



University of British Columbia
 CPSC 314 Computer Graphics
 May-June 2005

Tamara Munzner
Sampling, Virtual Trackball, Hidden Surfaces

Week 5, Tue Jun 7

<http://www.ugrad.cs.ubc.ca/~cs314/Vmay2005>

News

- Midterm handed back
 - solutions posted
 - distribution posted
 - all grades so far posted
- P1 Hall of Fame posted
- P3 grading
 - after 3:20
- P4 proposals
 - email or conversation to all

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H3 Corrections/Clarifications

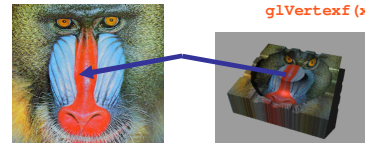
- Q1 should be from +infinity, not -infinity
- Q 2-4 correction for point B
- Q7 clarified: only x and y coordinates are given for P
- Q8 is deleted

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Review: Texture Coordinates

- texture image: 2D array of color values (*texels*)
- assigning *texture coordinates* (s,t) at vertex with object coordinates (x,y,z,w)
 - use interpolated (s,t) for texel lookup at each pixel
 - use value to modify a polygon's color
 - or other surface property
 - specified by programmer or artist

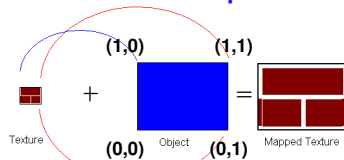
`glTexCoord2f (s, t)`
`glVertexf (x, y, z, w)`



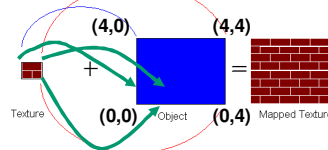
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Review: Tiled Texture Map

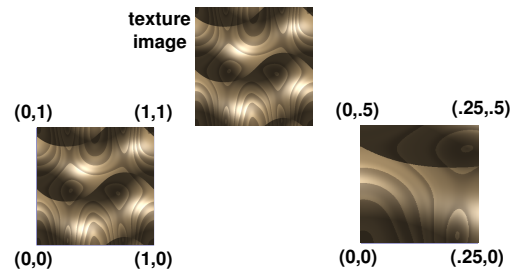
`glTexCoord2d(1, 1);`
`glVertex3d (x, y, z);`



`glTexCoord2d(4, 4);`
`glVertex3d (x, y, z);`



Review: Fractional Texture Coordinates



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Review: Texture

- action when s or t is outside [0...1] interval
 - tiling
 - clamping
- functions
 - replace/decal
 - modulate
 - blend
- texture matrix stack


```
glMatrixMode( GL_TEXTURE );
```

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Review: Basic OpenGL Texturing

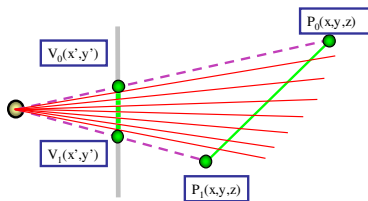
- setup
 - generate identifier: `glGenTextures`
 - load image data: `glTexImage2D`
 - set texture parameters (tile/clamp/...): `glTexParameteri`
 - set texture drawing mode (modulate/replace/...): `glTexEnvf`
- drawing
 - enable: `glEnable`
 - bind specific texture: `glBindTexture`
 - specify texture coordinates before each vertex: `glTexCoord2f`

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Review: Perspective Correct Interpolation

- screen space interpolation incorrect

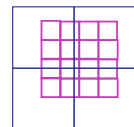
$$s = \frac{\alpha \cdot s_0 / w_0 + \beta \cdot s_1 / w_1 + \gamma \cdot s_2 / w_2}{\alpha / w_0 + \beta / w_1 + \gamma / w_2}$$



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Review: Reconstruction

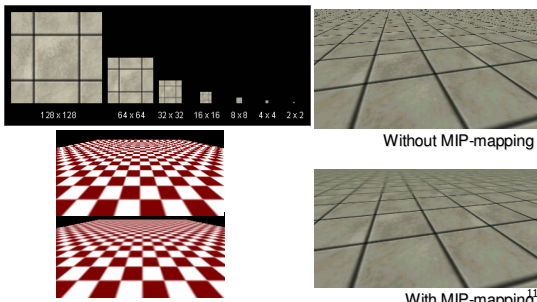
- how to deal with:
 - pixels that are much larger than texels?
 - apply filtering, "averaging"
 - pixels that are much smaller than texels?
 - interpolate



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Review: MIPmapping

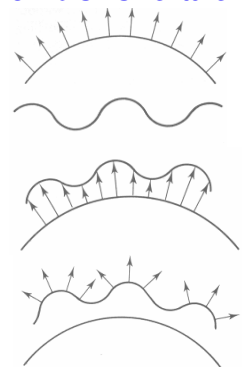
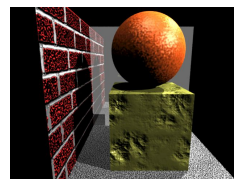
- image pyramid, precompute averaged versions



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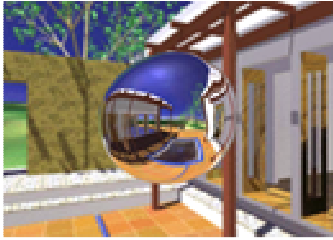
Review: Bump Mapping: Normals As Texture

- create illusion of complex geometry model
- control shape effect by locally perturbing surface normal



Review: Environment Mapping

- cheap way to achieve reflective effect
 - generate image of surrounding
 - map to object as texture



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Review: Sphere Mapping

- texture is distorted fish-eye view
 - point camera at mirrored sphere
 - spherical texture coordinates



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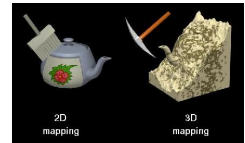
Review: Cube Mapping

- 6 planar textures, sides of cube
 - point camera outwards to 6 faces
 - use largest magnitude of vector to pick face
 - other two coordinates for (s,t) texel location



Review: Volumetric Texture

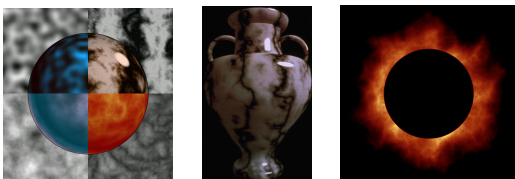
- define texture pattern over 3D domain - 3D space containing the object
 - texture function can be digitized or **procedural**
 - for each point on object compute texture from point location in space
- 3D function $\rho(x,y,z)$



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Review: Perlin Noise: Procedural Textures

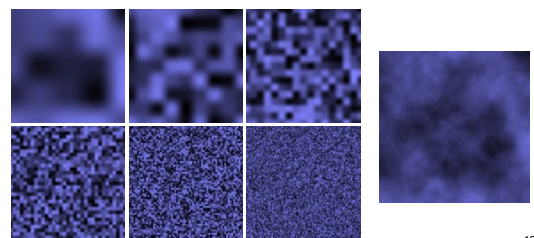
```
function marble(point)
  x = point.x + turbulence(point);
  return marble_color(sin(x))
```



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Review: Perlin Noise

- coherency: smooth not abrupt changes
- turbulence: multiple feature sizes



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Review: Generating Coherent Noise

- just three main ideas
 - nice interpolation
 - use vector offsets to make grid irregular
 - optimization
 - sneaky use of 1D arrays instead of 2D/3D one

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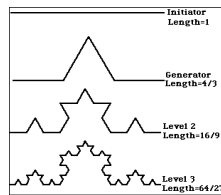
Review: Procedural Modeling

- textures, geometry
 - nonprocedural: explicitly stored in memory
- procedural approach
 - compute something on the fly
 - not load from disk
 - often less memory cost
 - visual richness
 - adaptable precision
- noise, fractals, particle systems

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Review: Language-Based Generation

- L-Systems
 - F: forward, R: right, L: left
 - Koch snowflake:
 - F = FLFRRFLF
 - Mariano's Bush:
 - F=FF-[-F+F+F]+[+F-F-F]
 - angle 16



<http://spanky.triumf.ca/www/fractint/l/sys/plants.html>

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Correction/Review: Fractal Terrain

- 1D: midpoint displacement
 - divide in half, randomly displace
 - scale variance by half
- 2D: diamond-square
 - generate new value at midpoint
 - average corner values + random displacement
 - scale variance by half each time

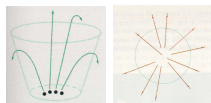


<http://www.gameprogrammer.com/fractal.html>

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Review: Particle Systems

- changeable/fluid stuff
 - fire, steam, smoke, water, grass, hair, dust, waterfalls, fireworks, explosions, flocks
- life cycle
 - generation, dynamics, death
- rendering tricks
 - avoid hidden surface computations



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Sampling

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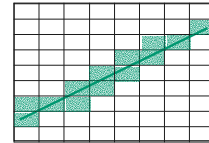
Samples

- most things in the real world are **continuous**
- everything in a computer is **discrete**
- the process of mapping a continuous function to a discrete one is called **sampling**
- the process of mapping a discrete function to a continuous one is called **reconstruction**
- the process of mapping a continuous variable to a discrete one is called **quantization**
- rendering an image requires sampling and quantization
- displaying an image involves reconstruction

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Line Segments

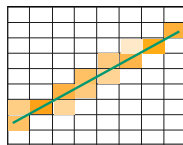
- we tried to sample a line segment so it would map to a 2D raster display
- we quantized the pixel values to 0 or 1
- we saw stair steps, or jaggies



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Line Segments

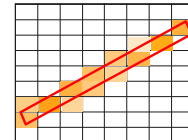
- instead, quantize to many shades
- but what sampling algorithm is used?



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Unweighted Area Sampling

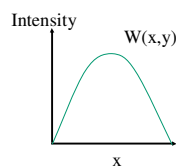
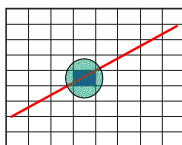
- shade pixels wrt area covered by thickened line
- equal areas cause equal intensity, regardless of distance from pixel center to area
 - rough approximation formulated by dividing each pixel into a finer grid of pixels
- primitive cannot affect intensity of pixel if it does not intersect the pixel



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Weighted Area Sampling

- intuitively, pixel cut through the center should be more heavily weighted than one cut along corner
- weighting function, $W(x,y)$
 - specifies the contribution of primitive passing through the point (x, y) from pixel center



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Images

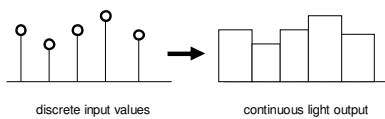
- an image is a 2D function $I(x, y)$ that specifies intensity for each point (x, y)



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Image Sampling and Reconstruction

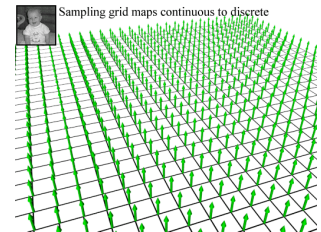
- convert **continuous** image to **discrete** set of samples
- display hardware **reconstructs** samples into continuous image
 - finite sized source of light for each pixel



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Point Sampling an Image

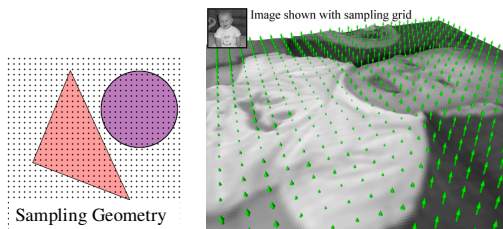
- simplest sampling is on a grid
- sample depends solely on value at grid points



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Point Sampling

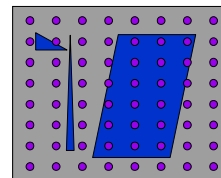
- multiply sample grid by image intensity to obtain a discrete set of points, or samples.



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Sampling Errors

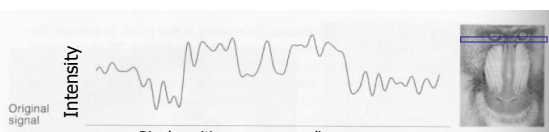
- some objects missed entirely, others poorly sampled
 - could try unweighted or weighted area sampling
 - but how can we be sure we show everything?
- need to think about entire class of solutions!



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Image As Signal

- image as spatial signal
- 2D raster image
 - discrete sampling of 2D spatial signal
- 1D slice of raster image
 - discrete sampling of 1D spatial signal



Examples from Foley, van Dam, Feiner, and Hughes 35

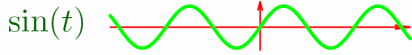
Sampling Theory

- how would we generate a signal like this out of simple building blocks?
- theorem
 - any signal can be represented as an (infinite) sum of sine waves at different frequencies

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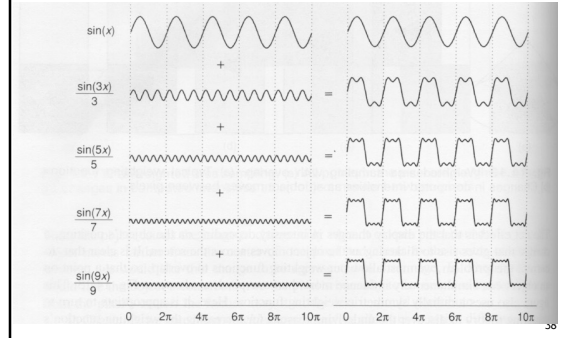
Sampling Theory in a Nutshell

- terminology
 - bandwidth – length of repeated sequence on infinite signal
 - frequency – $1/\text{bandwidth}$ (number of repeated sequences in unit length)
- example – sine wave
 - bandwidth = 2π
 - frequency = $1/2\pi$



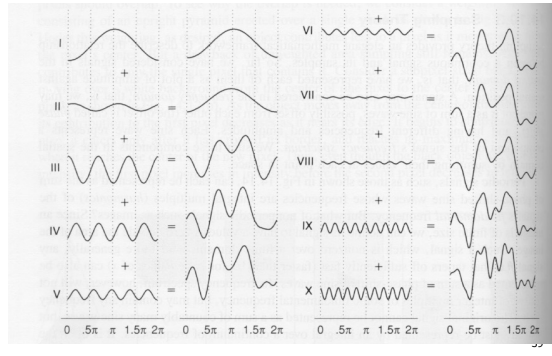
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Summing Waves I



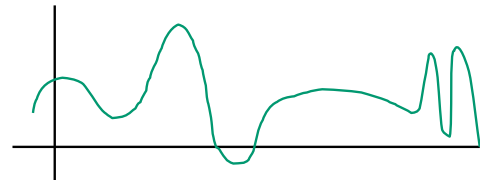
38

Summing Waves II



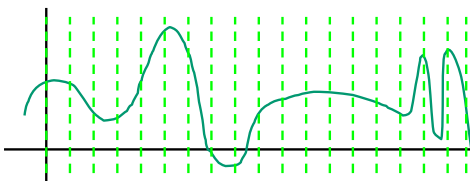
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1D Sampling and Reconstruction



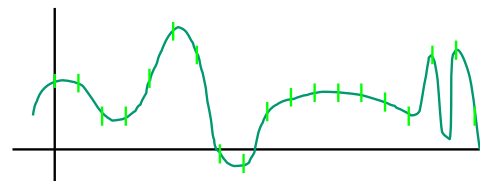
40

1D Sampling and Reconstruction

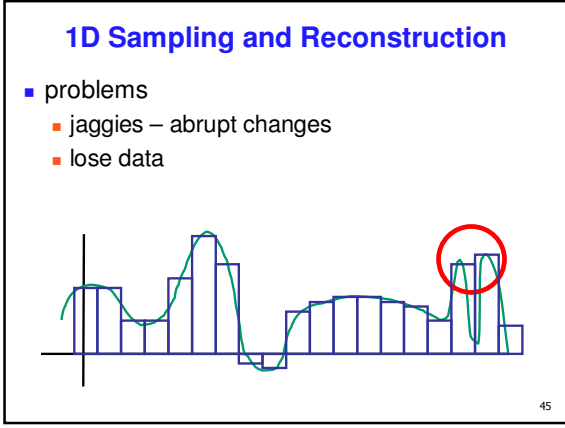
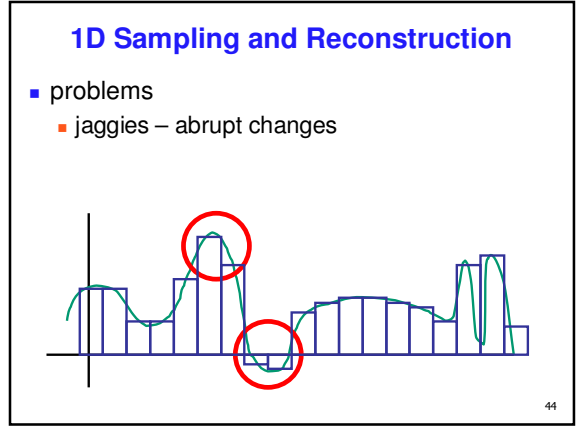
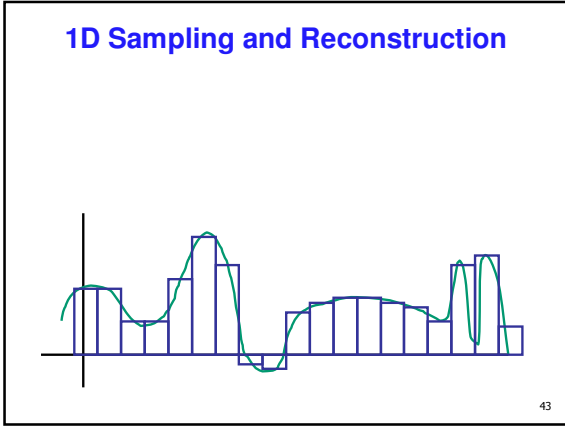


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1D Sampling and Reconstruction



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Sampling Theorem

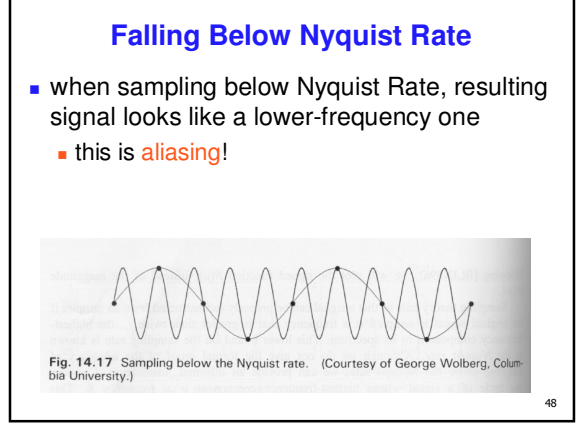
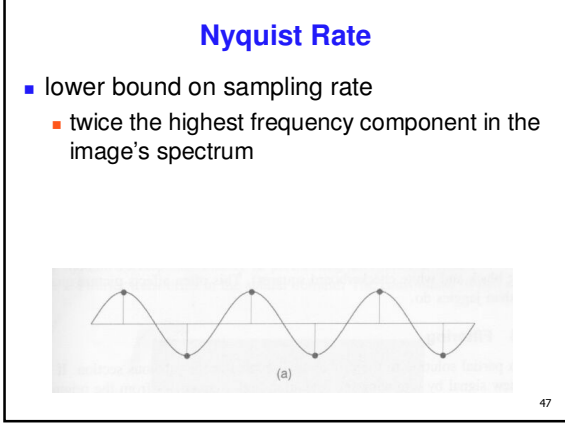
continuous signal can be completely recovered from its samples

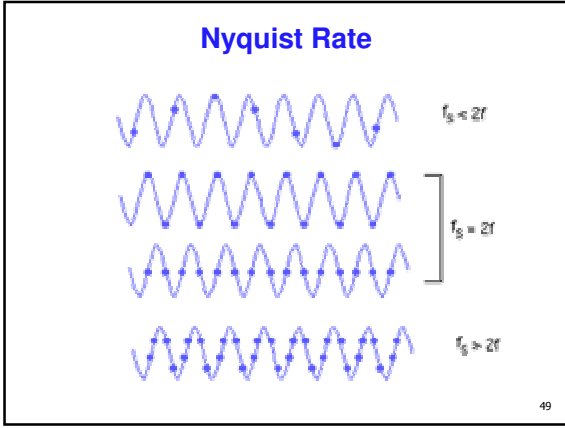
iff

sampling rate greater than twice maximum frequency present in signal

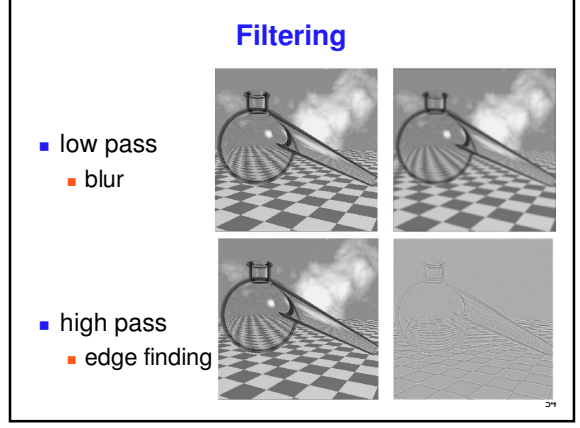
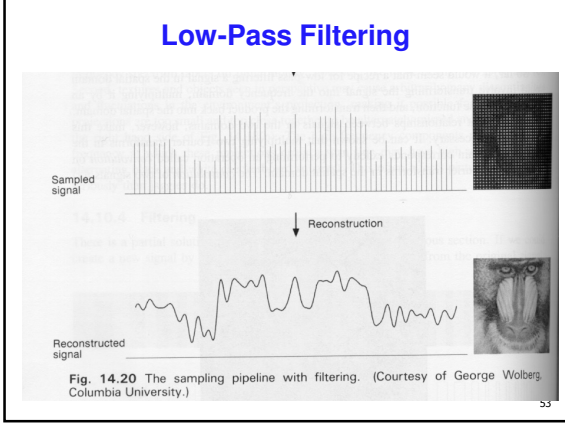
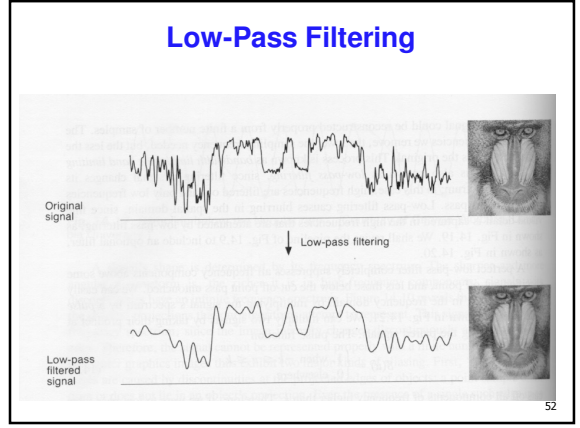
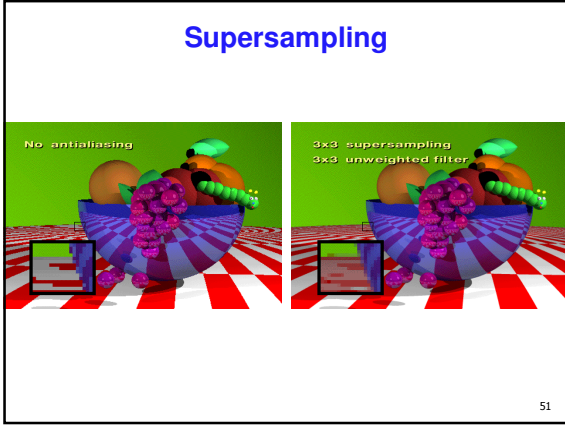
- Claude Shannon

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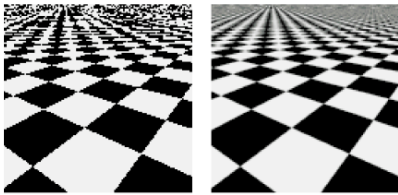


- ### Aliasing
- incorrect appearance of high frequencies as low frequencies
 - to avoid: **antialiasing**
 - supersample
 - sample at higher frequency
 - low pass filtering
 - remove high frequency function parts
 - aka prefiltering, band-limiting
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Previous Antialiasing Example

- texture mipmapping: low pass filter



(a)

(b)

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Virtual Trackball

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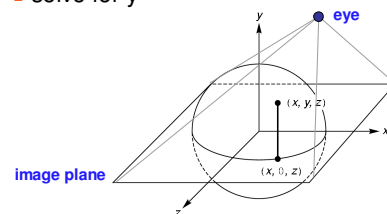
Virtual Trackball

- interface for spinning objects around
 - drag mouse to control rotation of view volume
- rolling glass trackball
 - center at screen origin, surrounds world
 - hemisphere “sticks up” in z, out of screen
 - rotate ball = spin world

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Virtual Trackball

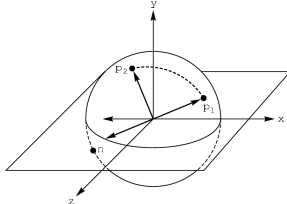
- know screen click: $(x, 0, z)$
- want to infer point on trackball: (x, y, z)
 - ball is unit sphere, so $\|x, y, z\| = 1.0$
 - solve for y



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Trackball Rotation

- correspondence:
 - moving point on plane from $(x, 0, z)$ to $(a, 0, c)$
 - moving point on ball from $\mathbf{p}_1 = (x, y, z)$ to $\mathbf{p}_2 = (a, b, c)$
- correspondence:
 - translating mouse from \mathbf{p}_1 (mouse down) to \mathbf{p}_2 (mouse up)
 - rotating about the axis $\mathbf{n} = \mathbf{p}_1 \times \mathbf{p}_2$



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Trackball Computation

- user defines two points
 - place where first clicked $\mathbf{p}_1 = (x, y, z)$
 - place where released $\mathbf{p}_2 = (a, b, c)$
- create plane from vectors between points, origin
 - axis of rotation is plane normal: cross product
 - $(\mathbf{p}_1 - \mathbf{o}) \times (\mathbf{p}_2 - \mathbf{o})$: $\mathbf{p}_1 \times \mathbf{p}_2$ if origin = $(0,0,0)$
 - amount of rotation depends on angle between lines
 - $\mathbf{p}_1 \cdot \mathbf{p}_2 = |\mathbf{p}_1| |\mathbf{p}_2| \cos \theta$
 - $|\mathbf{p}_1 \times \mathbf{p}_2| = |\mathbf{p}_1| |\mathbf{p}_2| \sin \theta$
- compute rotation matrix, use to rotate world

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Visibility

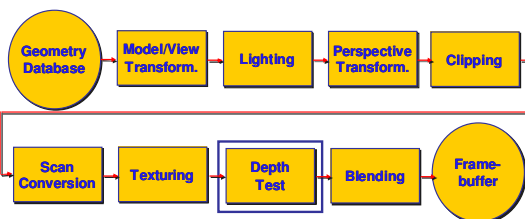
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Reading

- FCG Chapter 7

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Rendering Pipeline



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Covered So Far

- modeling transformations
- viewing transformations
- projection transformations
- clipping
- scan conversion
- lighting
- shading
- we now know everything about how to draw a polygon on the screen, except *visible surface determination*

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Invisible Primitives

- *why might a polygon be invisible?*
 - polygon outside the *field of view / frustum*
 - solved by *clipping*
 - polygon is *backfacing*
 - solved by *backface culling*
 - polygon is *occluded* by object(s) nearer the viewpoint
 - solved by *hidden surface removal*
- for efficiency reasons, we want to avoid spending work on polygons outside field of view or backfacing
- for efficiency and correctness reasons, we need to know when polygons are occluded

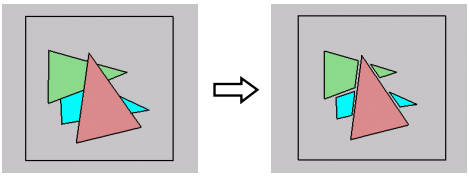
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Hidden Surface Removal

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Occlusion

- for most interesting scenes, some polygons overlap

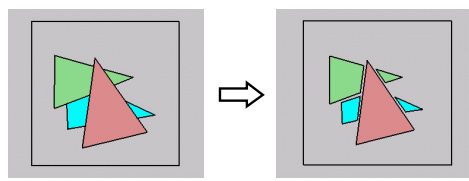


- to render the correct image, we need to determine which polygons occlude which

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Painter's Algorithm

- simple: render the polygons from back to front, "painting over" previous polygons




- draw blue, then green, then orange
- will this work in the general case?

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Painter's Algorithm: Problems


- intersecting polygons* present a problem
- even non-intersecting polygons can form a cycle with no valid visibility order:



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Analytic Visibility Algorithms

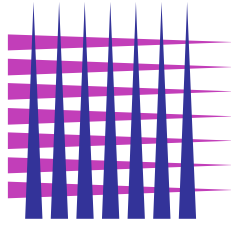
- early visibility algorithms computed the set of visible polygon *fragments* directly, then rendered the fragments to a display:



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Analytic Visibility Algorithms

- what is the minimum worst-case cost of computing the fragments for a scene composed of n polygons?*
- answer:
 $O(n^2)$



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Analytic Visibility Algorithms

- so, for about a decade (late 60s to late 70s) there was intense interest in finding efficient algorithms for *hidden surface removal*
- we'll talk about two:
 - Binary Space Partition (BSP) Trees*
 - Warnock's Algorithm*

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Binary Space Partition Trees (1979)

- BSP Tree: partition space with binary tree of planes
 - idea: divide space recursively into half-spaces by choosing splitting planes that separate objects in scene
 - preprocessing: create binary tree of planes
 - runtime: correctly traversing this tree enumerates objects from back to front

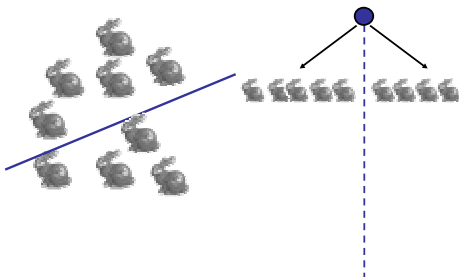
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Creating BSP Trees: Objects



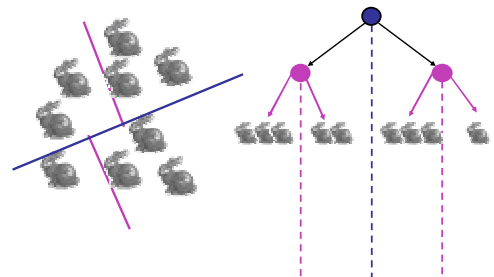
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Creating BSP Trees: Objects



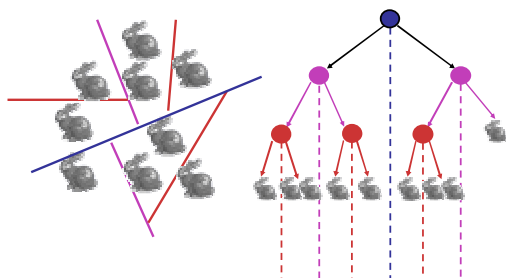
75

Creating BSP Trees: Objects



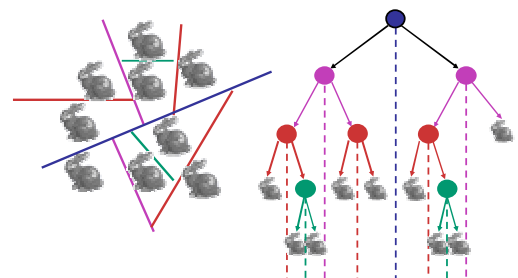
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Creating BSP Trees: Objects



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Creating BSP Trees: Objects



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Splitting Objects

- no bunnies were harmed in previous example
- but what if a splitting plane passes through an object?
 - split the object; give half to each node



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Traversing BSP Trees

- tree creation independent of viewpoint
 - preprocessing step
- tree traversal uses viewpoint
 - runtime, happens for many different viewpoints
- each plane divides world into near and far
 - for given viewpoint, decide which side is near and which is far
 - check which side of plane viewpoint is on independently for each tree vertex
 - tree traversal differs depending on viewpoint!
- recursive algorithm
 - recurse on far side
 - draw object
 - recurse on near side

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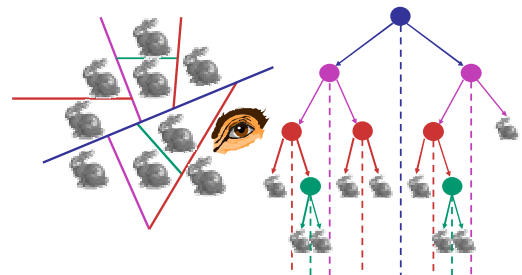
Traversing BSP Trees

query: given a viewpoint, produce an ordered list of (possibly split) objects from back to front:

```
renderBSP(BSPtree *T)
  BSPtree *near, *far;
  if (eye on left side of T->plane)
    near = T->left; far = T->right;
  else
    near = T->right; far = T->left;
  renderBSP(far);
  if (T is a leaf node)
    renderObject(T)
  renderBSP(near);
```

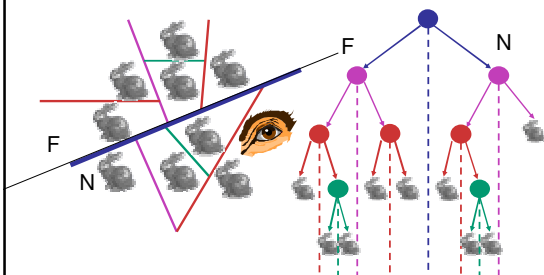
81

BSP Trees : Viewpoint A



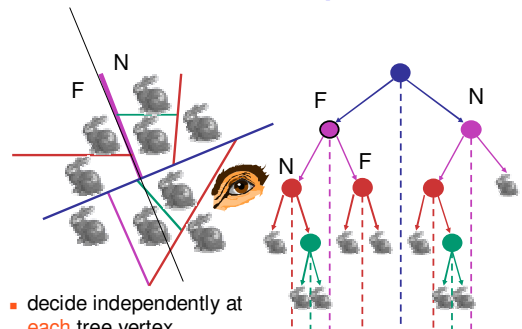
82

BSP Trees : Viewpoint A



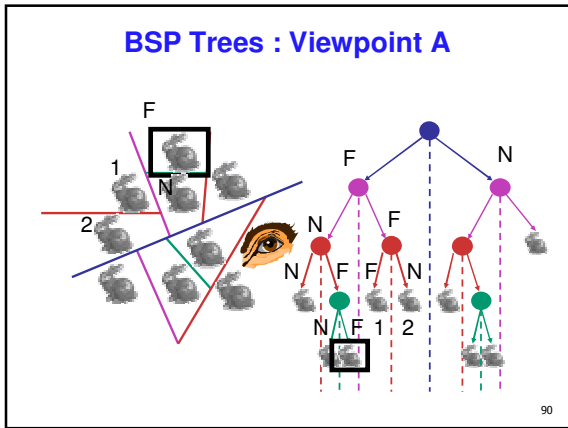
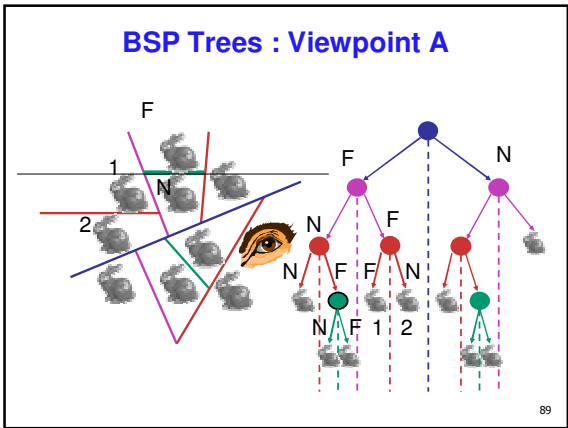
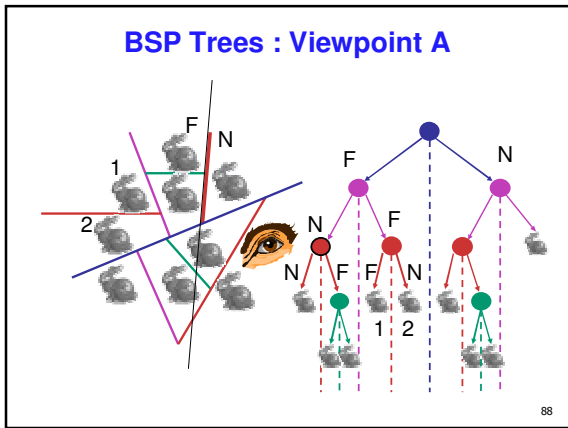
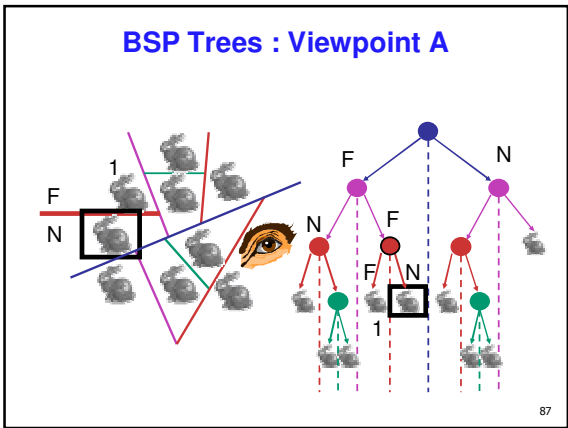
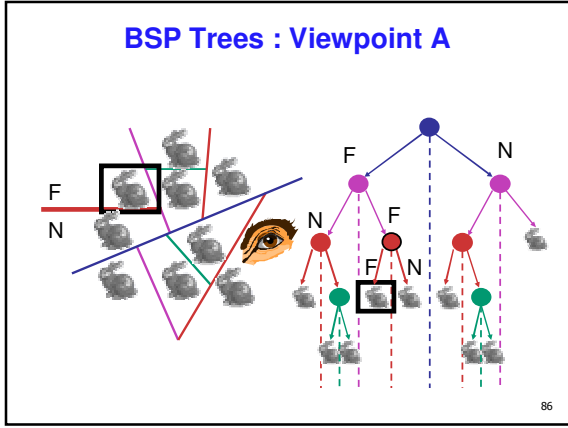
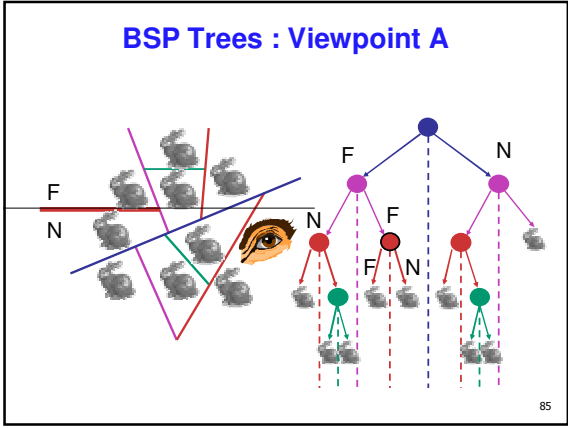
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BSP Trees : Viewpoint A



- decide independently at each tree vertex
- not just left or right child!

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BSP Tree Traversal: Polygons

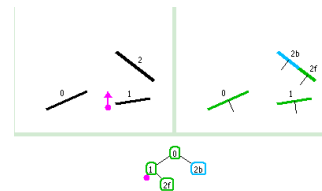
- split along the plane defined by any polygon from scene
- classify all polygons into positive or negative half-space of the plane
 - if a polygon intersects plane, split polygon into two and classify them both
- recurse down the negative half-space
- recurse down the positive half-space

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BSP Demo

- useful demo:

<http://symbolcraft.com/graphics/bsp>



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Summary: BSP Trees

- pros:
 - simple, elegant scheme
 - correct version of painter's algorithm back-to-front rendering approach
 - was very popular for video games (but getting less so)
- cons:
 - slow to construct tree: $O(n \log n)$ to split, sort
 - splitting increases polygon count: $O(n^2)$ worst-case
 - computationally intense preprocessing stage restricts algorithm to static scenes

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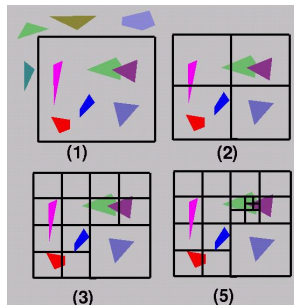
Warnock's Algorithm (1969)

- based on a powerful general approach common in graphics
 - if the situation is too complex, **subdivide**
- BSP trees was object space approach
- Warnock is image space approach

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Warnock's Algorithm

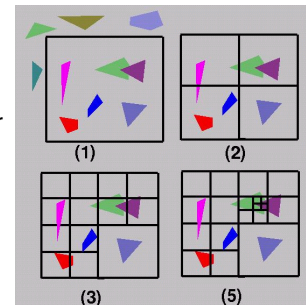
- start with root viewport and list of all objects
- recursion:
 - clip objects to viewport
 - if only 0 or 1 objects
 - done
 - else
 - subdivide to new smaller viewports
 - distribute objects to new viewpoints
 - recurse



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Warnock's Algorithm

- termination
 - viewport is single pixel
 - explicitly check for object occlusion



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Warnock's Algorithm

- pros:
 - very elegant scheme
 - extends to any primitive type
- cons:
 - hard to embed hierarchical schemes in hardware
 - complex scenes usually have small polygons and high **depth complexity** (number of polygons that overlap a single pixel)
 - thus most screen regions come down to the single-pixel case

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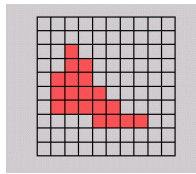
The Z-Buffer Algorithm (mid-70's)

- both BSP trees and Warnock's algorithm were proposed when memory was expensive
 - first 512x512 framebuffer was >\$50,000!
- Ed Catmull proposed a radical new approach called **z-buffering**.
- the big idea:
 - resolve visibility **independently at each pixel**

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The Z-Buffer Algorithm

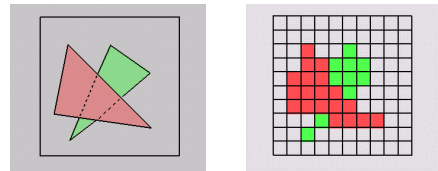
- we know how to rasterize polygons into an image discretized into pixels:



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The Z-Buffer Algorithm

- what happens if multiple primitives occupy the same pixel on the screen?
 - which is allowed to paint the pixel?



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The Z-Buffer Algorithm

- idea: retain depth after projection transform
 - each vertex maintains z coordinate
 - relative to eye point
 - can do this with canonical viewing volumes

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The Z-Buffer Algorithm

- augment color framebuffer with **Z-buffer** or **depth buffer** which stores Z value at each pixel
 - at frame beginning, initialize all pixel depths to ∞
 - when rasterizing, interpolate depth (Z) across polygon
 - check Z-buffer before storing pixel color in framebuffer and storing depth in Z-buffer
 - don't write pixel if its Z value is more distant than the Z value already stored there

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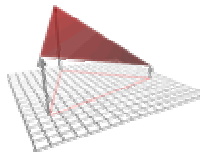
Interpolating Z

- edge equations: Z just another planar parameter:

- $z = (-D - Ax - By) / C$
- if walking across scanline by (D_x)
 $Z_{new} = Z_{old} - (A/C)(D_x)$

- total cost:

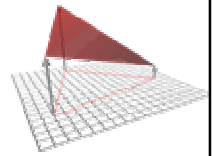
- 1 more parameter to increment in inner loop
- 3x3 matrix multiply for setup



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Interpolating Z

- edge walking
 - just interpolate Z along edges and across spans
- barycentric coordinates
 - interpolate Z like other parameters



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Z-Buffer

- store (r,g,b,z) for each pixel
- typically 8+8+8+24 bits, can be more

```

for all i,j {
  Depth[i,j] = MAX_DEPTH
  Image[i,j] = BACKGROUND_COLOUR
}
for all polygons P {
  for all pixels in P {
    if (Z_pixel < Depth[i,j]) {
      Image[i,j] = C_pixel
      Depth[i,j] = Z_pixel
    }
  }
}
    
```

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Depth Test Precision

- reminder: projective transformation maps eye-space z to generic z -range (NDC)
- simple example:

$$T \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

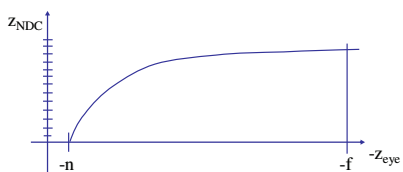
- thus:

$$z_{NDC} \equiv \frac{a \cdot z_{eye} + b}{z_{eye}} \equiv a + \frac{b}{z_{eye}}$$

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Depth Test Precision

- therefore, depth-buffer essentially stores $1/z$, rather than z !
- issue with integer depth buffers
 - high precision for near objects
 - low precision for far objects



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Depth Test Precision

- low precision can lead to **depth fighting** for far objects
 - two different depths in eye space get mapped to same depth in framebuffer
 - which object "wins" depends on drawing order and scan-conversion
- gets worse for larger ratios $f:n$
 - rule of thumb: $f:n < 1000$ for 24 bit depth buffer
- with 16 bits cannot discern millimeter differences in objects at 1 km distance

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Z-Buffer Algorithm Questions

- how much memory does the Z-buffer use?
- does the image rendered depend on the drawing order?
- does the time to render the image depend on the drawing order?
- how does Z-buffer load scale with visible polygons? with framebuffer resolution?

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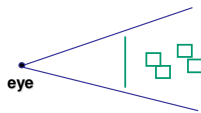
Z-Buffer Pros

- simple!!!
- easy to implement in hardware
 - hardware support in all graphics cards today
- polygons can be processed in arbitrary order
- easily handles polygon interpenetration
- enables **deferred shading**
 - rasterize shading parameters (e.g., surface normal) and only shade final visible fragments

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Z-Buffer Cons

- poor for scenes with high depth complexity
 - need to render all polygons, even if most are invisible



- shared edges are handled inconsistently
 - *ordering dependent*

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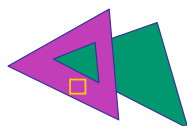
Z-Buffer Cons

- requires lots of memory
 - (e.g. 1280x1024x32 bits)
- requires fast memory
 - Read-Modify-Write in inner loop
- hard to simulate translucent polygons
 - we throw away color of polygons behind closest one
 - works if polygons ordered back-to-front
 - extra work throws away much of the speed advantage

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Hidden Surface Removal

- two kinds of visibility algorithms
 - object space methods
 - image space methods



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Object Space Algorithms

- determine visibility on object or polygon level
 - using camera coordinates
- resolution independent
 - explicitly compute visible portions of polygons
- early in pipeline
 - after clipping
- requires depth-sorting
 - painter's algorithm
 - BSP trees

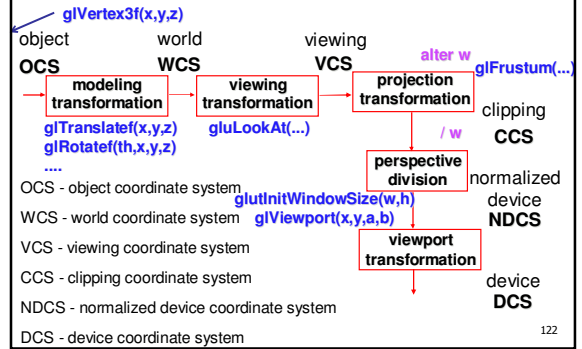
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Image Space Algorithms

- perform visibility test for in screen coordinates
 - limited to resolution of display
 - Z-buffer: check every pixel independently
 - Warnock: check up to single pixels if needed
- performed late in rendering pipeline

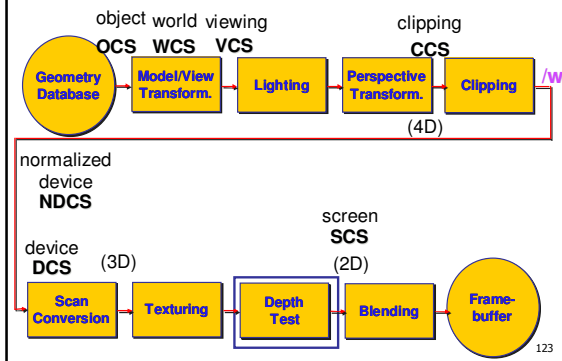
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Projective Rendering Pipeline



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Rendering Pipeline



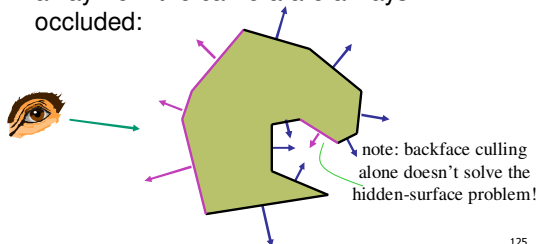
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Backface Culling

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Back-Face Culling

- on the surface of a closed orientable manifold, polygons whose normals point away from the camera are always occluded:



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Back-Face Culling

- not rendering backfacing polygons improves performance
 - by how much?
 - reduces by about half the number of polygons to be considered for each pixel
 - optimization when appropriate

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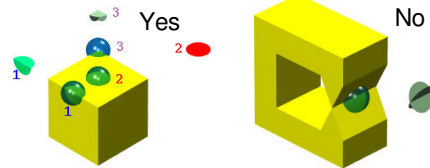
Back-Face Culling

- most objects in scene are typically "solid"
- rigorously: **orientable closed manifolds**
 - **orientable**: must have two distinct sides
 - cannot self-intersect
 - a sphere is orientable since has two sides, 'inside' and 'outside'.
 - a Mobius strip or a Klein bottle is not orientable
 - **closed**: cannot "walk" from one side to the other
 - sphere is closed manifold
 - plane is not



Back-Face Culling

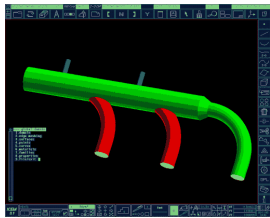
- most objects in scene are typically "solid"
- rigorously: **orientable closed manifolds**
 - **manifold**: local neighborhood of all points isomorphic to disc
 - boundary partitions space into interior & exterior



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Manifold

- examples of *manifold* objects:
 - sphere
 - torus
 - well-formed CAD part



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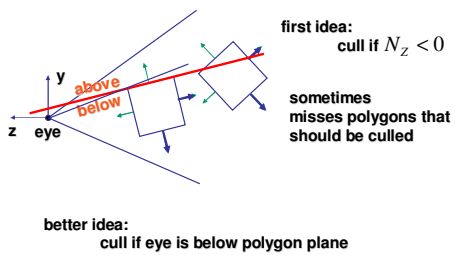
Back-Face Culling

- examples of non-manifold objects:
 - a single polygon
 - a terrain or height field
 - polyhedron w/ missing face
 - anything with cracks or holes in boundary
 - one-polygon thick lampshade



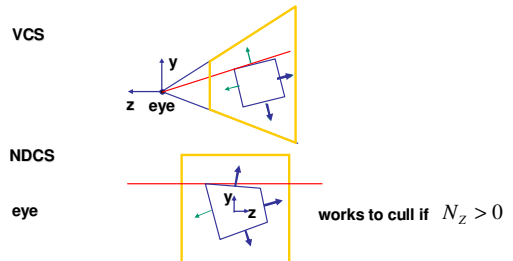
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Back-face Culling: VCS



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Back-face Culling: NDCS



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