1) $A = P_1 + t_1(P_2 - P_1)$

$$\begin{pmatrix} X_a \\ 20 \end{pmatrix} = \begin{pmatrix} 14 \\ 18 \end{pmatrix} + t_1 \begin{pmatrix} 30-14 \\ 32-18 \end{pmatrix}$$

$X_a = 14 + t_1(16)$

$20 = 18 + t_1(14) \to 2 = 14t_1 \to t_1 = \frac{1}{7}$

$X_a = 14 + (\frac{1}{7})(16) = \frac{98+16}{7} = \frac{114}{7}$

$B = P_3 + t_2(P_2 - P_3)$

$$\begin{pmatrix} X_b \\ 20 \end{pmatrix} = \begin{pmatrix} 42 \\ 12 \end{pmatrix} + t_2 \begin{pmatrix} 30-42 \\ 32-12 \end{pmatrix}$$

$X_b = 42 + t_2(-12)$

$20 = 12 + t_2(20) \to 8 = 20t_2 \to t_2 = \frac{2}{5}$

$X_b = 42 + (\frac{2}{5})(-12) = \frac{208-24}{5} = \frac{186}{5}$

$P = A + t_3(B - A)$

$$\begin{pmatrix} 25 \\ 20 \end{pmatrix} = \begin{pmatrix} \frac{114}{7} \\ 20 \end{pmatrix} + t_3 \begin{pmatrix} \frac{186}{5} - \frac{114}{7} \\ 20 - 20 \end{pmatrix}$$

$25 = \frac{114}{7} + t_3(\frac{186}{5} - \frac{114}{7})$

$\frac{61}{7} = t_3(\frac{732}{35}) \to t_3 = \frac{61}{7}(\frac{35}{732}) = \frac{305}{732} = \frac{5}{12}$

$Z_a = 52 + (\frac{1}{7})(10-52)$
$= 52 + (-6) = 46$

$Z_b = 32 + (\frac{2}{5})(10-32)$
$= 32 - \frac{214}{5} = \frac{116}{5}$

$Z_p = 46 + (\frac{5}{12})(\frac{116}{5} - 46)$
$= 46 + (\frac{5}{12})(\frac{-114}{5}) = 46 + (\frac{-57}{6}) = 46 + (\frac{-19}{2}) = \frac{73}{2}$

2 $n = (P_2 - P_1) \times (P_3 - P_1)$

$$= \begin{pmatrix} 30-14 \\ 32-18 \\ 10-52 \end{pmatrix} \times \begin{pmatrix} 42-14 \\ 12-18 \\ 32-52 \end{pmatrix} = \begin{pmatrix} 16 \\ 14 \\ -42 \end{pmatrix} \times \begin{pmatrix} 28 \\ -6 \\ -20 \end{pmatrix} = \begin{pmatrix} 14(-20)-(-6)(-42) \\ (-42)(28)-(-20)(16) \\ (16)(-6)-(14)(28) \end{pmatrix}$$

$$= \begin{pmatrix} -532 \\ -856 \\ -488 \end{pmatrix}$$

$\|n\| = \sqrt{(-532)^2 + (-856)^2 + (-488)^2}$
$= 1120$

$$n/|n| = \begin{pmatrix} -532/1120 \\ -856/1120 \\ -488/1120 \end{pmatrix} = \begin{pmatrix} -133/280 \\ -107/140 \\ -61/140 \end{pmatrix}$$

$$n \cdot (P - P_1) = 0$$

$$\begin{pmatrix} -133/280 \\ -107/140 \\ -61/140 \end{pmatrix} \cdot \begin{pmatrix} 25 - 14 \\ 20 - 18 \\ z - 52 \end{pmatrix} = \begin{pmatrix} -133/280 \\ -107/140 \\ -61/140 \end{pmatrix} \cdot \begin{pmatrix} 11 \\ 2 \\ z - 52 \end{pmatrix}$$

$$= \left(\frac{-133}{280}\right)(11) + \left(\frac{-107}{140}\right)(2) + \left(\frac{-61}{140}\right)(z - 52) = 0$$

$$\left(\frac{-61}{140}\right)(z - 52) = \left(\frac{133}{280}\right)(11) + \left(\frac{107}{140}\right)(2) = \frac{1891}{280}$$

$$z - 52 = \frac{1891}{280}\left(\frac{140}{-61}\right) = \frac{-31}{2}$$

$$z = \frac{-31}{2} + 52 = \frac{73}{2}$$

3 from 1)

$$A = P_1 + t_1(P_2 - P_1) \qquad B = P_3 + t_2(P_2 - P_3) \qquad P = A + t_3(B - A)$$

$$A = (1 - t_1)P_1 + t_1 P_2 \qquad B = (1 - t_2)P_3 + t_2 P_2 \qquad P = (1 - t_3)A + t_3 B$$

$$P = (1 - t_3)\left[(1 - t_1)P_1 + t_1 P_2\right] + t_3\left[(1 - t_2)P_3 + t_2 P_2\right]$$

$$= (1 - t_3)(1 - t_1)P_1 + (1 - t_3)t_1 P_2 + t_3(1 - t_2)P_3 + t_2 t_3 P_2$$

$$= (1 - t_3)(1 - t_1)P_1 + \left[(1 - t_3)t_1 + t_2 t_3\right]P_2 + t_3(1 - t_2)P_3$$

$$= \left(1 - \tfrac{5}{12}\right)\left(1 - \tfrac{1}{7}\right)P_1 + \left[\left(1 - \tfrac{5}{12}\right)\left(\tfrac{1}{7}\right) + \left(\tfrac{2}{5}\right)\left(\tfrac{5}{12}\right)\right]P_2 + \tfrac{5}{12}\left(1 - \tfrac{2}{5}\right)P_3$$

$$= \left(\tfrac{7}{12}\right)\left(\tfrac{6}{7}\right)P_1 + \left[\left(\tfrac{7}{12}\right)\left(\tfrac{1}{7}\right) + \tfrac{1}{6}\right]P_2 + \tfrac{5}{12}\left(\tfrac{3}{5}\right)P_3$$

$$= \tfrac{1}{2}P_1 + \tfrac{1}{4}P_2 + \tfrac{1}{4}P_3$$

$$\alpha = \tfrac{1}{2}, \quad \beta = \tfrac{1}{4}, \quad \gamma = \tfrac{1}{4}$$

$$z_p = \alpha z_1 + \beta z_2 + \gamma z_3$$
$$= \tfrac{1}{2}(52) + \tfrac{1}{4}(10) + \tfrac{1}{4}(32)$$
$$= 26 + \tfrac{5}{2} + 8 = \tfrac{73}{2}$$

$$r_p = \alpha r_1 + \beta r_2 + \gamma r_3$$
$$= \tfrac{1}{2}(1) + \left(\tfrac{1}{4}\right)(0.3) + \tfrac{1}{4}(0)$$
$$= 0.5 + 0.075 = 0.575$$

$$g_p = \alpha g_1 + \beta g_2 + \gamma g_3$$
$$= \left(\tfrac{1}{2}\right)(1) + \left(\tfrac{1}{4}\right)(0.2) + \left(\tfrac{1}{4}\right)(0.5)$$
$$= 0.5 + 0.05 + 0.125 = 0.675$$

$$b_p = \alpha b_1 + \beta b_2 + \gamma b_3$$
$$= \left(\tfrac{1}{2}\right)(0) + \left(\tfrac{1}{4}\right)(0.8) + \left(\tfrac{1}{4}\right)(0)$$
$$= 0.2$$

3 Another method

$P = \alpha P_1 + \beta P_2 + \gamma P_3$

$$\begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} = \alpha \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \beta \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} + \gamma \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}$$

we have

$$x_p = \alpha x_1 + \beta x_2 + \gamma x_3$$
$$y_p = \alpha y_1 + \beta y_2 + \gamma y_3$$
$$\gamma = 1 - \alpha - \beta$$

$$\Rightarrow x_p = \alpha x_1 + \beta x_2 + (1 - \alpha - \beta) x_3$$
$$= \alpha (x_1 - x_3) + \beta (x_2 - x_3) + x_3$$
$$y_p = \alpha (y_1 - y_3) + \beta (y_2 - y_3) + y_3$$

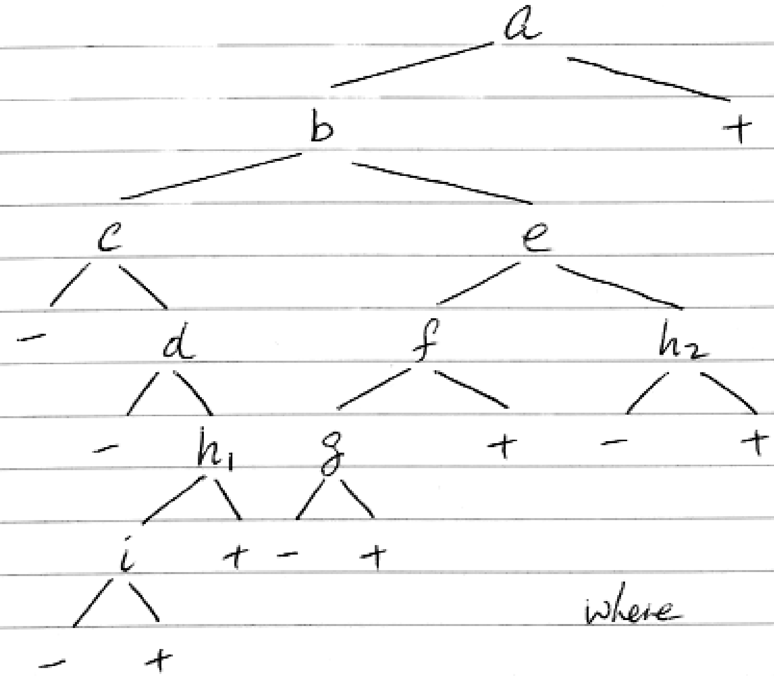
$$\therefore 25 = \alpha(14 - 42) + \beta(30 - 42) + 42 \Rightarrow 25 = -28\alpha - 12\beta + 42$$
$$20 = \alpha(18 - 12) + \beta(32 - 12) + 12 \qquad 20 = 6\alpha + 20\beta + 12$$

$$\Rightarrow 28\alpha + 12\beta = 17 \quad \Rightarrow \quad 140\alpha + 60\beta = 85$$
$$6\alpha + 20\beta = 8 \qquad\qquad 18\alpha + 60\beta = 24$$
$$122\alpha = 61 \quad \Rightarrow \quad \alpha = \frac{1}{2}$$

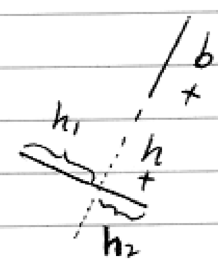
$$6\left(\frac{1}{2}\right) + 20\beta = 8 \Rightarrow 3 + 20\beta = 8 \Rightarrow 20\beta = 5 \Rightarrow \beta = \frac{1}{4}$$
$$\gamma = 1 - \alpha - \beta = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\therefore \alpha = \frac{1}{2}, \ \beta = \frac{1}{4}, \ \gamma = \frac{1}{4}$$

4,



where



5, We draw far objects first.

(-a) a (+a)

(+b) b (-b) a (+a)

(-e) e (+e) b (-b) a (+a)

(+f) f (-f) e (+e) b (-b) a (+a)

~~(+g) g (-g) f (-f) e (+e) b (-b) a (+a)~~

f (+g) g (-g) e (+e) b (-b) a (+a)

f g e (+h_2) h_2 (-h_2) b (-b) a (+a)

f g e h_2 b (+c) c (-c) a (+a)

f g e h_2 b (+d) d (-d) c (-c) a (+a)

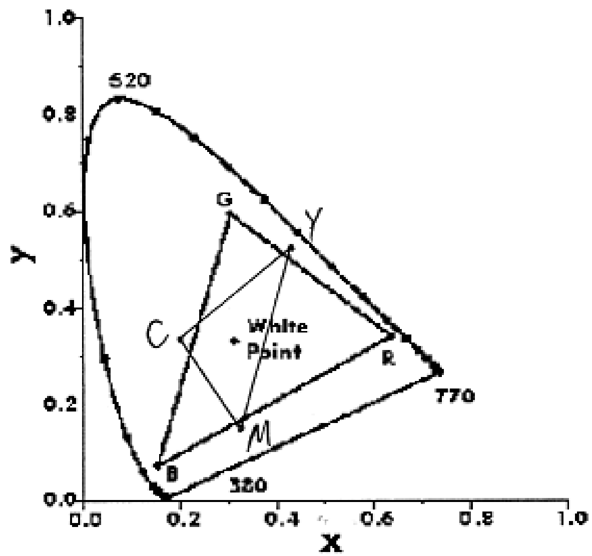
f g e h_2 b (+h_1) h_1 (-h_1) d (-d) c (-c) a (+a)

f g e h_2 b h_1 (+i) i (-i) d (-d) c (-c) a (+a)

f g e h_2 b h_1 i d c a

Final order : f g e h_2 b h_1 i d c a

7.

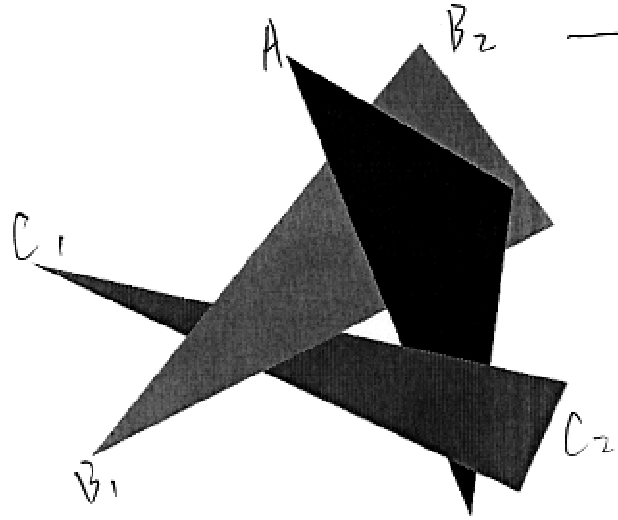


The gamut of CMY is shown in left figure. Since monitor is additive color mixing, the gamut is just the triangle of points C, M and Y.

This would not make a good monitor because

1. The gamut of CMY is much smaller than that of RGB
2. CMY can't make pure Red, Green or Blue colors, which are very important for display.

6.



BSP tree will choose a polygon as root, then split the other two polygons and add parts to the front/behind list.

For example, we choose polygon A as the root for the left figure. Suppose the plane of A intersects with polygon B and C. Then these two polygons are splitted into two parts, which are added to the front/behind list of A for recursive processing.

8. Here are some reasons:

1. Different type of monitor: $RGB_{CRT} \neq RGB_{LCD}$
2. The system Gamma and Gamut are different
3. The brightness and contrast settings are different
4. The environment, such as the ambient light, is different.

9. $P_1(-4,-5,7)$, $P_2(0,-6,-10)$, $P_3(0,-4,-12)$

$b=-1 \quad t=1 \quad l=-1 \quad r=1 \quad n=3 \quad far=9$

plane equation:

left: $x + l * z/n = x - z/3 = 0$

right: $-(x + z/3) = 0$

top: $-(y + t * z/n) = -(y + z/3) = 0$

bottom: $y - z/3 = 0$

near: $-(z+3) = 0$

far: $z + 9 = 0$

Clipping:

$\boxed{\text{Top}}$ P_1: $-(-5 + 7/3) > 0$ Inside

P_2: $-(-6 - 10/3) > 0$ Inside

P_3: $-(-4 - 12/3) > 0$ Inside

output $\overrightarrow{P_1P_2}$, $\overrightarrow{P_2P_3}$, $\overrightarrow{P_3P_1}$

$\boxed{\text{Bottom}}$ P_1: $-5 - 7/3 < 0$ outside

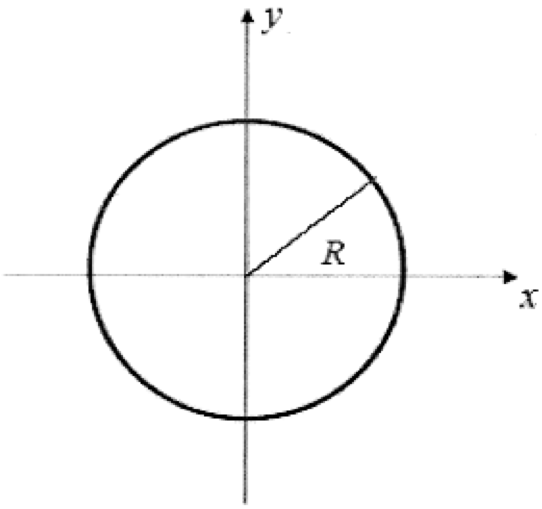
P_2: $-6 + 10/3 < 0$ outside

P_3: $-4 + 12/3 = 0$ outside

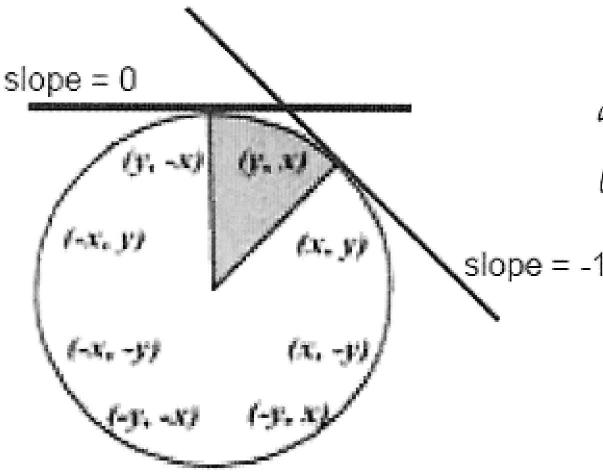
output none

After clipping of bottom plane, nothing left in the frustum.

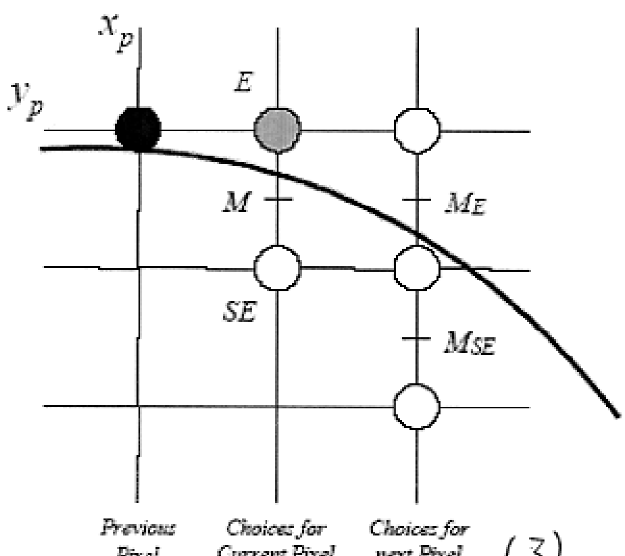
10.



(1)



(2)



(3)

Assume the circle is $x^2+y^2=R^2$

For the function $d = F(x,y) = x^2+y^2-R^2$

we have $\begin{cases} d=0 : & (x,y) \text{ is on the circle} \\ d>0 : & (x,y) \text{ is outside} \\ d<0 : & (x,y) \text{ is inside} \end{cases}$

Since the symmetry of a circle, we only need to compute 1/8 of the whole circle (Figure 2), starting from (0, R) to the point where $x \geq y$

Similar to Bresenham algorithm, we only need to consider the E and SE directions for next step, depending on which pixel is closer to the circle. That is, whether the midpoint of E and SE is inside or outside the circle (figure 3)

The computation of d_{old}, d_{new} and $d_{initial}$ is:

$$d_{old} = F(x_p+1, y_p-\tfrac{1}{2}) = (x_p+1)^2+(y_p-\tfrac{1}{2})^2-R^2$$

$$d_{new} = \begin{cases} \text{next} = E : & F(x_p+2, y_p-\tfrac{1}{2}) \\ & = d_{old}+2x_p+3 \\ \text{next} = SE : & F(x_p+2, y_p-\tfrac{3}{2}) \\ & = d_{old}+2x_p-2y_p+5 \end{cases}$$

$$d_{initial} = F(x_0+1, y_0-\tfrac{1}{2}) = 5/4 - R$$

Reference: http://www.cs.sfu.ca/CourseCentral/361/hyounesy/Lectures/09-raster.pdf

10. (Continue)

What we need is integer only algorithm. But $d_{initial}$ is not an integer! There are two methods to solve this. One is multiply 4 for each expression of d, the other is just using $1-R$ for $d_{initial}$ since ~~the sign of~~

$$\begin{cases} d \le 0 \iff d - \frac{1}{4} \le 0 \\ d > 0 \iff d - \frac{1}{4} > 0 \end{cases}$$

The pseudo code for drawing a circle is

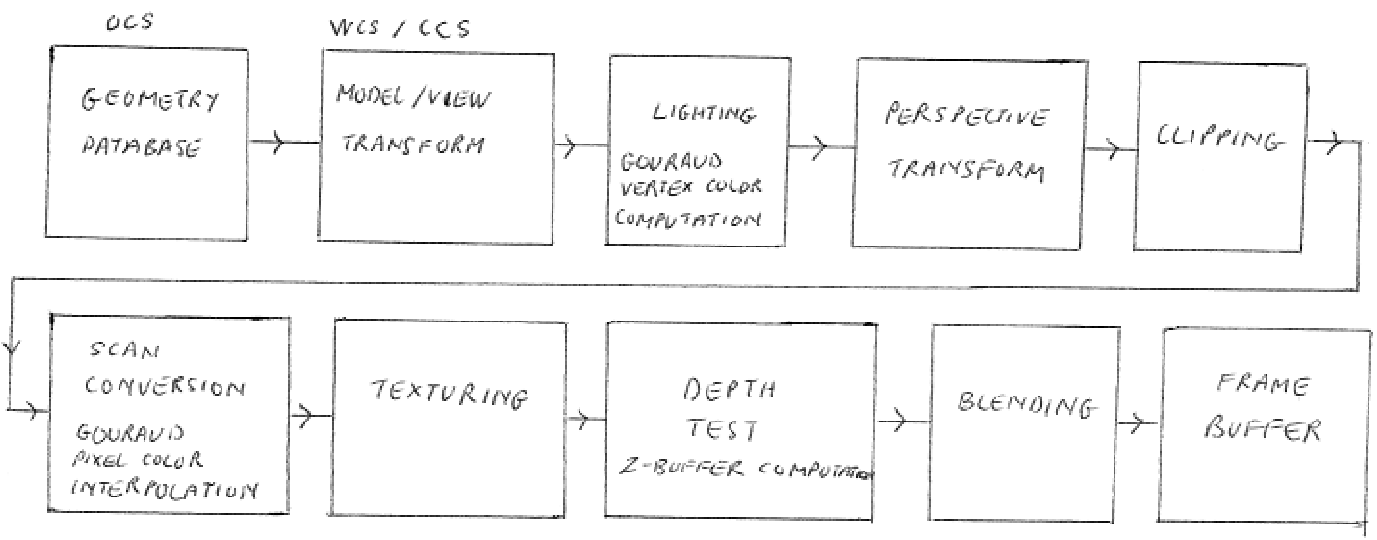
```
x = 0 ; y = R ; d = d_initial
draw8waypoint (x, y)
while (x < y) {
    if (d < 0) {
        E direction
    } else {
        SE direction
    }
    draw 8 way point (x, y)
}
```

CPSC 414, WRITTEN HOMEWORK 2 SOLUTIONS

Q11. FACES THAT WOULD BE REMOVED BY BACKFACE CULLING ARE:

D, F, G & I

Q12 GRAPHICS PIPELINE



Q13 GIMBAL LOCK:

IT IS A PROBLEM THAT CAN OCCUR WHILE SPECFYING ROTATIONS USING EULER ANGLES. IF DUE TO A SEQUENCE OF ROTATIONS ABOUT THE THREE AXES, TWO OF THE AXES GET ALIGNED THEN WE ARRIVE AT A CONFIGURATION CALLED GIMBAL LOCK. ONE DEGREE OF FREEDOM IS LOST AS ROTATION ABOUT ONE OF THESE AXES RESULTS IN ROTATION ABOUT THE OTHER AXIS.

TWO METHODS THAT AVOID GIMBAL LOCKS ARE:

AXIS-ANGLE ROTATION

QUATERNION ROTATION

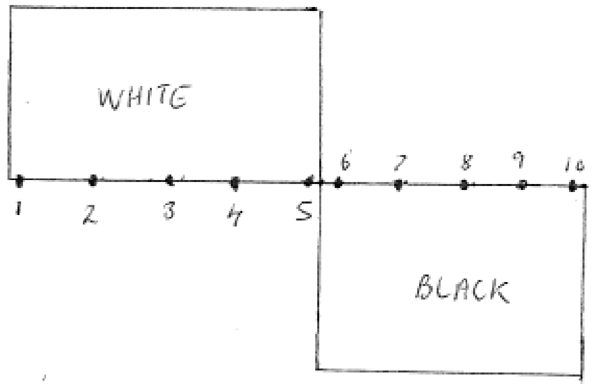
Q14 SITUATION A : 10,000 POLYGONS & 640 X 480 PIXEL FRAMEBUFFER
SITUATION B : 1,000 POLYGONS & 4800 X 2400 PIXEL FRAMEBUFFER

HIDDEN SURFACE REMOVAL CAN BE DONE BOTH AT THE OBJECT LEVEL (POLYGONS) AND IMAGE LEVEL (PIXELS). IN CLASS, WE SAW A FEW OBJECT LEVEL ALGORITHMS FOR RESOLVING VISIBILITY SUCH AS THE BINARY SPACE PARTITION (BSP) TREES AND THE WARNOCK'S ALGORITHM AS WELL AS THE Z-BUFFER (IMAGE LEVEL) ALGORITHM.

IN SITUATION A, THERE ARE MANY MORE POLYGONS AND COMPARATIVELY FEW PIXELS. MOST POLYGONS COULD BE OCCLUDED AS DUE TO A LIMITED NUMBER OF PIXELS, MANY POLYGONS COULD PROJECT TO THE SAME PIXELS. IN SUCH A SITUATION IT WOULD MAKE SENSE TO RESOLVE VISIBILITY AT THE PIXEL LEVEL BY USING THE Z-BUFFER ALGORITHM. MOST POLYGONS WOULD FAIL THE DEPTH TEST ELIMINATING THE NEED TO MODIFY THE FRAME BUFFER. ALSO, RESOLVING VISIBILITY AT THE OBJECT LEVEL IS TYPICALLY SCENE DEPENDENT AND REQUIRES MANAGING DATA STRUCTURES SUCH AS BSP TREES. IF THE SCENE CHANGES, RECOMPUTATION OF THE TREE IS EXPESIVE FOR A HIGH POLYGON COUNT.

IN SITUATION B, THE TRADE-OFF IS REVERSED. A FEW POLYGONS SHOULD BE EASY TO MANAGE AT THE OBJECT LEVEL. ALSO, WITH MANY PIXELS TO PROJECT TO, THERE WILL POTENTIALLY BE MANY UPDATES TO THE DEPTH & FRAMEBUFFER. ALSO THERE IS THIS HUGE MEMORY REQUIREMENT FOR SUCH LARGE DEPTH & FRAME BUFFERS.

Q15.



THE FIGURE DEPICTS THE RESPONSE OF THE CHECKER BOARD SIGNAL WITH WHITE BEING HIGH AND BLACK BEING LOW. THE PERIOD OF THIS SIGAL IS 10 PIXELS. ACCORDING TO THE NYQUIST RATE, WE NEED TO SAMPLE THIS SIGNAL AT TWICE THE PERIOD OF ITS HIGHEST FREQUENCY (HERE THERE IS ONLY ONE FREQUENCY). THUS WE NEED TO SAMPLE EVERY 5 PIXELS TO BE AT THE NYQUIST RATE. NOTE THAT BY SAMPLING AT EVERY 5 PIXELS YOU DO NOT RECONSTRUCT THE ENTIRE SIGNAL, JUST THAT WHITE AND BLACK PIXELS ALTERNATE IN YOUR SIGNAL. THEREBY SAMPLING AT NYQUIST RATE TELLS YOU THE ESSENTIAL CHARACTERISTICS OF THE SIGNAL.