

University of British Columbia CPSC 314 Computer Graphics May-June 2005

## Tamara Munzner Sampling, Virtual Trackball, Hidden Surfaces

## Week 5, Tue Jun 7

http://www.ugrad.cs.ubc.ca/~cs314/Vmay2005

# News

- Midterm handed back
  - solutions posted
  - distribution posted
  - all grades so far posted
- P1 Hall of Fame posted
- P3 grading
  - after 3:20
- P4 proposals
  - email or conversation to all

# H3 Corrections/Clarifications

- Q1 should be from +infinity, not -infinity
- Q 2-4 correction for point B
- Q7 clarified: only x and y coordinates are given for P
- Q8 is deleted

### **Review: Texture Coordinates**

- texture image: 2D array of color values (texels)
- assigning texture coordinates (s,t) at vertex with object coordinates (x,y,z,w)
  - use interpolated (s,t) for texel lookup at each pixel
  - use value to modify a polygon's color
    - or other surface property
  - specified by programmer or artist



glTexCoord2f(s,t)

#### **Review: Tiled Texture Map**



#### **Review: Fractional Texture Coordinates**

texture image

(0,1) (1,1)



(0,0) (1,0)

(0,.5) (.25,.5)



(0,0) (.25,0)

## **Review: Texture**

- action when s or t is outside [0...1] interval
  - tiling
  - clamping
- functions
  - replace/decal
  - modulate
  - blend
- texture matrix stack

glMatrixMode( GL\_TEXTURE );

# **Review: Basic OpenGL Texturing**

- setup
  - generate identifier: glGenTextures
  - Ioad image data: glTexImage2D
  - set texture parameters (tile/clamp/...): glTexParameteri
  - set texture drawing mode (modulate/replace/...): glTexEnvf
- drawing
  - enable: glEnable
  - bind specific texture: glBindTexture
  - specify texture coordinates before each vertex: glTexCoord2f

#### **Review: Perspective Correct Interpolation**

screen space interpolation incorrect

 $s = \frac{\alpha \cdot s_0 / w_0 + \beta \cdot s_1 / w_1 + \gamma \cdot s_2 / w_2}{\alpha / w_0 + \beta / w_1 + \gamma / w_2}$ 



### **Review: Reconstruction**

- how to deal with:
  - pixels that are much larger than texels?
    - apply filtering, "averaging"



- pixels that are much smaller than texels ?
  - interpolate



## **Review: MIPmapping**

image pyramid, precompute averaged versions





Without MIP-mapping





With MIP-mapping<sup>11</sup>

#### **Review: Bump Mapping: Normals As Texture**

- create illusion of complex geometry model
- control shape effect by locally perturbing surface normal







#### **Review: Environment Mapping**

- cheap way to achieve reflective effect
  - generate image of surrounding
  - map to object as texture



#### **Review: Sphere Mapping**

- texture is distorted fish-eye view
  - point camera at mirrored sphere
  - spherical texture coordinates



#### **Review: Cube Mapping**

6 planar textures, sides of cube

- point camera outwards to 6 faces
  - use largest magnitude of vector to pick face
  - other two coordinates for (s,t) texel location





#### **Review: Volumetric Texture**

- define texture pattern over 3D domain - 3D space containing the object
  - texture function can be digitized or procedural
  - for each point on object compute texture from point location in space
- 3D function  $\rho(x,y,z)$



#### **Review: Perlin Noise: Procedural Textures**

function marble(point)
x = point.x + turbulence(point);
return marble\_color(sin(x))







#### **Review: Perlin Noise**

- coherency: smooth not abrupt changes
- turbulence: multiple feature sizes



## **Review: Generating Coherent Noise**

- just three main ideas
  - nice interpolation
  - use vector offsets to make grid irregular
  - optimization
    - sneaky use of 1D arrays instead of 2D/3D one

## **Review: Procedural Modeling**

- textures, geometry
  - nonprocedural: explicitly stored in memory
- procedural approach
  - compute something on the fly
    - not load from disk
  - often less memory cost
  - visual richness
    - adaptable precision
- noise, fractals, particle systems

## **Review: Language-Based Generation**

#### L-Systems

- F: forward, R: right, L: left
- Koch snowflake:
  F = FLFRRFLF
- Mariano's Bush:
   F=FF-[-F+F+F]+[+F-F-F]
  - angle 16

http://spanky.triumf.ca/www/fractint/lsys/plants.html





# **Correction/Review: Fractal Terrain**

- 1D: midpoint displacement
  - divide in half, randomly displace
  - scale variance by half
- 2D: diamond-square
  - generate new value at midpoint
  - average corner values + random displacement
    - scale variance by half each time







## **Review: Particle Systems**

- changeable/fluid stuff
  - fire, steam, smoke, water, grass, hair, dust, waterfalls, fireworks, explosions, flocks
- life cycle
  - generation, dynamics, death
- rendering tricks
  - avoid hidden surface computations



# Sampling

# **Samples**

- most things in the real world are continuous
- everything in a computer is discrete
- the process of mapping a continuous function to a discrete one is called sampling
- the process of mapping a discrete function to a continuous one is called reconstruction
- the process of mapping a continuous variable to a discrete one is called quantization
- rendering an image requires sampling and quantization
- displaying an image involves reconstruction

## **Line Segments**

- we tried to sample a line segment so it would map to a 2D raster display
- we quantized the pixel values to 0 or 1
- we saw stair steps, or jaggies



## **Line Segments**

- instead, quantize to many shades
- but what sampling algorithm is used?



## **Unweighted Area Sampling**

- shade pixels wrt area covered by thickened line
- equal areas cause equal intensity, regardless of distance from pixel center to area
  - rough approximation formulated by dividing each pixel into a finer grid of pixels
- primitive cannot affect intensity of pixel if it does not intersect the pixel



# **Weighted Area Sampling**

- intuitively, pixel cut through the center should be more heavily weighted than one cut along corner
- weighting function, W(x,y)
  - specifies the contribution of primitive passing through the point (x, y) from pixel center





#### Images

#### an image is a 2D function I(x, y) that specifies intensity for each point (x, y)



## **Image Sampling and Reconstruction**

- convert continuous image to discrete set of samples
- display hardware reconstructs samples into continuous image
  - finite sized source of light for each pixel



# **Point Sampling an Image**

- simplest sampling is on a grid
- sample depends solely on value at grid points



# **Point Sampling**

#### multiply sample grid by image intensity to obtain a discrete set of points, or samples.





## **Sampling Errors**

some objects missed entirely, others poorly sampled

- could try unweighted or weighted area sampling
- but how can we be sure we show everything?
- need to think about entire class of solutions!



## **Image As Signal**

- image as spatial signal
- 2D raster image
  - discrete sampling of 2D spatial signal
- ID slice of raster image
  - discrete sampling of 1D spatial signal

Intensity Origina signal Pixel position across scanline

Examples from Foley, van Dam, Feiner, and Hughes 35

# **Sampling Theory**

- how would we generate a signal like this out of simple building blocks?
- theorem
  - any signal can be represented as an (infinite) sum of sine waves at different frequencies
# **Sampling Theory in a Nutshell**

#### terminology

- bandwidth length of repeated sequence on infinite signal
- frequency 1/bandwidth (number of repeated sequences in unit length)
- example sine wave
  - bandwidth =  $2\pi$
  - frequency =  $1/2\pi$



## **Summing Waves I**



# **Summing Waves II**











#### problems

jaggies – abrupt changes



#### problems

- jaggies abrupt changes
- Iose data



# **Sampling Theorem**

continuous signal can be completely recovered from its samples

iff

sampling rate greater than twice maximum frequency present in signal

- Claude Shannon

## **Nyquist Rate**

Iower bound on sampling rate

twice the highest frequency component in the image's spectrum



## **Falling Below Nyquist Rate**

- when sampling below Nyquist Rate, resulting signal looks like a lower-frequency one
  - this is aliasing!



### **Nyquist Rate**



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# Aliasing

- incorrect appearance of high frequencies as low frequencies
- to avoid: antialiasing
  - supersample
    - sample at higher frequency
  - Iow pass filtering
    - remove high frequency function parts
    - aka prefiltering, band-limiting

## Supersampling



### **Low-Pass Filtering**



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### **Low-Pass Filtering**



# Filtering



low passblur

high passedge finding

### **Previous Antialiasing Example**

#### texture mipmapping: low pass filter



### **Virtual Trackball**

# **Virtual Trackball**

- interface for spinning objects around
  - drag mouse to control rotation of view volume
- rolling glass trackball
  - center at screen origin, surrounds world
  - hemisphere "sticks up" in z, out of screen
  - rotate ball = spin world

## **Virtual Trackball**

- know screen click: (x, 0, z)
- want to infer point on trackball: (x,y,z)
  - ball is unit sphere, so ||x, y, z|| = 1.0
  - solve for y



## **Trackball Rotation**

- correspondence:
  - moving point on plane from (x, 0, z) to (a, 0, c)
  - moving point on ball from  $\mathbf{p_1} = (x, y, z)$  to  $\mathbf{p_2} = (a, b, c)$
- correspondence:
  - translating mouse from p<sub>1</sub> (mouse down) to p<sub>2</sub> (mouse up)
  - rotating about the axis  $\mathbf{n} = \mathbf{p}_1 \times \mathbf{p}_2$



# **Trackball Computation**

user defines two points

• place where first clicked  $\mathbf{p}_1 = (x, y, z)$ 

place where released p<sub>2</sub> = (a, b, c)

create plane from vectors between points, origin

axis of rotation is plane normal: cross product

•  $(\mathbf{p}_{1} - \mathbf{o}) \times (\mathbf{p}_{2} - \mathbf{o})$ :  $\mathbf{p}_{1} \times \mathbf{p}_{2}$  if origin = (0,0,0)

- amount of rotation depends on angle between lines
  - **p**<sub>1</sub> **p**<sub>2</sub> =  $|\mathbf{p}_1| |\mathbf{p}_2| \cos \theta$
  - $|\mathbf{p}_1 \times \mathbf{p}_2| = |\mathbf{p}_1| |\mathbf{p}_2| \sin \theta$

compute rotation matrix, use to rotate world

## Visibility

### Reading

#### FCG Chapter 7

## **Rendering Pipeline**



# **Covered So Far**

- modeling transformations
- viewing transformations
- projection transformations
- clipping
- scan conversion
- lighting
- shading

we now know everything about how to draw a polygon on the screen, except visible surface determination

## **Invisible Primitives**

why might a polygon be invisible?

- polygon outside the *field of view / frustum* 
  - solved by clipping
- polygon is *backfacing* 
  - solved by backface culling
- polygon is occluded by object(s) nearer the viewpoint
  - solved by hidden surface removal
- for efficiency reasons, we want to avoid spending work on polygons outside field of view or backfacing
- for efficiency and correctness reasons, we need to know when polygons are occluded

### **Hidden Surface Removal**

# Occlusion

 for most interesting scenes, some polygons overlap



to render the correct image, we need to determine which polygons occlude which

# **Painter's Algorithm**

simple: render the polygons from back to front, "painting over" previous polygons



- draw blue, then green, then orange
- will this work in the general case?

## **Painter's Algorithm: Problems**

- intersecting polygons present a problem
- even non-intersecting polygons can form a cycle with no valid visibility order:



## **Analytic Visibility Algorithms**

 early visibility algorithms computed the set of visible polygon *fragments* directly, then rendered the fragments to a display:



# **Analytic Visibility Algorithms**

- what is the minimum worst-case cost of computing the fragments for a scene composed of n polygons?
- answer:
  O(n<sup>2</sup>)



## **Analytic Visibility Algorithms**

- so, for about a decade (late 60s to late 70s) there was intense interest in finding efficient algorithms for hidden surface removal
- we'll talk about two:
  - Binary Space Partition (BSP) Trees
  - Warnock's Algorithm
# **Binary Space Partition Trees (1979)**

- BSP Tree: partition space with binary tree of planes
  - idea: divide space recursively into half-spaces by choosing splitting planes that separate objects in scene
  - preprocessing: create binary tree of planes
  - runtime: correctly traversing this tree enumerates objects from back to front











# **Splitting Objects**

- no bunnies were harmed in previous example
- but what if a splitting plane passes through an object?
  - split the object; give half to each node



# **Traversing BSP Trees**

- tree creation independent of viewpoint
  - preprocessing step
- tree traversal uses viewpoint
  - runtime, happens for many different viewpoints
- each plane divides world into near and far
  - for given viewpoint, decide which side is near and which is far
    - check which side of plane viewpoint is on independently for each tree vertex
    - tree traversal differs depending on viewpoint!
  - recursive algorithm
    - recurse on far side
    - draw object
    - recurse on near side

### **Traversing BSP Trees**

query: given a viewpoint, produce an ordered list of (possibly split) objects from back to front:

```
renderBSP(BSPtree *T)
BSPtree *near, *far;
if (eye on left side of T->plane)
    near = T->left; far = T->right;
else
    near = T->right; far = T->left;
renderBSP(far);
 if (T is a leaf node)
    renderObject(T)
 renderBSP(near);
```







not just left or right child!

























### **BSP Tree Traversal: Polygons**

- split along the plane defined by any polygon from scene
- classify all polygons into positive or negative half-space of the plane
  - if a polygon intersects plane, split polygon into two and classify them both
- recurse down the negative half-space
- recurse down the positive half-space



#### useful demo:

http://symbolcraft.com/graphics/bsp



# **Summary: BSP Trees**

- pros:
  - simple, elegant scheme
  - correct version of painter's algorithm back-to-front rendering approach
  - was very popular for video games (but getting less so)
- CONS:
  - slow to construct tree: O(n log n) to split, sort
  - splitting increases polygon count: O(n<sup>2</sup>) worst-case
  - computationally intense preprocessing stage restricts algorithm to static scenes

# Warnock's Algorithm (1969)

- based on a powerful general approach common in graphics
  - if the situation is too complex, subdivide
- BSP trees was object space approach
- Warnock is image space approach

# Warnock's Algorithm

- start with root viewport and list of all objects
- recursion:
  - clip objects to viewport
  - if only 0 or 1 objects
    - done
  - else
    - subdivide to new smaller viewports
    - distribute objects to new viewpoints
    - recurse



# Warnock's Algorithm

#### termination

- viewport is single pixel
- explicitly check for object occlusion



# Warnock's Algorithm

#### pros:

- very elegant scheme
- extends to any primitive type
- CONS:
  - hard to embed hierarchical schemes in hardware
  - complex scenes usually have small polygons and high depth complexity (number of polygons that overlap a single pixel)
    - thus most screen regions come down to the single-pixel case

# The Z-Buffer Algorithm (mid-70's)

- both BSP trees and Warnock's algorithm were proposed when memory was expensive
  - first 512x512 framebuffer was >\$50,000!
- Ed Catmull proposed a radical new approach called z-buffering.
- the big idea:
  - resolve visibility independently at each pixel

we know how to rasterize polygons into an image discretized into pixels:



- what happens if multiple primitives occupy the same pixel on the screen?
  - which is allowed to paint the pixel?



idea: retain depth after projection transform

- each vertex maintains z coordinate
  - relative to eye point
- can do this with canonical viewing volumes

- augment color framebuffer with Z-buffer or depth buffer which stores Z value at each pixel
  - at frame beginning, initialize all pixel depths to ∞
  - when rasterizing, interpolate depth (Z) across polygon
  - check Z-buffer before storing pixel color in framebuffer and storing depth in Z-buffer
  - don't write pixel if its Z value is more distant than the Z value already stored there
# **Interpolating Z**

edge equations: Z just another planar parameter:

• if walking across scanline by  $(D_x)$  $z_{new} = z_{old} - (A/C)(D_x)$ 

total cost:

- I more parameter to increment in inner loop
- 3x3 matrix multiply for setup

# **Interpolating Z**

- edge walking
  - just interpolate Z along edges and across spans
- barycentric coordinates
  - interpolate Z like other parameters



## **Z-Buffer**

store (r,g,b,z) for each pixel
typically 8+8+8+24 bits, can be more

```
for all i,j {
  Depth[i,j] = MAX_DEPTH
  Image[i,j] = BACKGROUND_COLOUR
}
for all polygons P {
  for all pixels in P {
    if (Z_pixel < Depth[i,j]) {
      Image[i,j] = C_pixel
      Depth[i,j] = Z_pixel
    }
  }
}</pre>
```

#### **Depth Test Precision**

- reminder: projective transformation maps eye-space z to generic z-range (NDC)
- simple example:



## **Depth Test Precision**

- therefore, depth-buffer essentially stores 1/z, rather than z!
- issue with integer depth buffers
  - high precision for near objects
  - Iow precision for far objects



# **Depth Test Precision**

- low precision can lead to depth fighting for far objects
  - two different depths in eye space get mapped to same depth in framebuffer
  - which object "wins" depends on drawing order and scan-conversion
- gets worse for larger ratios f:n
  - *rule of thumb:* f:n < 1000 *for 24 bit depth buffer*
- with 16 bits cannot discern millimeter differences in objects at 1 km distance

## **Z-Buffer Algorithm Questions**

- how much memory does the Z-buffer use?
- does the image rendered depend on the drawing order?
- does the time to render the image depend on the drawing order?
- how does Z-buffer load scale with visible polygons? with framebuffer resolution?

#### **Z-Buffer Pros**

- simple!!!
- easy to implement in hardware
  - hardware support in all graphics cards today
- polygons can be processed in arbitrary order
- easily handles polygon interpenetration
- enables deferred shading
  - rasterize shading parameters (e.g., surface normal) and only shade final visible fragments

#### **Z-Buffer Cons**

 poor for scenes with high depth complexity
 need to render all polygons, even if most are invisible



shared edges are handled inconsistently
 ordering dependent

# **Z-Buffer Cons**

- requires lots of memory
  - (e.g. 1280x1024x32 bits)
- requires fast memory
  - Read-Modify-Write in inner loop
- hard to simulate translucent polygons
  - we throw away color of polygons behind closest one
  - works if polygons ordered back-to-front
    - extra work throws away much of the speed advantage

#### **Hidden Surface Removal**

- two kinds of visibility algorithms
  - object space methods
  - image space methods



# **Object Space Algorithms**

- determine visibility on object or polygon level
  - using camera coordinates
- resolution independent
  - explicitly compute visible portions of polygons
- early in pipeline
  - after clipping
- requires depth-sorting
  - painter's algorithm
  - BSP trees

## **Image Space Algorithms**

- perform visibility test for in screen coordinates
  - Iimited to resolution of display
  - Z-buffer: check every pixel independently
  - Warnock: check up to single pixels if needed
- performed late in rendering pipeline

## **Projective Rendering Pipeline**



#### **Rendering Pipeline**



on the surface of a closed orientable manifold, polygons whose normals point away from the camera are always occluded:

> note: backface culling alone doesn't solve the hidden-surface problem!

- not rendering backfacing polygons improves performance
  - by how much?

reduces by about half the number of polygons to be considered for each pixel

optimization when appropriate

- most objects in scene are typically "solid"
- rigorously: orientable closed manifolds
  - orientable: must have two distinct sides
    - cannot self-intersect
    - a sphere is orientable since has two sides, 'inside' and 'outside'.
    - a Mobius strip or a Klein bottle is not orientable
  - closed: cannot "walk" from one side to the other
    - sphere is closed manifold
    - plane is not



- most objects in scene are typically "solid"
- rigorously: orientable closed manifolds
  - manifold: local neighborhood of all points isomorphic to disc
  - boundary partitions space into interior & exterior



# Manifold

- examples of *manifold* objects:
  - sphere
  - torus
  - well-formed
     CAD part



- examples of non-manifold objects:
  - a single polygon
  - a terrain or height field
  - polyhedron w/ missing face
  - anything with cracks or holes in boundary
  - one-polygon thick lampshade



#### **Back-face Culling: VCS**



first idea: cull if  $N_{\rm Z}<0$ 

sometimes misses polygons that should be culled

better idea: cull if eye is below polygon plane

#### **Back-face Culling: NDCS**

