1. (1 pt) Write down the 4x4 matrix for rotating an object by 90% around the z axis.

$$\begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. (2 pts) Describe in words what this matrix does (be specific about the order of operations)

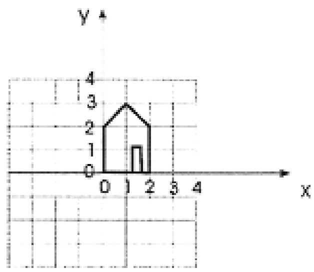
$$\begin{bmatrix} .707 & 0 & .707 & 0 \\ 0 & 2 & 0 & 0 \\ -.707 & 0 & .707 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .707 & 0 & .707 & 0 \\ 0 & 1 & 0 & 0 \\ -.707 & 0 & .707 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 2 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

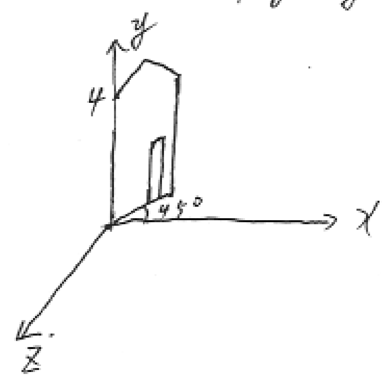
scale in y by 2, then rotate (y, 45°)

$$\text{OR} = \begin{bmatrix} 1 & & & \\ & 2 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} .707 & 0 & .707 & 0 \\ 0 & 1 & 0 & 0 \\ -.707 & 0 & .707 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

rotate (y, 45°), then scale in y by 2

3. (1 pt) Draw a picture of the object below transformed by the above matrix



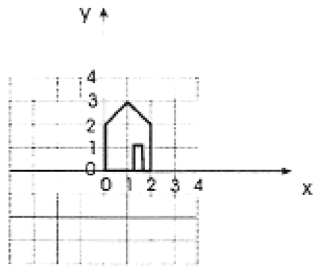


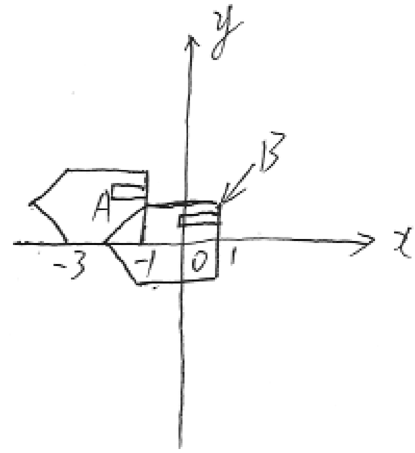
4. (1 pt) Give sequence of OpenGL commands necessary to implement the above transformation.

```
glRotatef(45, 0, 1, 0)
glScalef(1, 2, 1)
```

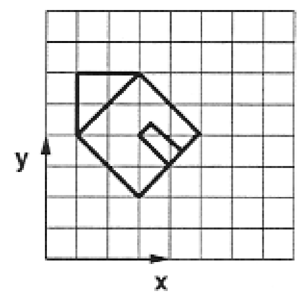
5. (2 pts) Draw houseA and houseB, transformed by the appropriate OpenGL commands. The untransformed house is below.

```
glIdentity();
glTranslate(1, 0, 0);
glRotate(90, 0, 0, 1);
glPushMatrix();
glTranslate(0, 2, 0);
drawHouseA();
glPopMatrix();
glTranslate(-1, 0, 0);
drawHouseB();
```





6. (1 pt) Give the series of affine transformations (assuming postmultiplying) needed to create the picture below, assuming the house started from the position shown in the above questions.

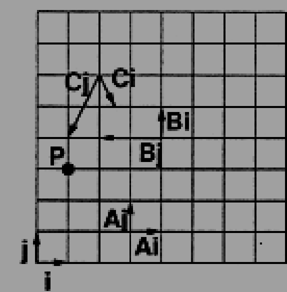


glTranslate (3, 2, 0)
glRotate (45, 0, 0, 1)
glScale ($\sqrt{2}$, $\sqrt{2}$, 1)

$$\begin{bmatrix} \text{Translate} \end{bmatrix} \begin{bmatrix} \text{Rotate} \end{bmatrix} \begin{bmatrix} \text{scale} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & & & 3 \\ & 1 & & 2 \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & & \\ \sin 45^\circ & \cos 45^\circ & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & & & \\ & \sqrt{2} & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

7. (1 pt) The point coordinate P, as shown below to the right, can be thought of as P = 1*i + 3*j, where i and j are basis vectors of unit length along the x and y axes, respectively. In effect, a coordinate system is defined by the location of its origin, and its basis vectors. Describe the point P in terms of the 3 other coordinate systems given below.



$$\left.\begin{array}{l} P = -2\,A_i + 2\,A_j \\ P = -1\,B_i + 1.5\,B_j \end{array}\right\} \tfrac{1}{2}$$

$$P = 0.5\,C_i + 1.25\,C_j \quad -\tfrac{1}{2}$$

$$\begin{pmatrix}1\\3\end{pmatrix} = a\begin{pmatrix}0.5\\-1\end{pmatrix} + b\begin{pmatrix}-1\\-2\end{pmatrix} + \begin{pmatrix}2\\6\end{pmatrix}$$

$1 = 0.5a - b + 2$

$3 = -a - 2b + 6$

$0.5a - b = -1$

$-a - 2b = -3$

$a - 2b = -2$

$-a - 2b = -3$

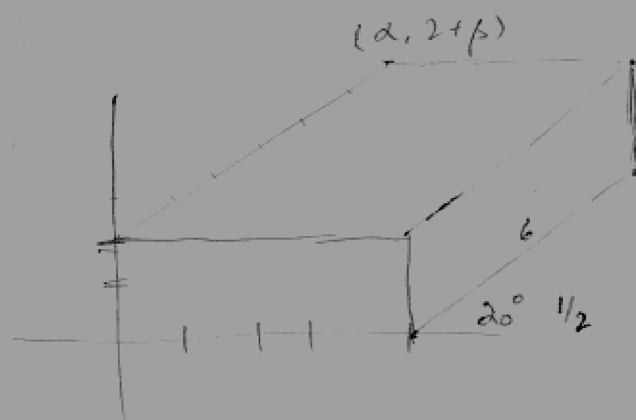
$2a = 1 \quad a = \frac{1}{2}$

$-\frac{1}{2} - 2b = -3 \quad 2b = \frac{5}{2} \quad b = \frac{5}{4}$

8. (1 pt) Normalize the homogenous point (2,4,6,2).

$$(1, 2, 3, 1) = \left(\frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \frac{2}{2}\right)$$

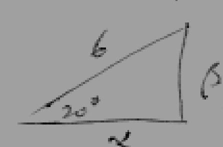
9. (1 pt) Draw the cavalier projection of a box of size x=4, y=2, z=6. Use a 20° projection (that is, the z axis in the scene should make a 20° angle with the x axis in the projection). The drawing doesn't have to be exactly to scale, but you must label the point locations.



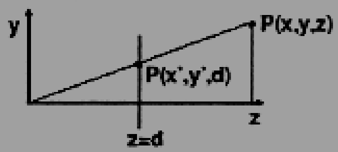
$\alpha = 6\cos 20° = 5.64$

$\beta = 6\sin 20° = 2.05$

$\frac{1}{2}$



10. (2 pts) Derive a 4x4 matrix that when applied to the point $(x, y, z, 1)^T$ would result in the projection in the picture below. Show your work.

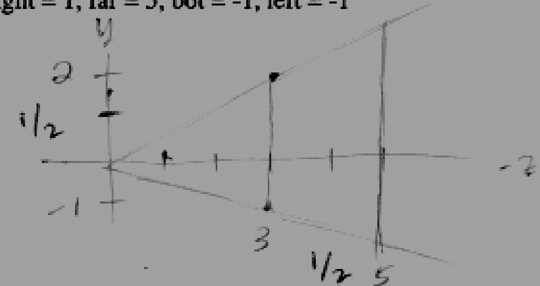


$$\frac{x'}{d} = \frac{x}{z} \qquad x' = \frac{xd}{z}$$

$$\frac{y'}{d} = \frac{y}{z} \qquad y' = \frac{yd}{z}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z/d \end{pmatrix} \rightarrow \begin{pmatrix} \frac{xd}{z} \\ \frac{yd}{z} \\ d \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ d \\ 1 \end{pmatrix}$$

11. (1 pt) Sketch a side view (yz plane) of the perspective view frustum, in VCS, that is specified by the following parameters: near = 3, top = 2, right = 1, far = 5, bot = -1, left = -1



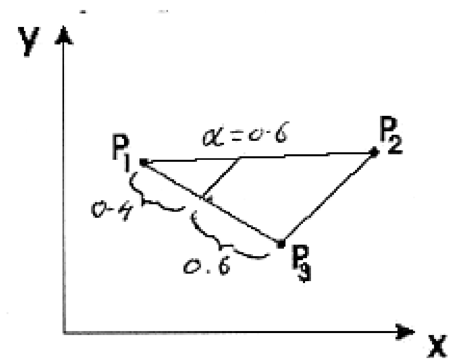
12. (1 pt) Write out the OpenGL perspective transformation matrix for the above configuration.

$$\begin{pmatrix} \frac{6}{2} & 0 & \frac{0}{2} & 0 \\ 0 & \frac{6}{3} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{8}{2} & -\frac{30}{2} \\ 0 & 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & \frac{1}{3} & 0 \\ 0 & 0 & -4 & -15 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

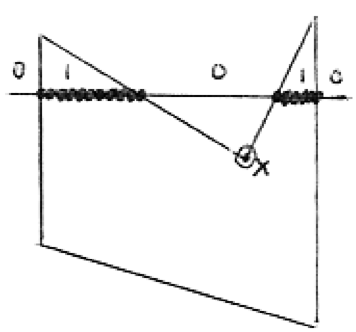
13. (1 pt) Briefly describe how to implement per-object picking using the back buffer.

STORE A UNIQUE COLOR FOR EACH OBJECT (PICKABLE) IN SCENE IN A TABLE. RENDER SCENE TO BACK BUFFER WITH SHADING TURNED OFF. READ BACK PIXEL AT CURSOR LOCATION AND CHECK AGAINST TABLE.

14. (1 pt) A point in a triangle can be expressed using barycentric coordinates as follows: $P = \alpha P_1 + \beta P_2 + \gamma P_3$, where $0 <= \alpha, \beta, \gamma <= 1$ and $-\alpha + \beta + gamma = 1$. Draw the line corresponding to $\alpha = .6$ on the following triangle which sits in the xy-plane.



15. (1 pt) Briefly describe how to use parity when scan converting a general polygon.



PARITY TEST:

SCAN ALONG EACH SCANLINE. ON ODD NUMBER OF EDGE CROSSINGS, RASTERIZE PIXELS. ON EVEN EDGE CROSSINS STOP RASTERIZING.

SPECIAL CASES:

(i) COUNT CONCAVE (SPLIT) VERTICES TWICE (e.g. X)

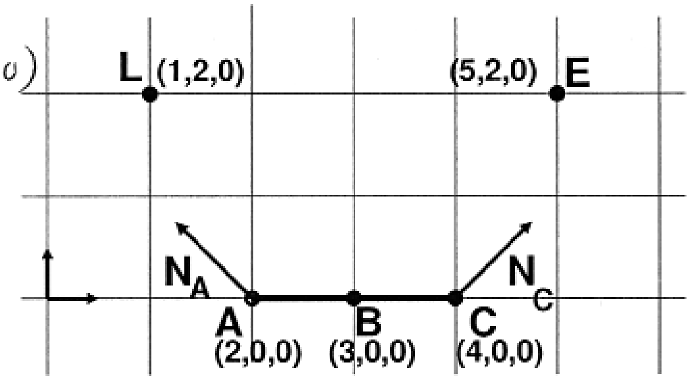
(ii) DON'T RASTERIZE BOTTOM HORIZONTAL EDGE.

In the problems below, use the Phong illumination model given by $I = I_a k_a + k_d I_L (N \cdot L) + k_s I_L (R \cdot V)^n$ with parameters $I_a = .8, I_L = 1.0, k_a = .2, k_d = .9, k_s = .5, n = 30$.

$l_A = (-1/\sqrt{5}, 2/\sqrt{5}, 0)$ $\quad$ $l_C = (-3/\sqrt{13}, 2/\sqrt{13}, 0)$

$n_A = (-1/\sqrt{2}, 1/\sqrt{2}, 0)$ $\quad$ $n_C = (1/\sqrt{2}, 1/\sqrt{2}, 0)$

$V_A = (3/\sqrt{13}, 2/\sqrt{13}, 0)$ $\quad$ $V_C = (1/\sqrt{5}, 2/\sqrt{5}, 0)$



16. (2 pts) Give the specular component of B, using the Gouraud shading model.

$R_A = 2 \times n_A \times (n_A \cdot l_A) - l_A = (-2/\sqrt{5}, 1/\sqrt{5}, 0) \quad \therefore R_A \cdot V_A = 0$

$\therefore \text{SPECULAR}_A = k_s \times I_\ell \times (R_A \cdot V_A)^n = 0$

$R_C = 2 \times n_C \times (n_C \cdot l_C) - l_C = -l_C \ [n_C \cdot l_C = 0] \quad \therefore R_C \cdot V_C = 0$

$\therefore \text{SPECULAR}_C = k_s \times I_\ell \times (R_C \cdot V_C)^n = 0$

$\therefore \text{SPECULAR}_B = \dfrac{\text{SPECULAR}_A + \text{SPECULAR}_C}{2} = 0$

17. (2 pts) Give the specular component of B, using the Phong shading model.

$n_B = \dfrac{n_A + n_C}{|n_A + n_C|} = (0, 1, 0)$

$l_B = (-2/\sqrt{8}, 2/\sqrt{8}, 0)$

$V_B = (2/\sqrt{8}, 2/\sqrt{8}, 0)$

$R_B = 2 \times n_B \times (n_B \cdot l_B) - l_B = (2/\sqrt{8}, 2/\sqrt{8}, 0)$

$\therefore R_B \cdot V_B = (4/8 + 4/8 + 0) = 1.0$

$\therefore \text{SPECULAR}_B = 0.5 \times 1.0 \times 1.0 = 0.5$