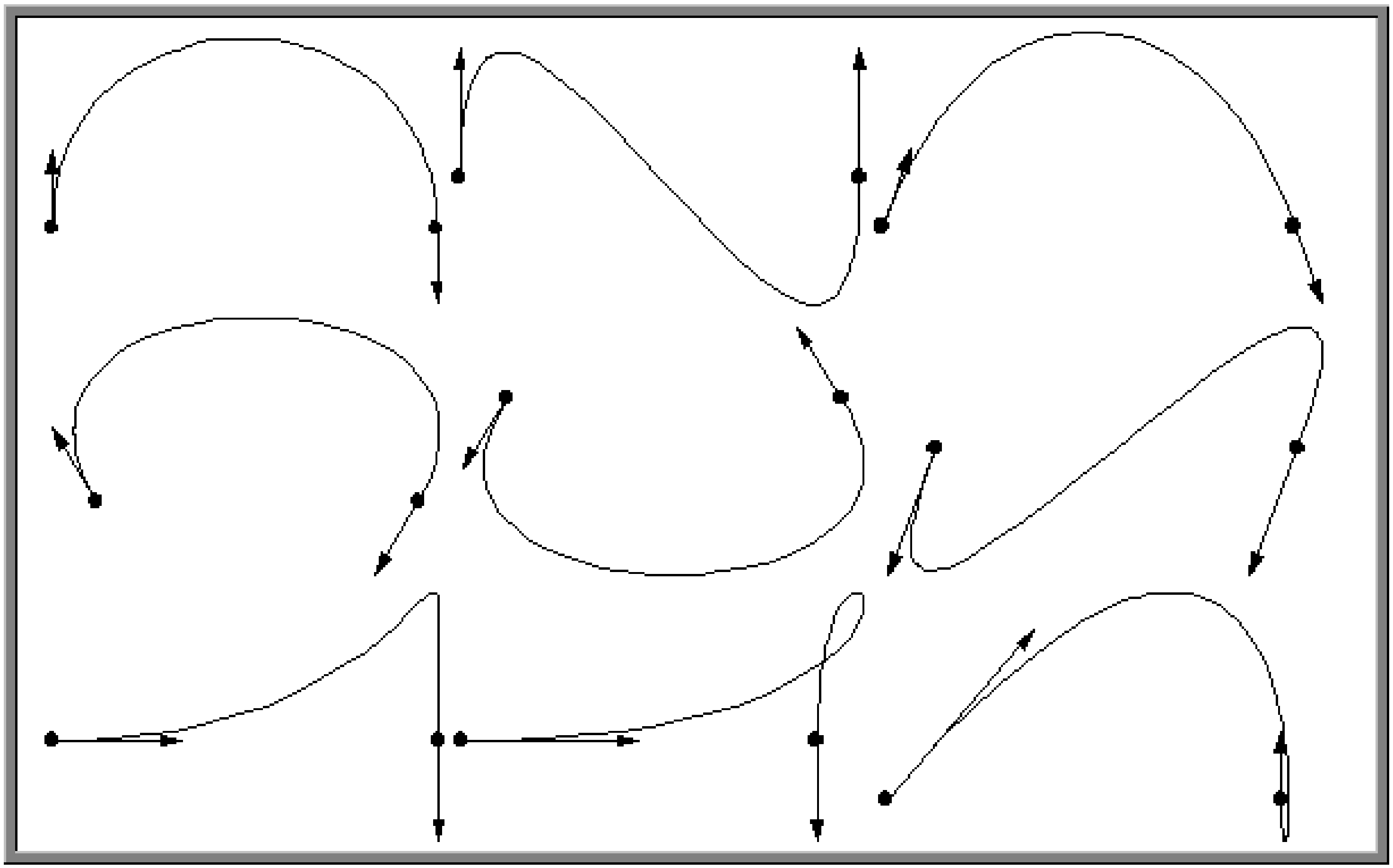
$$
\begin{aligned}
x = {} & x_1(-2t^3 + 3t^2) \\
& + x_0(2t^3 - 3t^2 + 1) \\
& + \mathrm{x}_1'(t^3 - t^2) \\
& + x_0'(t^3 - 2t^2 + t)
\end{aligned}
\quad \text{or} \quad
x = \begin{bmatrix} x_1 & x_0 & x_1' & x_0' \end{bmatrix}
\begin{bmatrix} -2 & 3 & 0 & 0 \\ 2 & -3 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \end{bmatrix}
\begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}
$$

Basis Functions

- a point on a Hermite curve is obtained by multiplying each control point by some function and summing
- functions are called *basis functions*



Sample Hermite Curves



Splines in 2D and 3D

- we have defined only 1D splines: $x=f(t:x_0,x_1,x'_0,x'_1)$
- for higher dimensions, define control points in higher dimensions (that is, as vectors)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_1 & x_0 & x_1' & x_0' \\ y_1 & y_0 & y_1' & y_0' \\ z_1 & z_0 & z_1' & z_0' \end{bmatrix} \begin{bmatrix} -2 & 3 & 0 & 0 \\ 2 & -3 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

Bezier Curves (1)

- different choices of basis functions give different curves
  - choice of basis determines how control points influence curve
  - in Hermite case, two control points define endpoints, and two more define parametric derivatives
- for Bezier curves, two control points define endpoints, and two control the tangents at the endpoints in a geometric way

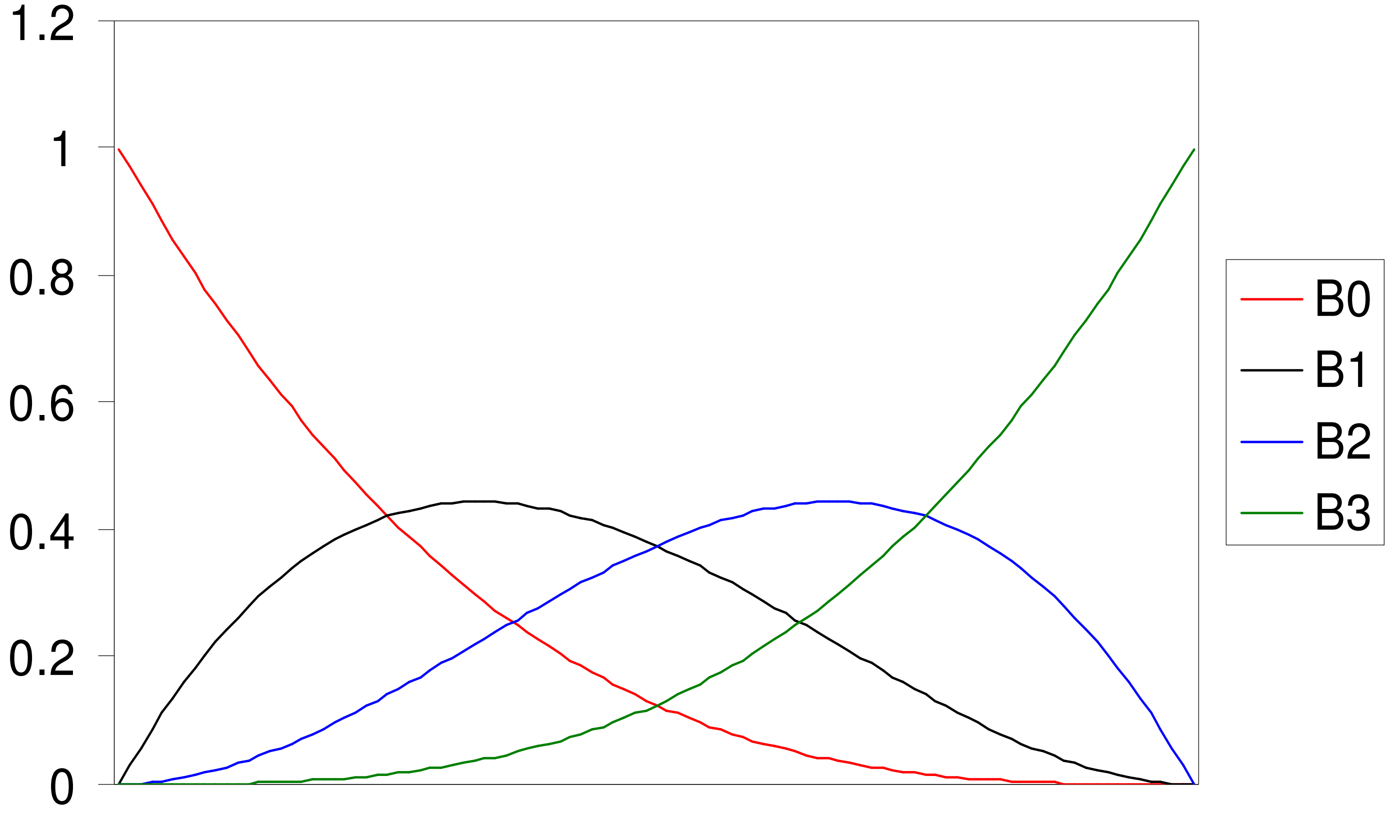
Bezier Curves (2)

- user supplies d control points, $\boldsymbol{p}_i$
- write the curve as:

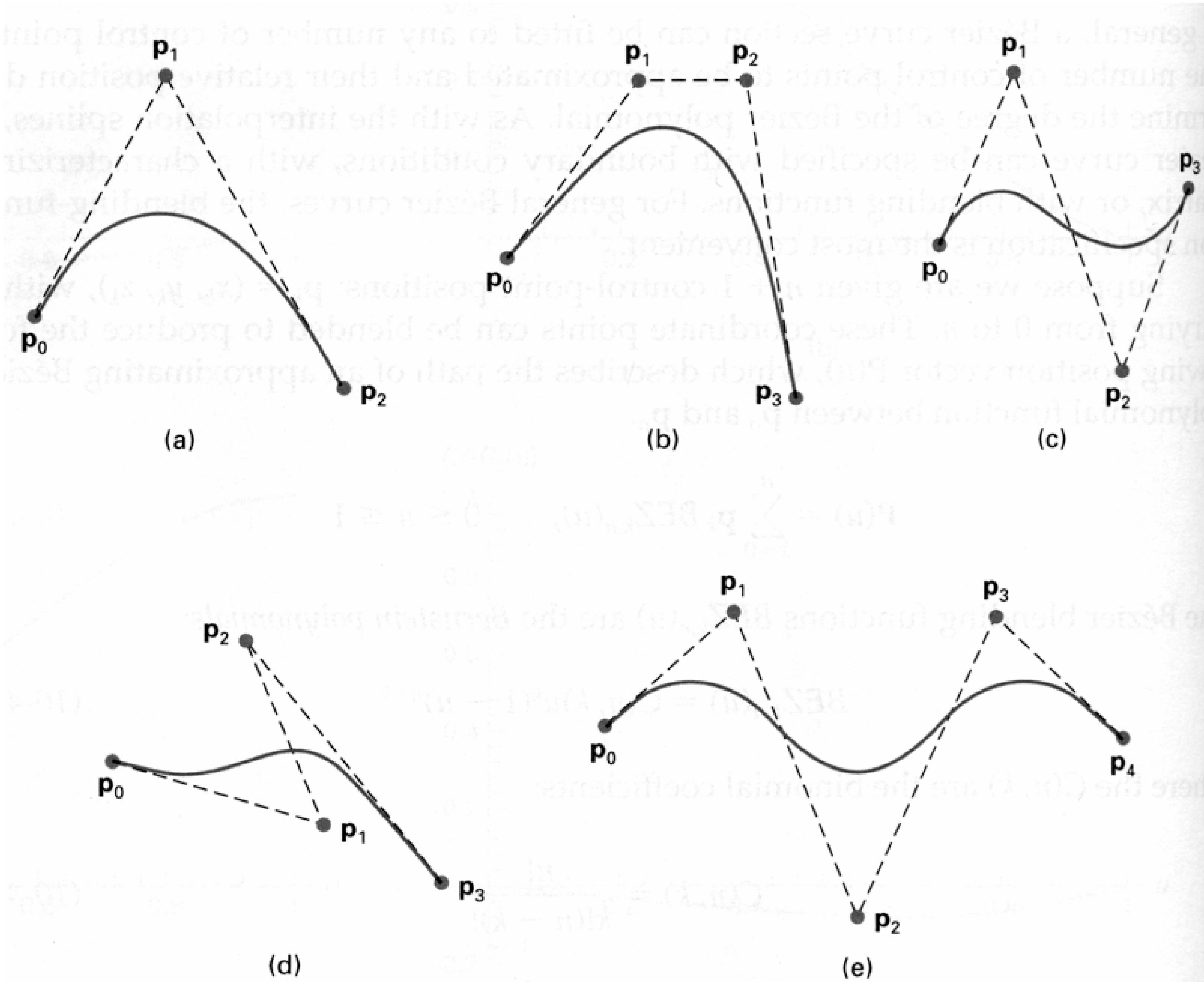
$$\mathbf{x}(t) = \sum_{i=0}^{d} \mathbf{p}_i B_i^d(t) \qquad B_i^d(t) = \binom{d}{i} t^i (1-t)^{d-i}$$

- functions B_i^d are the *Bernstein polynomials* of degree d
- this equation can be written as matrix equation also
  - there is a matrix to take Hermite control points to Bezier control points

Bezier Basis Functions for *d*=3



Some Bezier Curves

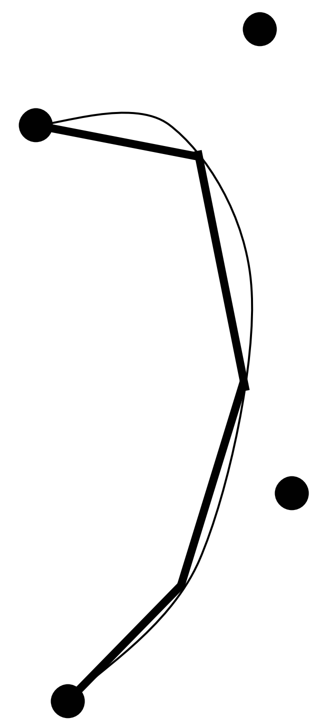


Bezier Curve Properties

- first and last control points are interpolated
- tangent to curve at first control point is along line joining first and second control points
- tangent at last control point is along line joining second last and last control points
- curve lies entirely within convex hull of its control points
  - Bernstein polynomials (the basis functions) sum to 1 and are everywhere positive
- can be rendered in many ways
  - convert to line segments with subdivision alg

Rendering Bezier Curves (1)

- evaluate curve at fixed set of parameter values and join points with straight lines
- advantage: very simple
- disadvantages:
  - expensive to evaluate the curve at many points
  - no easy way of knowing how fine to sample points, and maybe sampling rate must be different along curve
  - no easy way to adapt: hard to measure deviation of line segment from exact curve



Rendering Bezier Curves (2)

- recall that Bezier curve lies entirely within convex hull of its control vertices
- if control vertices are nearly collinear, then convex hull is good approximation to curve
- also, a cubic Bezier curve can be broken into two shorter cubic Bezier curves that exactly cover original curve
- suggests a rendering algorithm:
  – keep breaking curve into sub-curves
  – stop when control points of each sub-curve are nearly collinear
  – draw the control polygon - the polygon formed by control points

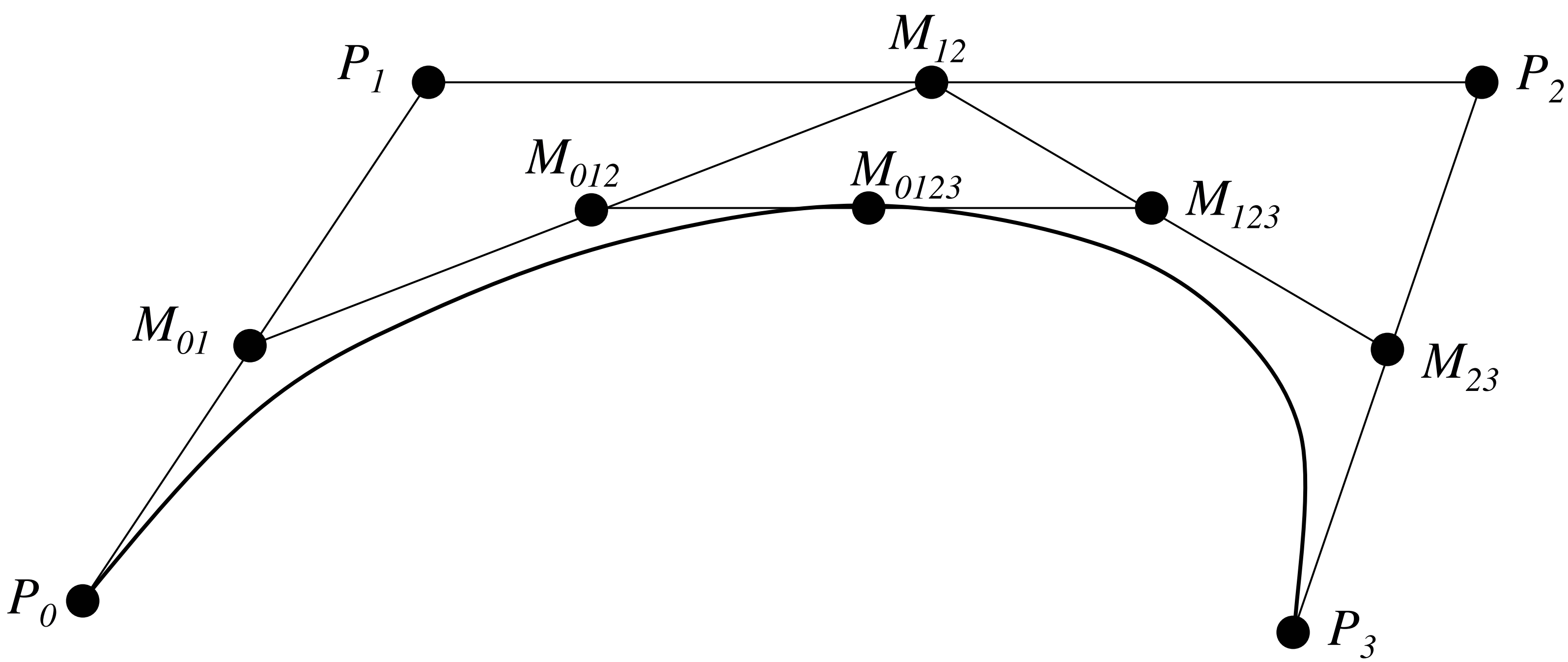
Sub-Dividing Bezier Curves

- step 1: find the midpoints of the lines joining the original control vertices. call them M_{01}, M_{12}, M_{23}
- step 2: find the midpoints of the lines joining M_{01}, M_{12} and M_{12}, M_{23}. call them M_{012}, M_{123}
- step 3: find the midpoint of the line joining M_{012}, M_{123}. call it M_{0123}

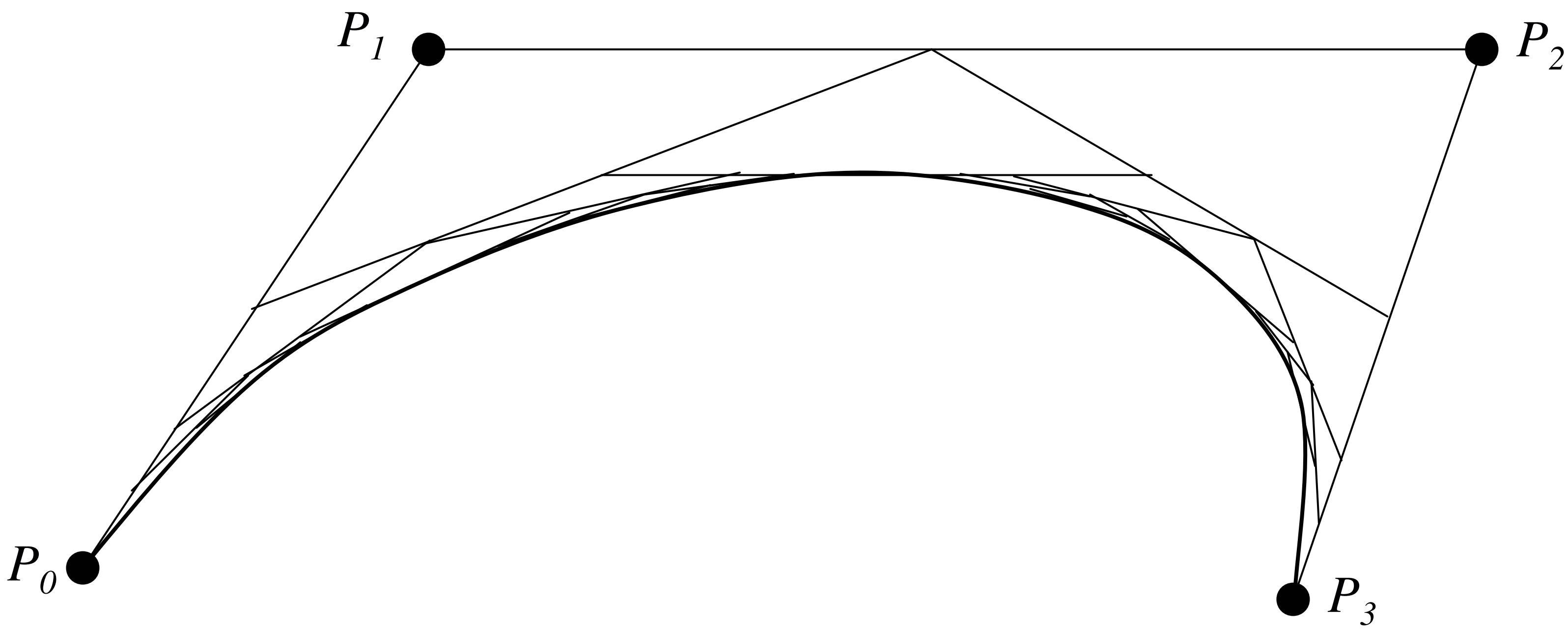
Sub-Dividing Bezier Curves

- curve with control points P_0, M_{01}, M_{012} and M_{0123} exactly follows the original curve from the point with t=0 to the point with t=0.5
- curve with control points M_{0123} , M_{123} , M_{23} and P_3 exactly follows the original curve from the point with t=0.5 to the point with t=1

Sub-Dividing Bezier Curves

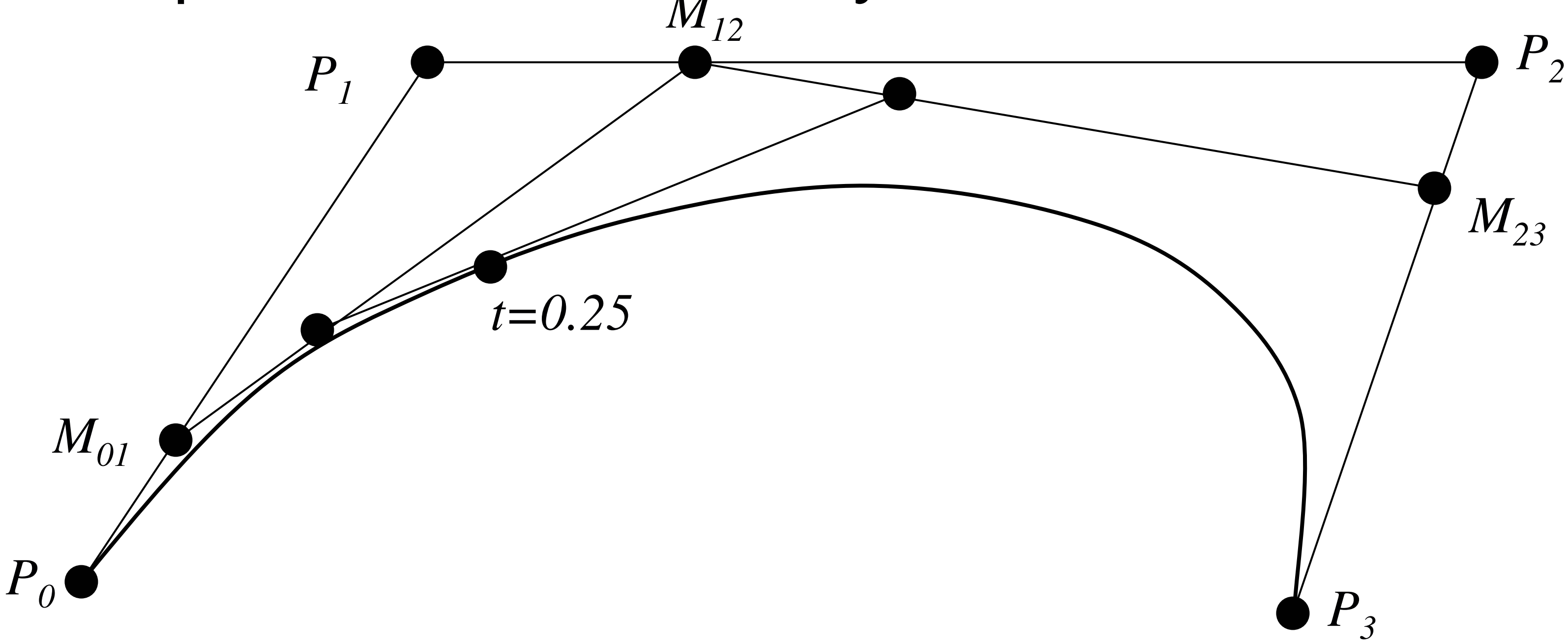


Sub-Dividing Bezier Curves



de Casteljau's Algorithm

- You can find the point on a Bezier curve for any parameter value *t* with a similar algorithm
- Say you want *t=0.25*, instead of taking midpoints take points 0.25 of the way



Invariance

- *translational invariance* means that translating control points and then evaluating curve is same as evaluating and then translating curve
- *rotational invariance* means that rotating control points and then evaluating curve is same as evaluating and then rotating curve
- these properties are essential for parametric curves used in graphics
- easy to prove that Bezier curves, Hermite curves and everything else we will study are translation and rotation invariant
- some forms of curves, *rational splines*, are also *perspective invariant*
  - can do perspective transform of control points and *then* evaluate curve