



University of British Columbia  
CPSC 314 Computer Graphics  
May-June 2005

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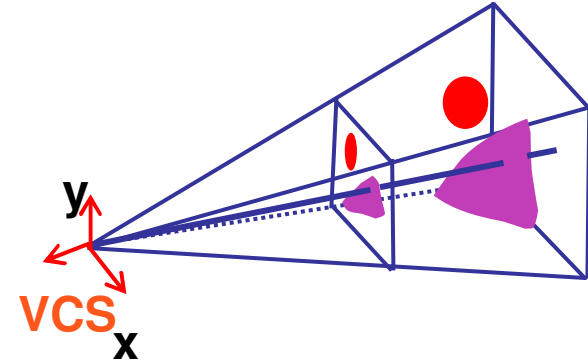
**Textures, Procedural Approaches,  
Sampling**

**Week 4, Thu Jun 2**

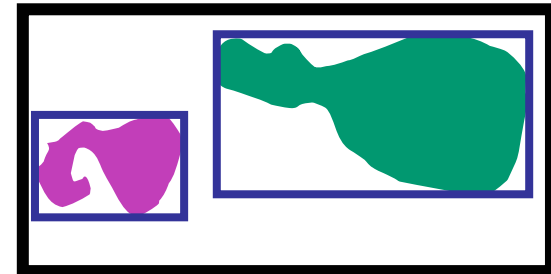
<http://www.ugrad.cs.ubc.ca/~cs314/Vmay2005>

# Review: Picking Methods

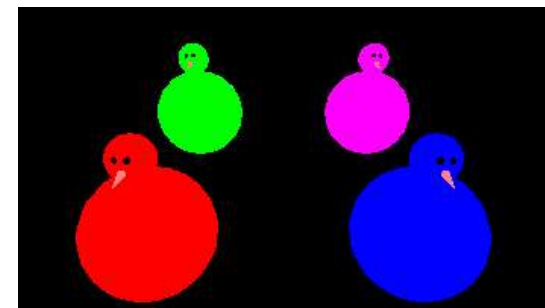
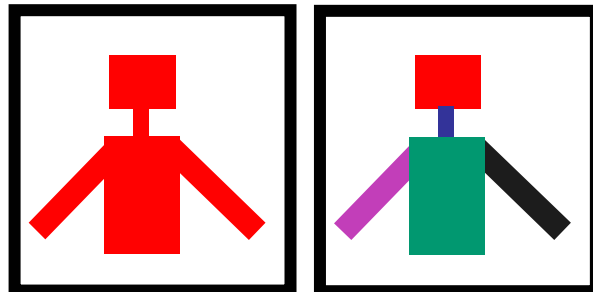
- manual ray intersection



- bounding extents

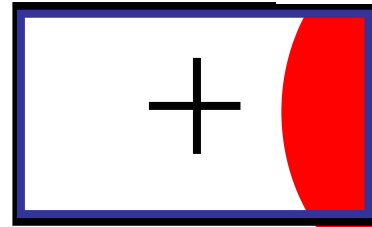
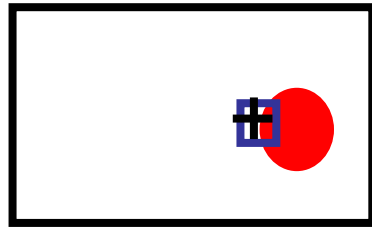


- backbuffer coding



# Review: Select/Hit Picking

- assign (hierarchical) integer key/name(s)
- small region around cursor as new viewport



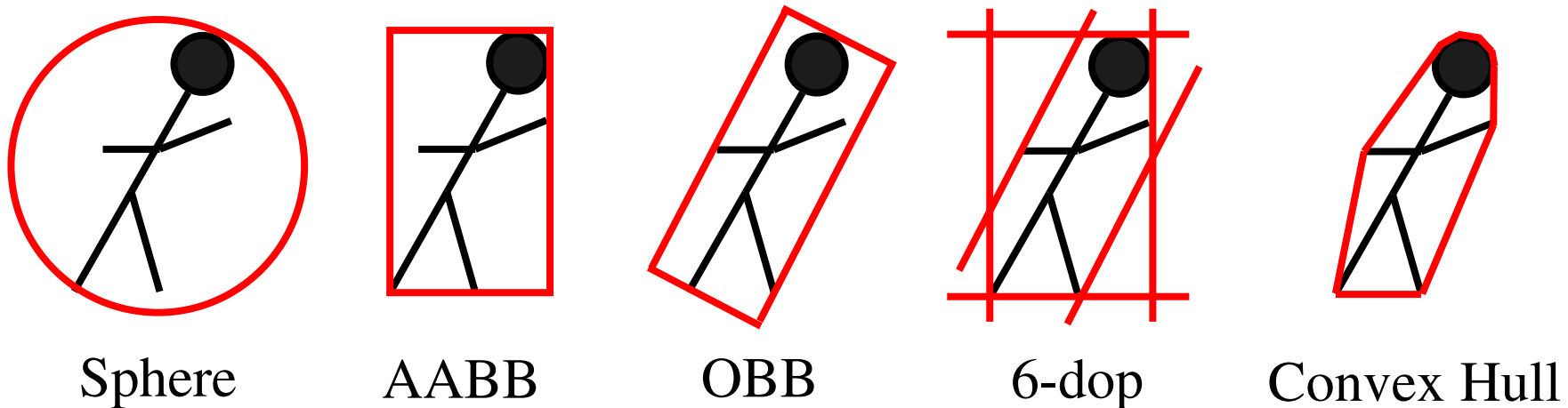
- redraw in selection mode
  - equivalent to casting pick “tube”
  - store keys, depth for drawn objects in hit list
- examine hit list
  - usually use frontmost, but up to application

# Review: Collision Detection

- boundary check
  - perimeter of world vs. viewpoint or objects
    - 2D/3D absolute coordinates for bounds
    - simple point in space for viewpoint/objects
- set of fixed barriers
  - walls in maze game
    - 2D/3D absolute coordinate system
- set of moveable objects
  - one object against set of items
    - missile vs. several tanks
  - multiple objects against each other
    - punching game: arms and legs of players
    - room of bouncing balls

# Review: Collision Proxy Tradeoffs

- **collision proxy (bounding volume)** is piece of geometry used to represent complex object for purposes of finding collision
- proxies exploit facts about human perception
  - we are bad at determining collision correctness
  - especially many things happening quickly

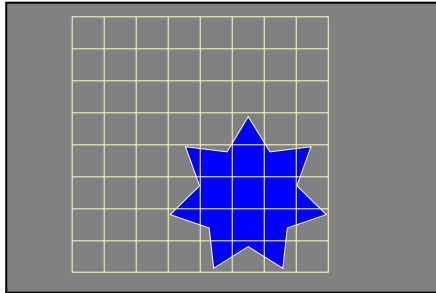


→ increasing complexity & tightness of fit

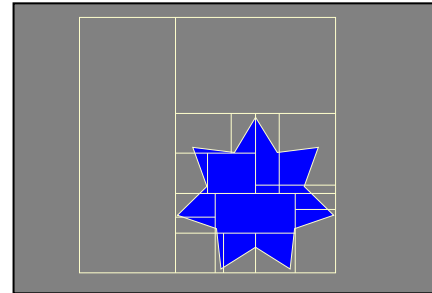
← decreasing cost of (overlap tests + proxy update)

# Review: Spatial Data Structures

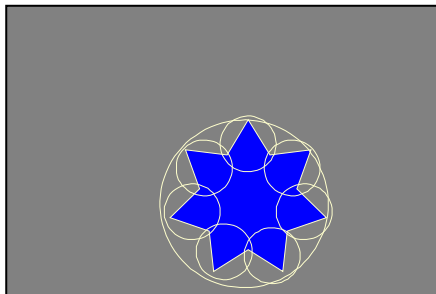
uniform grids



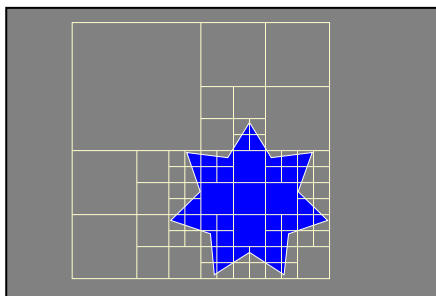
BSP trees



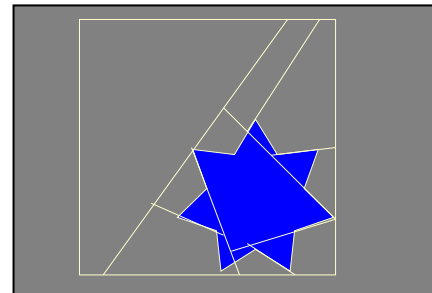
bounding volume hierarchies



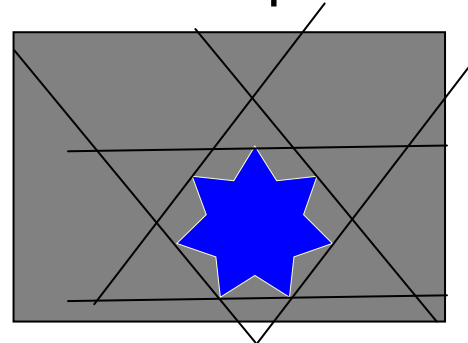
octrees



kd-trees



k-dops



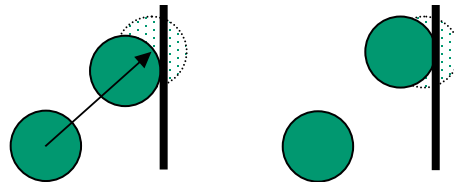
# Review: Exploiting Coherence

- player normally doesn't move far between frames
- track incremental changes, using previous results instead of doing full search each time
- keep track of entry and exit into cells through portals
  - probably the same cells they intersect now
  - or moved to neighbor

# Review: Precise Collisions

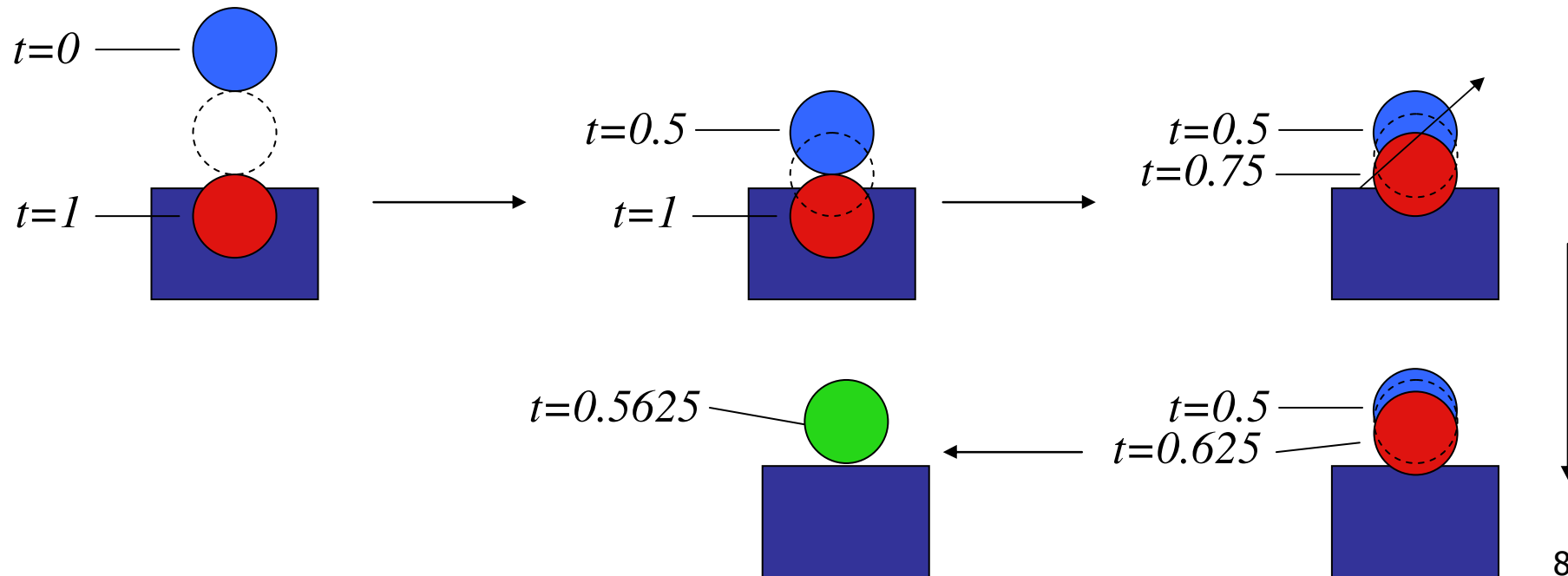
- hacked clean up

- simply move position so that objects just touch, leave time the same



- interval halving

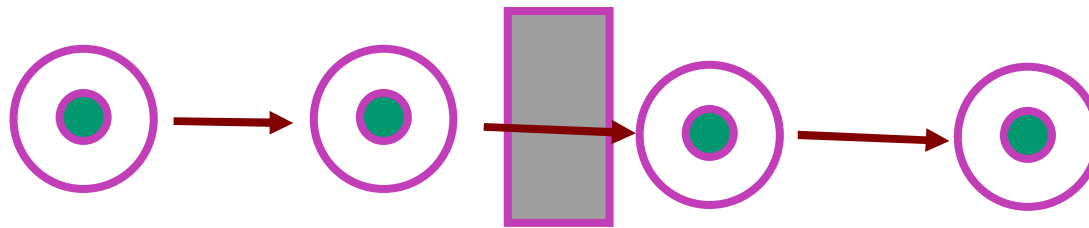
- binary search through time to find exact collision point and time



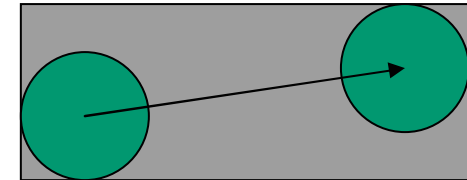


# Review: Fast-Moving Objects

- temporal sampling
  - aliasing: can miss collision completely!



- movement line
- conservative prediction
  - assume maximum velocity, smallest feature size
  - increase temporal and spatial sampling rate
- simple alternative: just miss the hard cases
  - player may not notice!



# Review: Collision Response

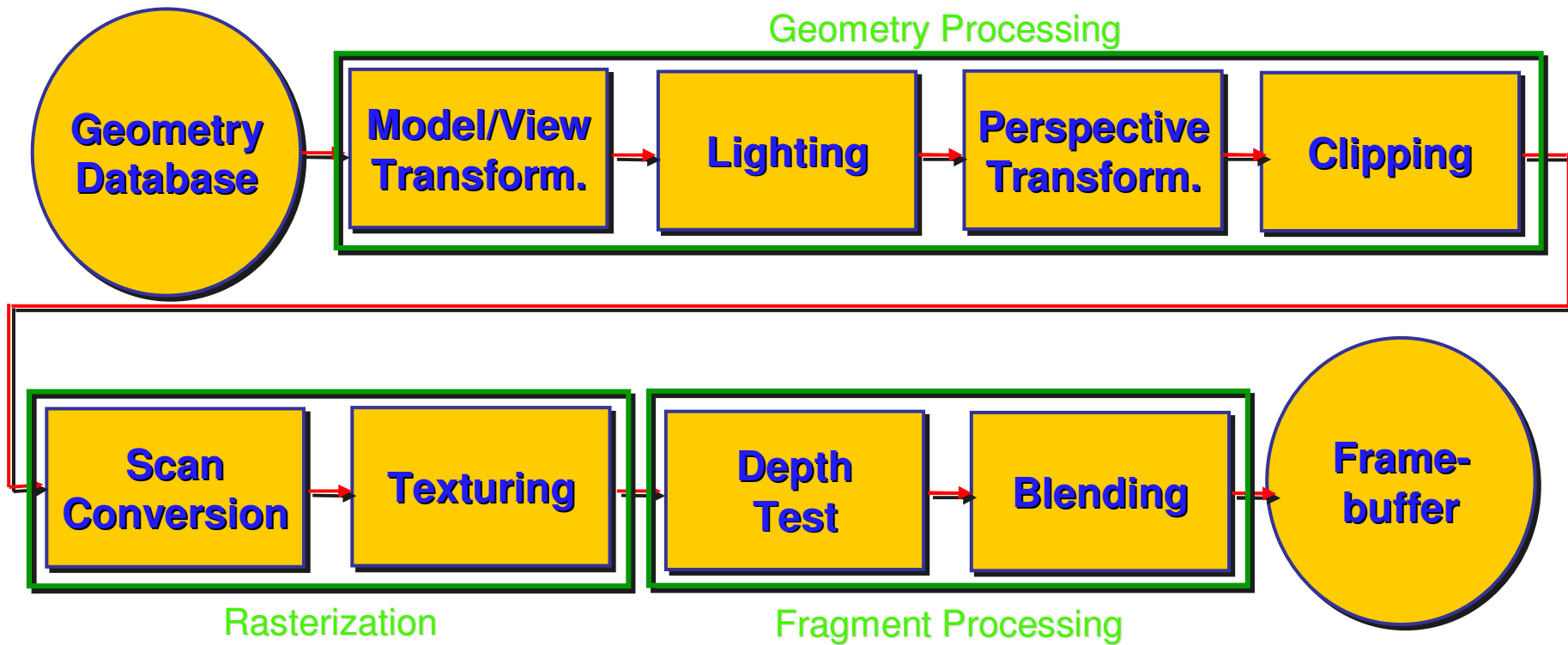
- frustrating to just stop player
  - often move tangentially to obstacle
- recursively to catch all collisions
- handling multiple simultaneous contacts

# Texturing

# Reading

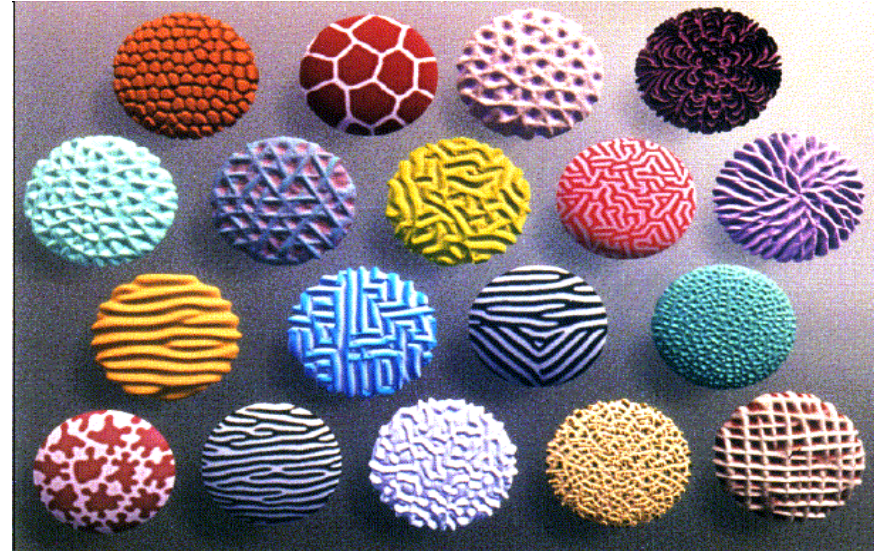
- FCG Chapter 10
- Red Book Chapter Texture Mapping

# Rendering Pipeline



# Texture Mapping

- real life objects have nonuniform colors, normals
- to generate realistic objects, reproduce coloring & normal variations = **texture**
- can often replace complex geometric details

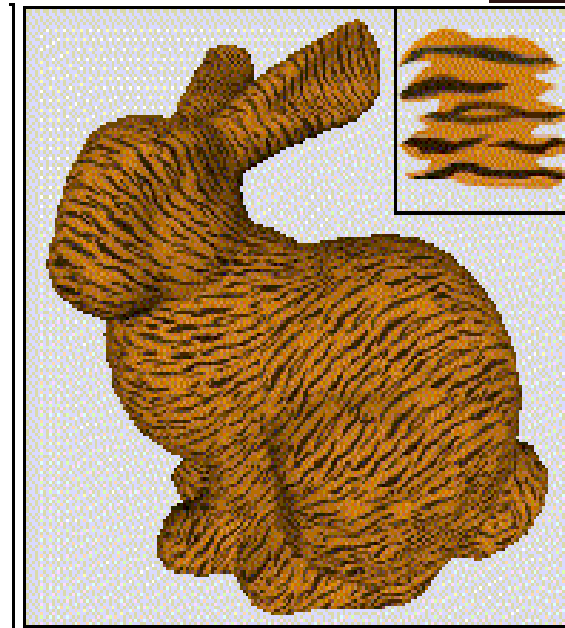
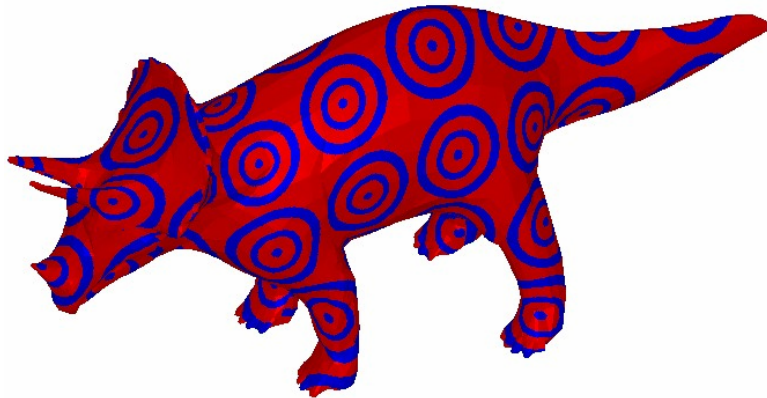


# Texture Mapping

- introduced to increase realism
  - lighting/shading models not enough
- hide geometric simplicity
  - images convey illusion of geometry
  - map a brick wall texture on a flat polygon
  - create bumpy effect on surface
- associate 2D information with 3D surface
  - point on surface corresponds to a point in texture
  - “paint” image onto polygon

# Color Texture Mapping

- define color (RGB) for each point on object surface
- two approaches
  - surface texture map
  - volumetric texture



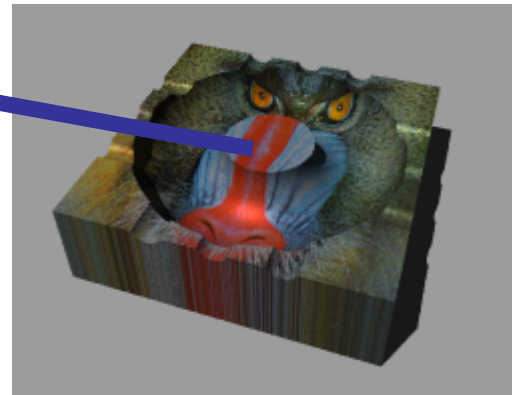
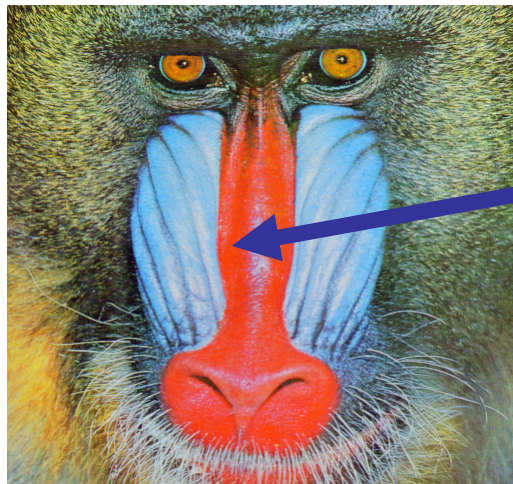


# Texture Coordinates

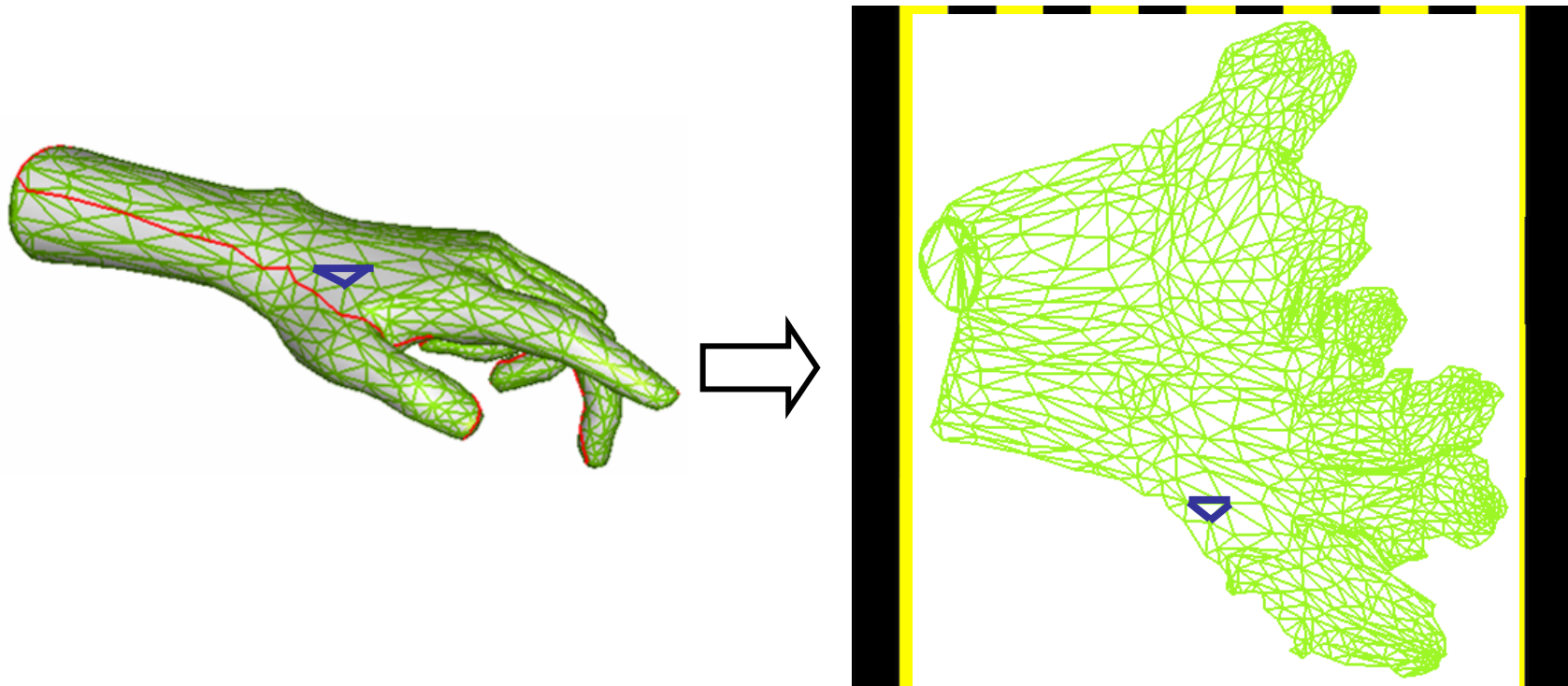
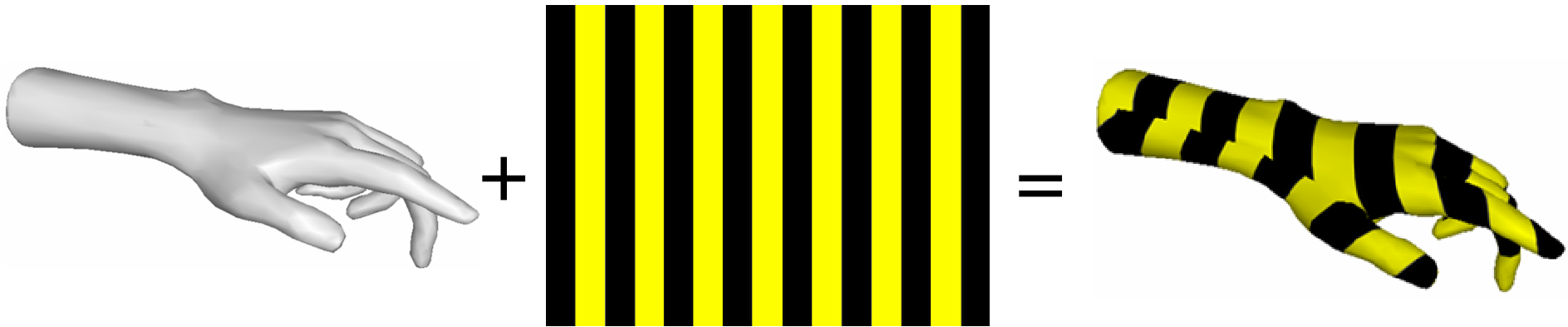
- texture image: 2D array of color values (**texels**)
- assigning **texture coordinates** (s,t) at vertex with object coordinates (x,y,z,w)
  - use interpolated (s,t) for texel lookup at each pixel
  - use value to modify a polygon's color
    - or other surface property
  - specified by programmer or artist

`glTexCoord2f (s, t)`

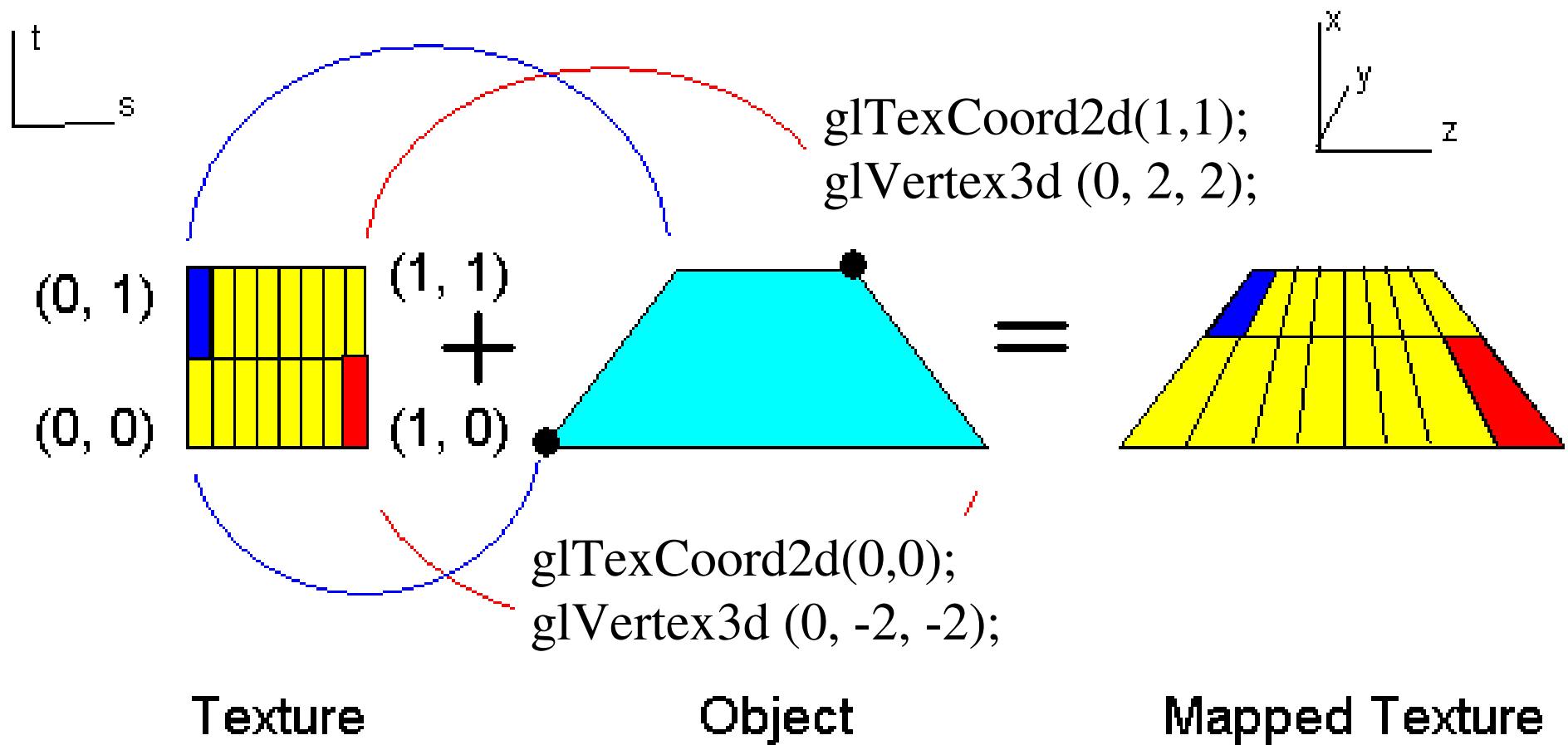
`glVertexf (x, y, z, w)`



# Texture Mapping Example

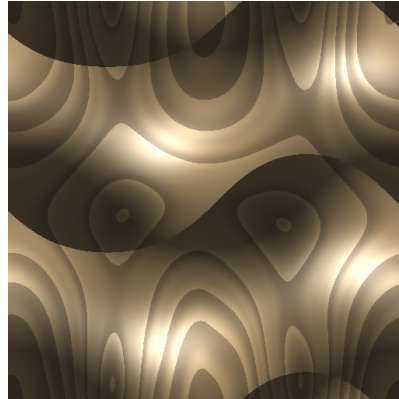


# Example Texture Map



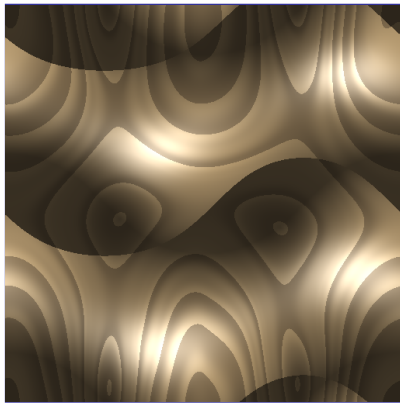
# Fractional Texture Coordinates

texture  
image



$(0,1)$

$(1,1)$

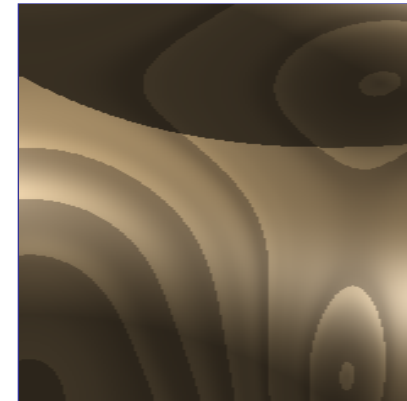


$(0,0)$

$(1,0)$

$(0,.5)$

$(.25,.5)$



$(0,0)$

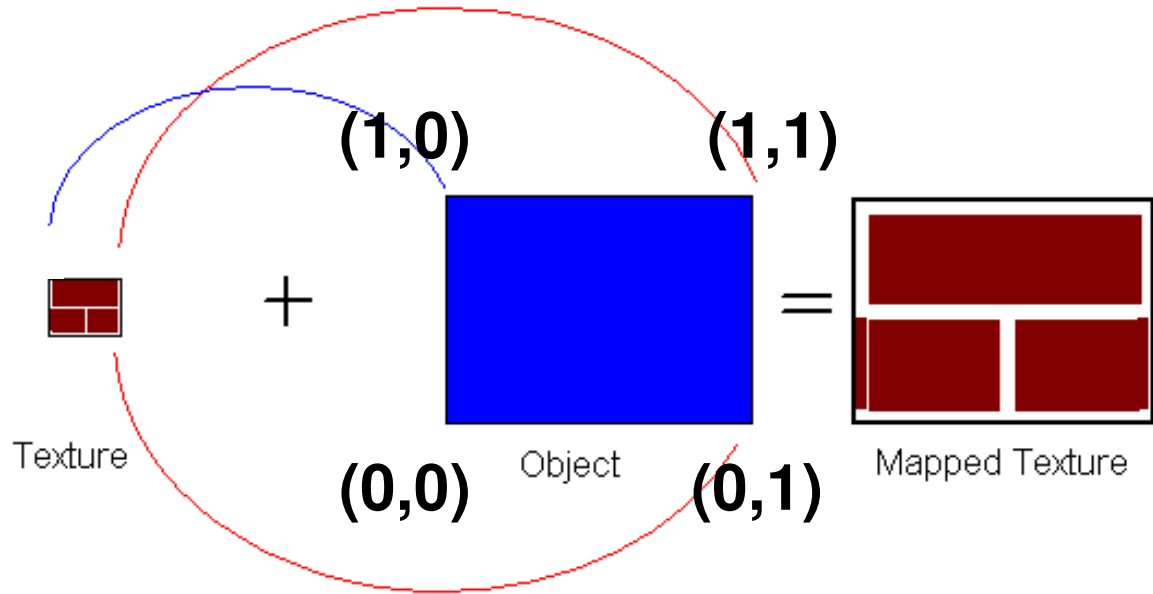
$(.25,0)$

# Texture Lookup: Tiling and Clamping

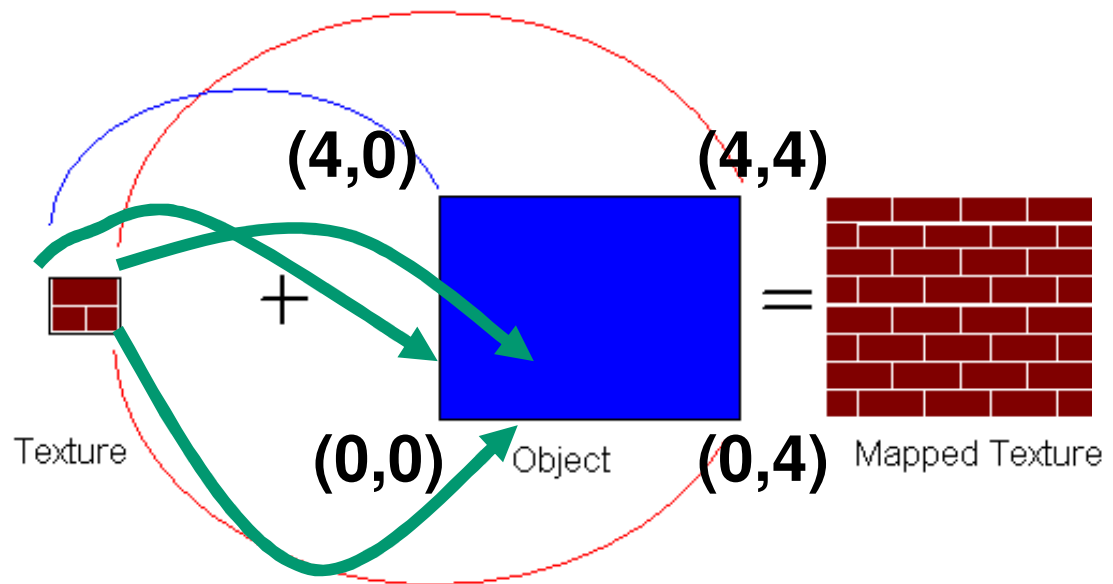
- what if  $s$  or  $t$  is outside the interval  $[0...1]$ ?
- multiple choices
  - use fractional part of texture coordinates
    - cyclic repetition of texture to tile whole surface  
`glTexParameteri( ..., GL_TEXTURE_WRAP_S, GL_REPEAT, GL_TEXTURE_WRAP_T, GL_REPEAT, ... )`
  - clamp every component to range  $[0...1]$ 
    - re-use color values from texture image border  
`glTexParameteri( ..., GL_TEXTURE_WRAP_S, GL_CLAMP, GL_TEXTURE_WRAP_T, GL_CLAMP, ... )`

# Tiled Texture Map

```
glTexCoord2d(1, 1);  
glVertex3d (x, y, z);
```



```
glTexCoord2d(4, 4);  
glVertex3d (x, y, z);
```



# Demo

# Texture Coordinate Transformation

- motivation
  - change scale, orientation of texture on an object
- approach
  - *texture matrix stack*
  - transforms specified (or generated) tex coords

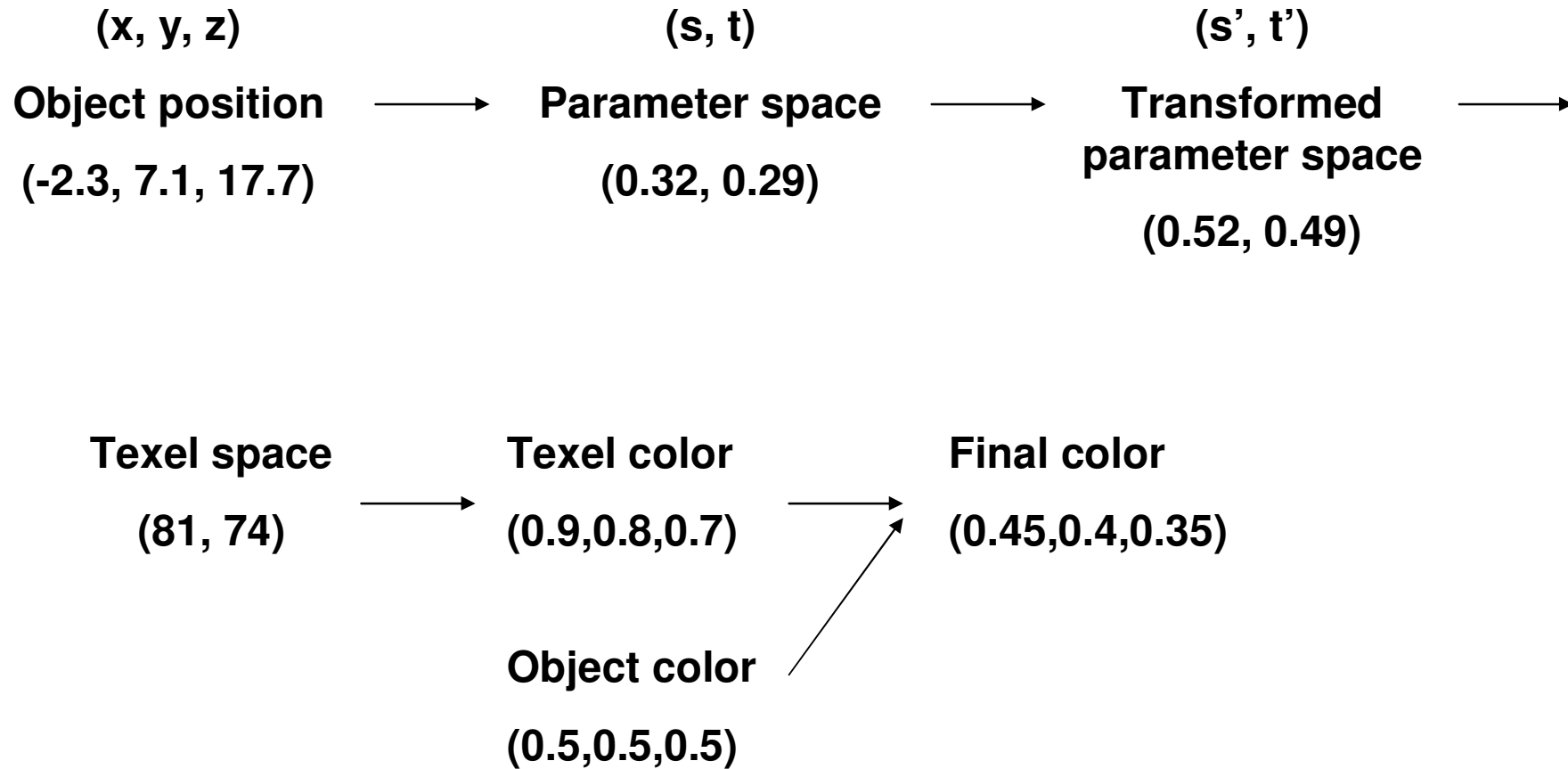
```
glMatrixMode ( GL_TEXTURE );  
glLoadIdentity ();  
glRotate ();  
  
...
```
  - more flexible than changing (s,t) coordinates
- [demo]



# Texture Functions

- once have value from the texture map, can:
  - directly use as surface color: `GL_REPLACE`
    - throw away old color, lose lighting effects
  - modulate surface color: `GL_MODULATE`
    - multiply old color by new value, keep lighting info
    - texturing happens **after** lighting, not relit
  - use as surface color, modulate alpha: `GL_DECAL`
    - like replace, but supports texture transparency
  - blend surface color with another: `GL_BLEND`
    - new value controls which of 2 colors to use
    - indirection, new value not used directly for coloring
- specify with `glTexEnvf (GL_TEXTURE_ENV, GL_TEXTURE_ENV_MODE, <mode>)`

# Texture Pipeline



# Texture Objects and Binding

- texture object
  - an OpenGL data type that keeps textures resident in memory and provides identifiers to easily access them
  - provides efficiency gains over having to repeatedly load and reload a texture
  - you can prioritize textures to keep in memory
  - OpenGL uses least recently used (LRU) if no priority is assigned
- texture binding
  - which texture to use right now
  - switch between preloaded textures

# Basic OpenGL Texturing

- create a texture object and fill it with texture data:
  - `glGenTextures(num, &indices)` to get identifiers for the objects
  - `glBindTexture(GL_TEXTURE_2D, identifier)` to bind
    - following texture commands refer to the bound texture
  - `glTexParameteri(GL_TEXTURE_2D, ..., ...)` to specify parameters for use when applying the texture
  - `glTexImage2D(GL_TEXTURE_2D, ...)` to specify the texture data (the image itself)
- enable texturing: `glEnable(GL_TEXTURE_2D)`
- state how the texture will be used:
  - `glTexEnvf(...)`
- specify texture coordinates for the polygon:
  - use `glTexCoord2f(s, t)` before each vertex:
    - `glTexCoord2f(0, 0); glVertex3f(x, y, z);`

# Low-Level Details

- large range of functions for controlling layout of texture data
  - state how the data in your image is arranged
  - e.g.: `glPixelStorei(GL_UNPACK_ALIGNMENT, 1)` tells OpenGL not to skip bytes at the end of a row
  - you must state how you want the texture to be put in memory: how many bits per “pixel”, which channels,...
- textures must be square and size a power of 2
  - common sizes are 32x32, 64x64, 256x256
  - smaller uses less memory, and there is a finite amount of texture memory on graphics cards
- ok to use texture template sample code for project 4
  - <http://nehe.gamedev.net/data/lessons/lesson.asp?lesson=09>

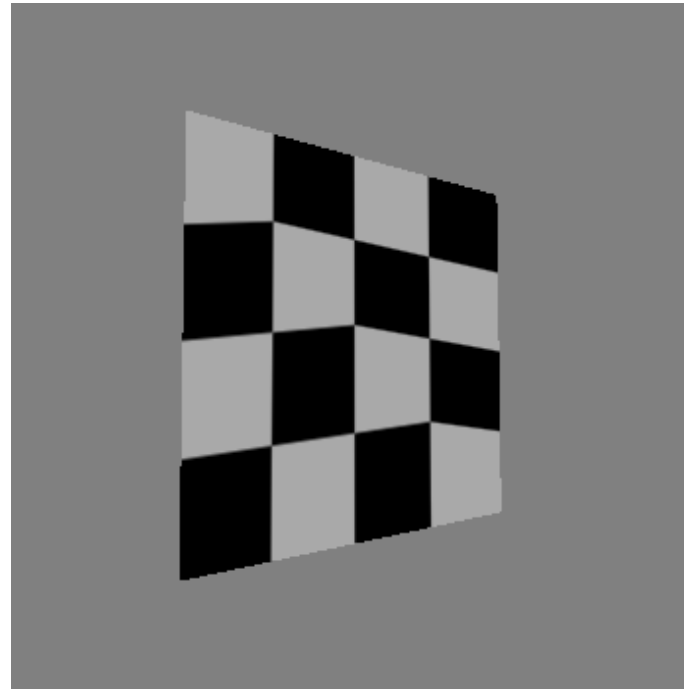
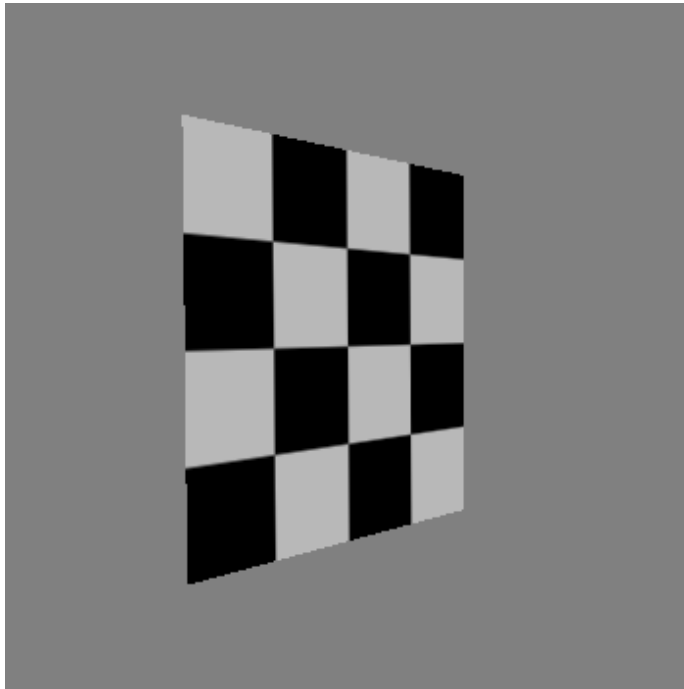
# Texture Mapping

- texture coordinates
  - specified at vertices

```
glTexCoord2f (s, t) ;  
glVertexf (x, y, z) ;
```
  - interpolated across triangle (like R,G,B,Z)
    - ...well not quite!

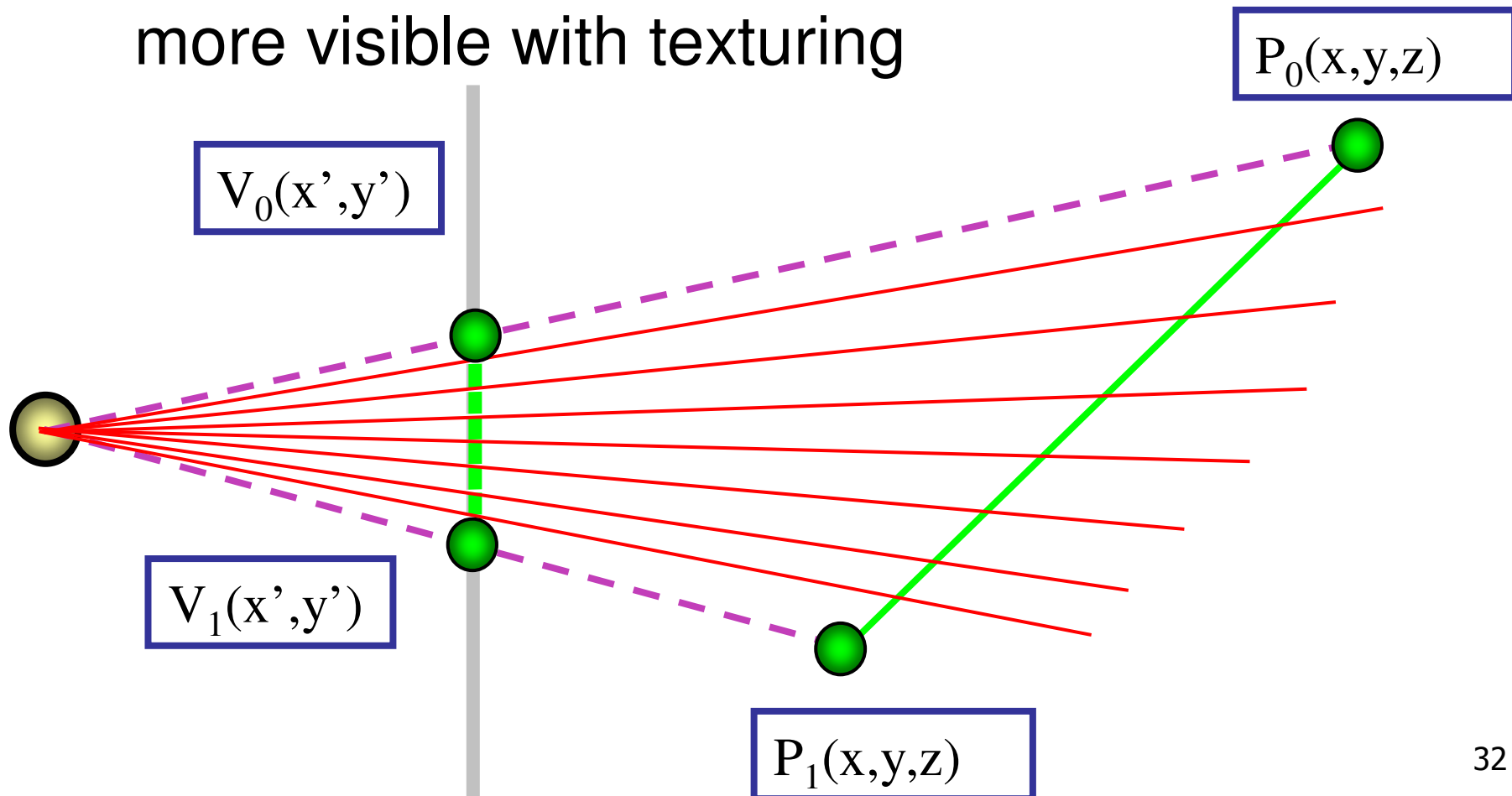
# Texture Mapping

- texture coordinate interpolation
  - perspective foreshortening problem



# Interpolation: Screen vs. World Space

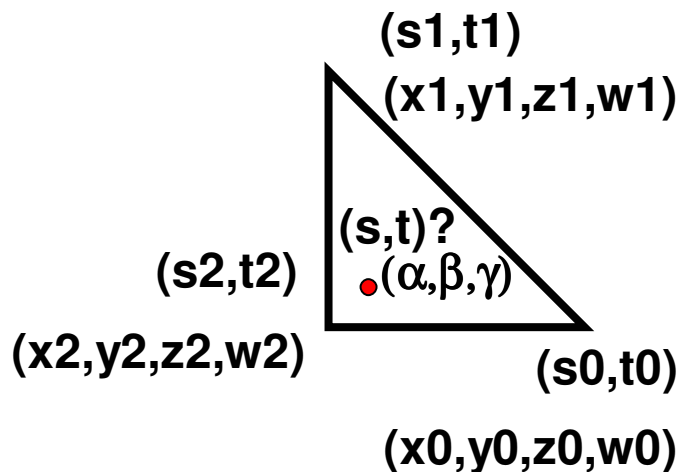
- screen space interpolation incorrect
  - problem ignored with shading, but artifacts more visible with texturing





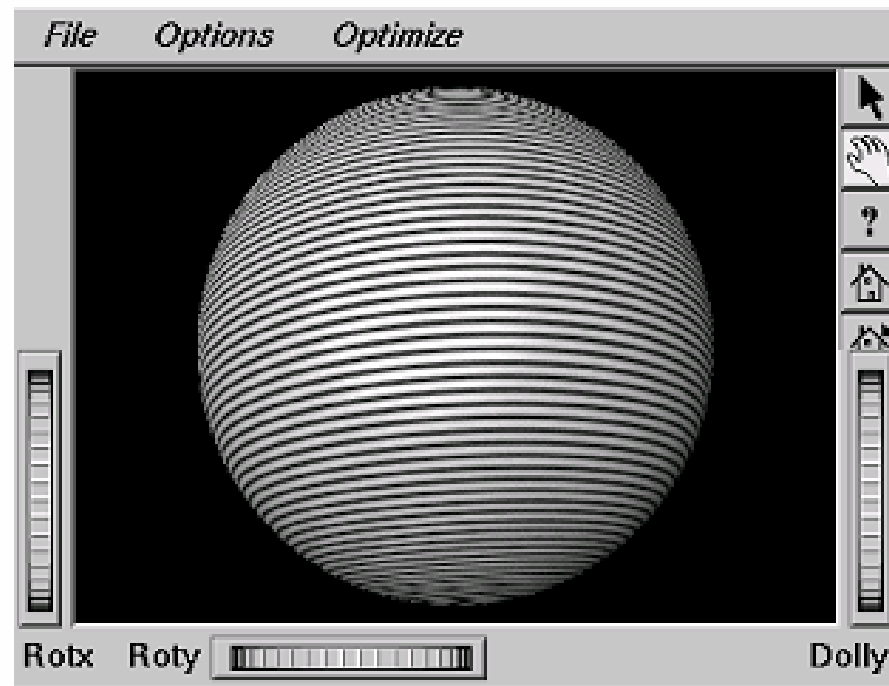
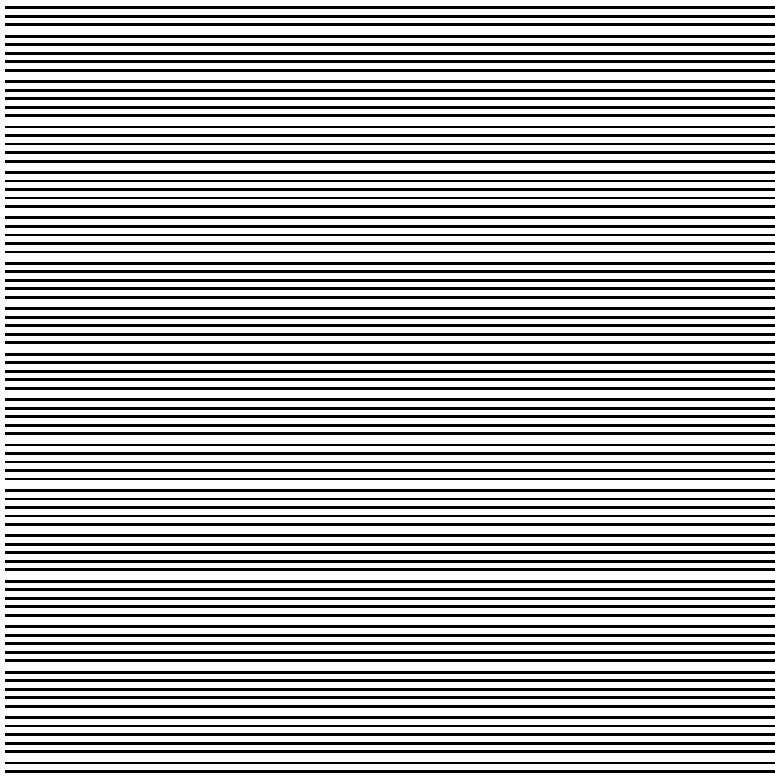
# Texture Coordinate Interpolation

- perspective correct interpolation
  - $\alpha, \beta, \gamma$  :
    - barycentric coordinates of a point **P** in a triangle
  - $s_0, s_1, s_2$  :
    - texture coordinates of vertices
  - $w_0, w_1, w_2$  :
    - homogeneous coordinates of vertices



$$s = \frac{\alpha \cdot s_0 / w_0 + \beta \cdot s_1 / w_1 + \gamma \cdot s_2 / w_2}{\alpha / w_0 + \beta / w_1 + \gamma / w_2}$$

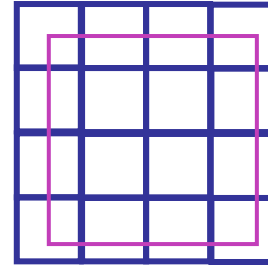
# Reconstruction



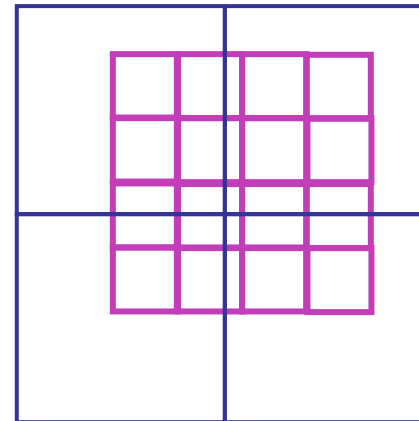
(image courtesy of Kiriakos Kutulakos, U Rochester)

# Reconstruction

- how to deal with:
  - pixels that are much larger than texels?
    - apply filtering, “averaging”

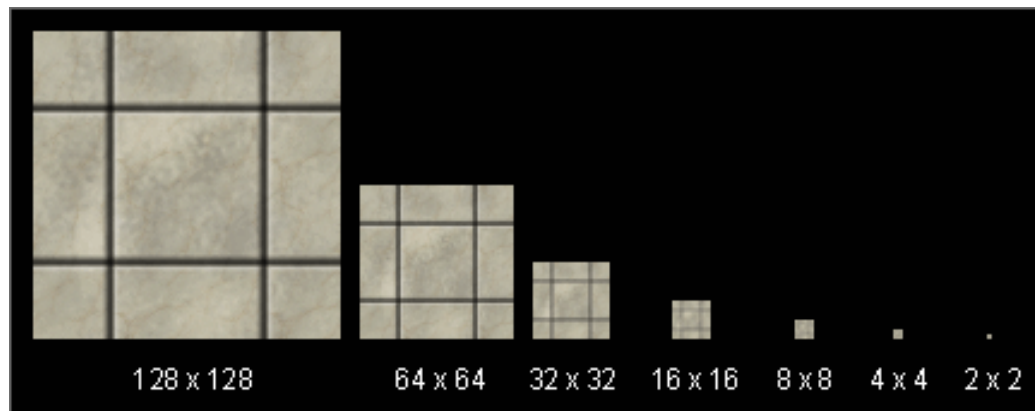


- pixels that are much smaller than texels ?
  - interpolate

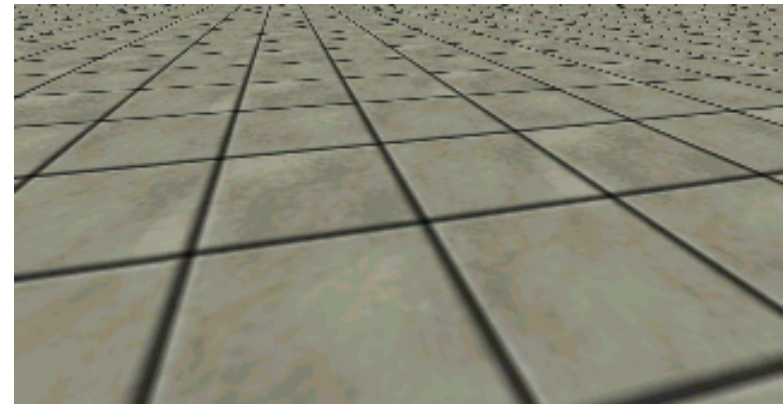
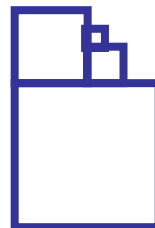


# MIPmapping

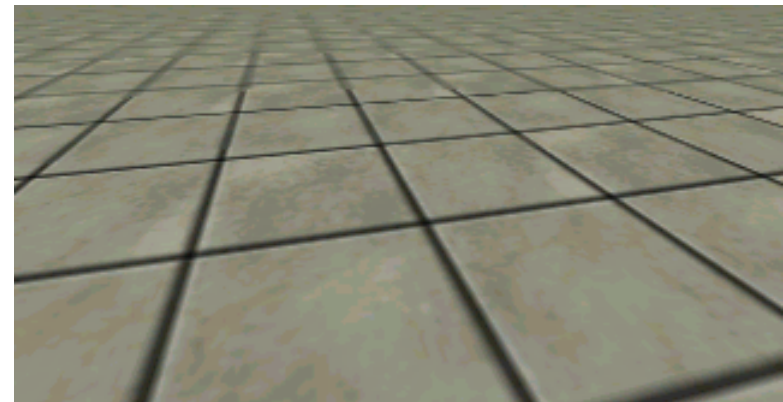
use “image pyramid” to precompute averaged versions of the texture



store whole pyramid in single block of memory



Without MIP-mapping

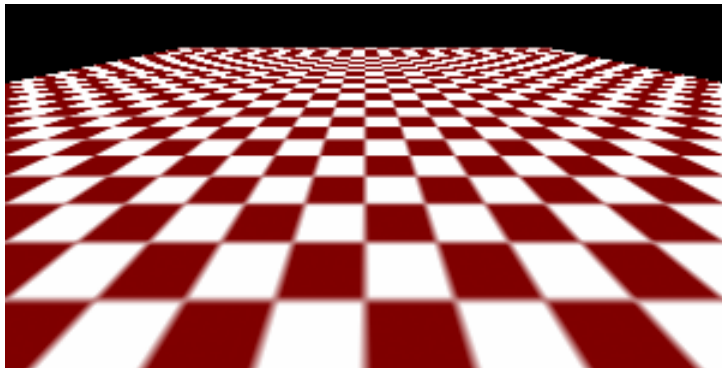


With MIP-mapping<sup>36</sup>

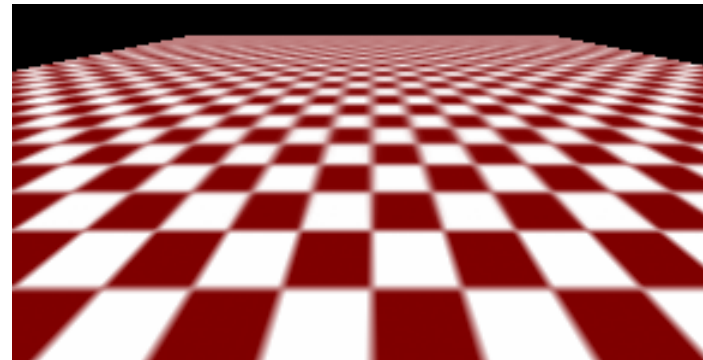
# MIPmaps

- **multum in parvo** -- many things in a small place
  - prespecify a series of prefiltered texture maps of decreasing resolutions
  - requires more texture storage
  - avoid shimmering and flashing as objects move
- `gluBuild2DMipmaps`
  - automatically constructs a family of textures from original texture size down to 1x1

without



with



# MIPmap storage

- only 1/3 more space required



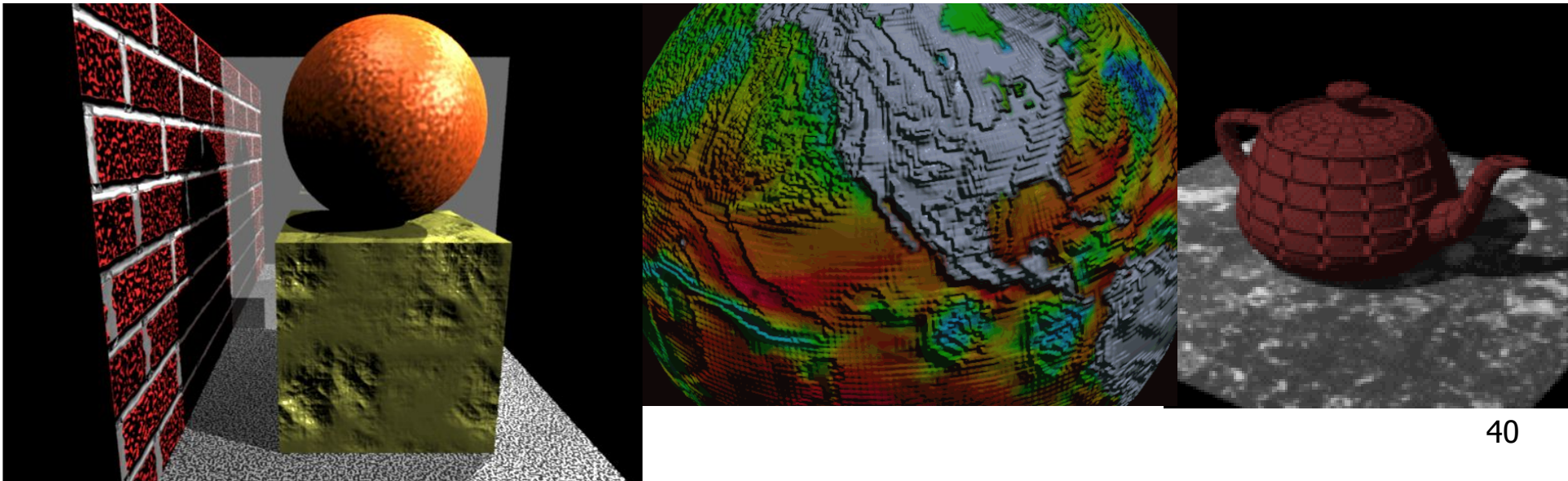
# Texture Parameters

- in addition to color can control other material/object properties
  - surface normal (bump mapping)
  - reflected color (environment mapping)



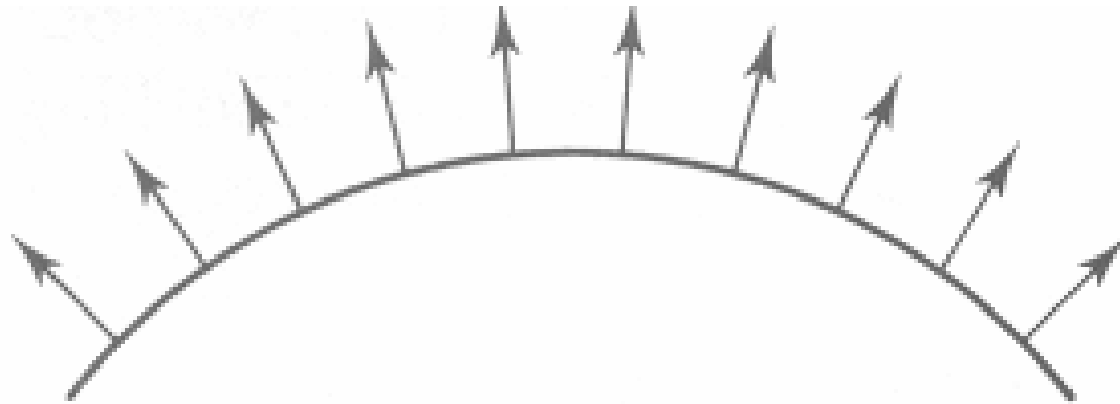
# Bump Mapping: Normals As Texture

- object surface often not smooth – to recreate correctly need complex geometry model
- can control shape “effect” by locally perturbing surface normal
  - random perturbation
  - directional change over region



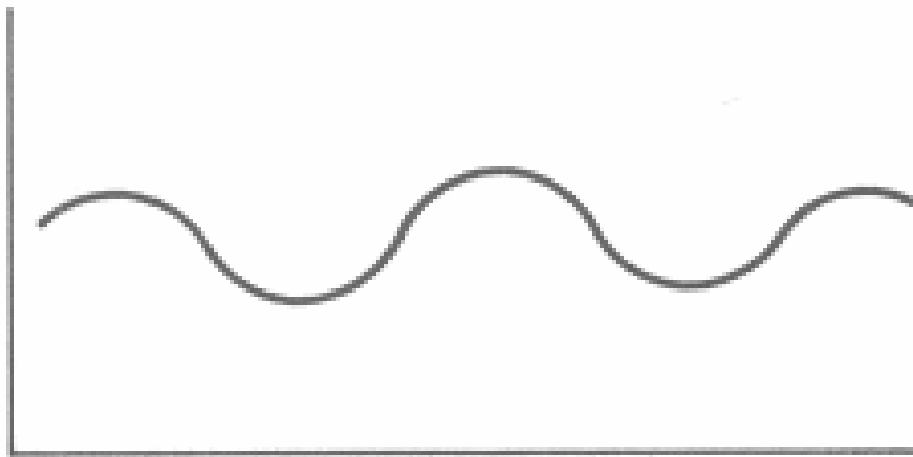


# Bump Mapping



$O(u)$

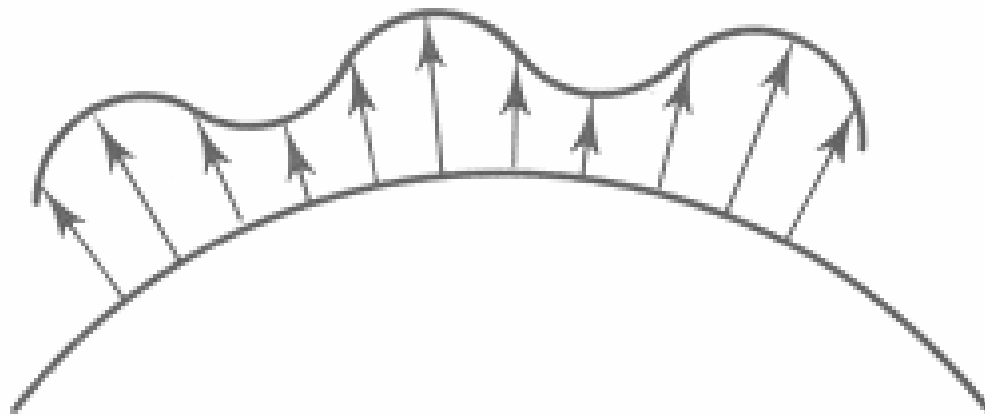
Original surface



$B(u)$

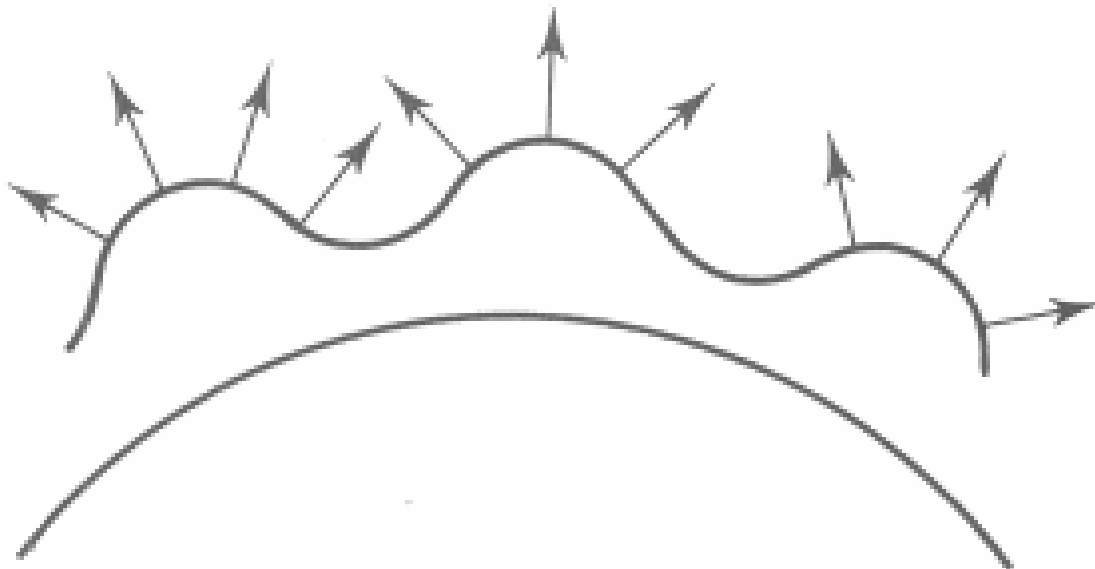
A bump map

# Bump Mapping



$O'(u)$

Lengthening or shortening  
 $O(u)$  using  $B(u)$

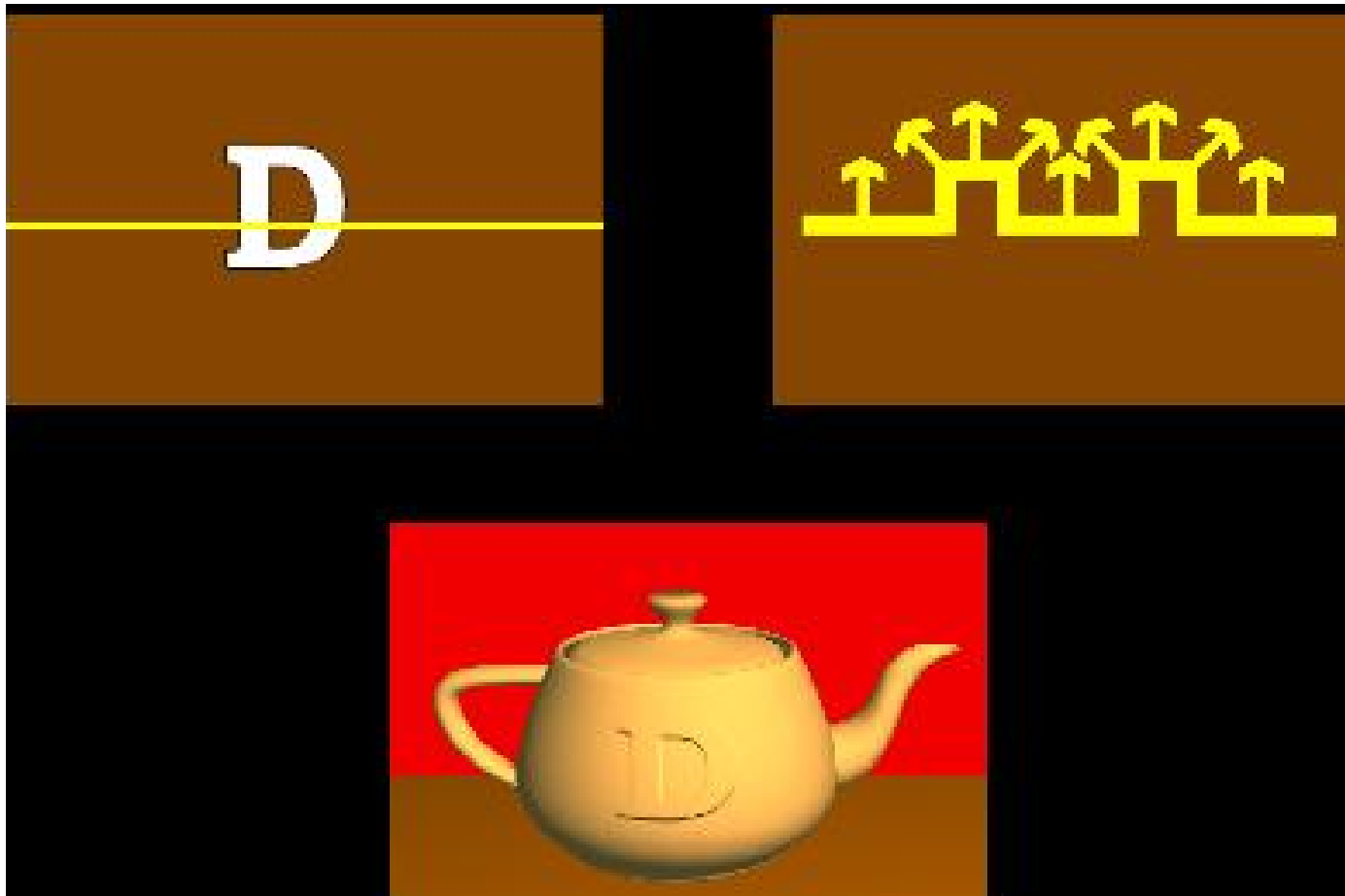


$N'(u)$

The vectors to the  
'new' surface

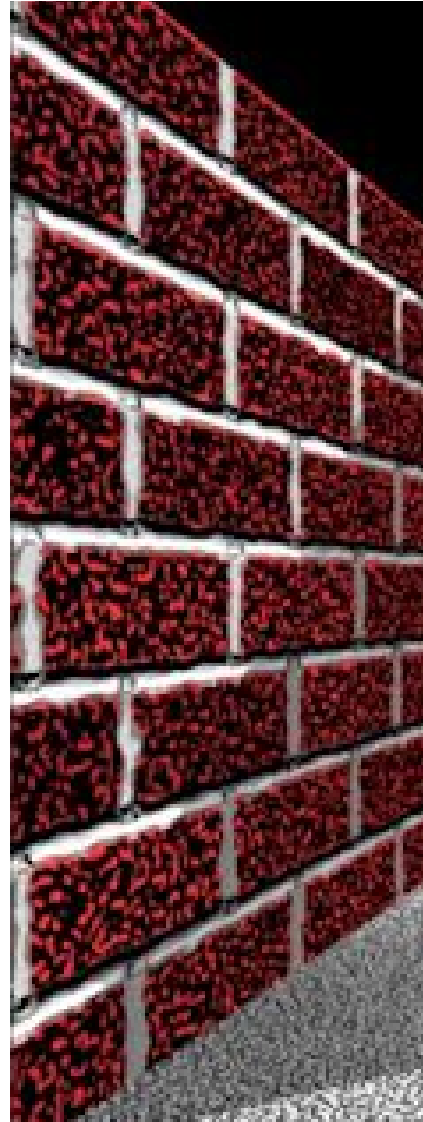
# Embossing

- at transitions
  - rotate point's surface normal by  $\theta$  or  $-\theta$



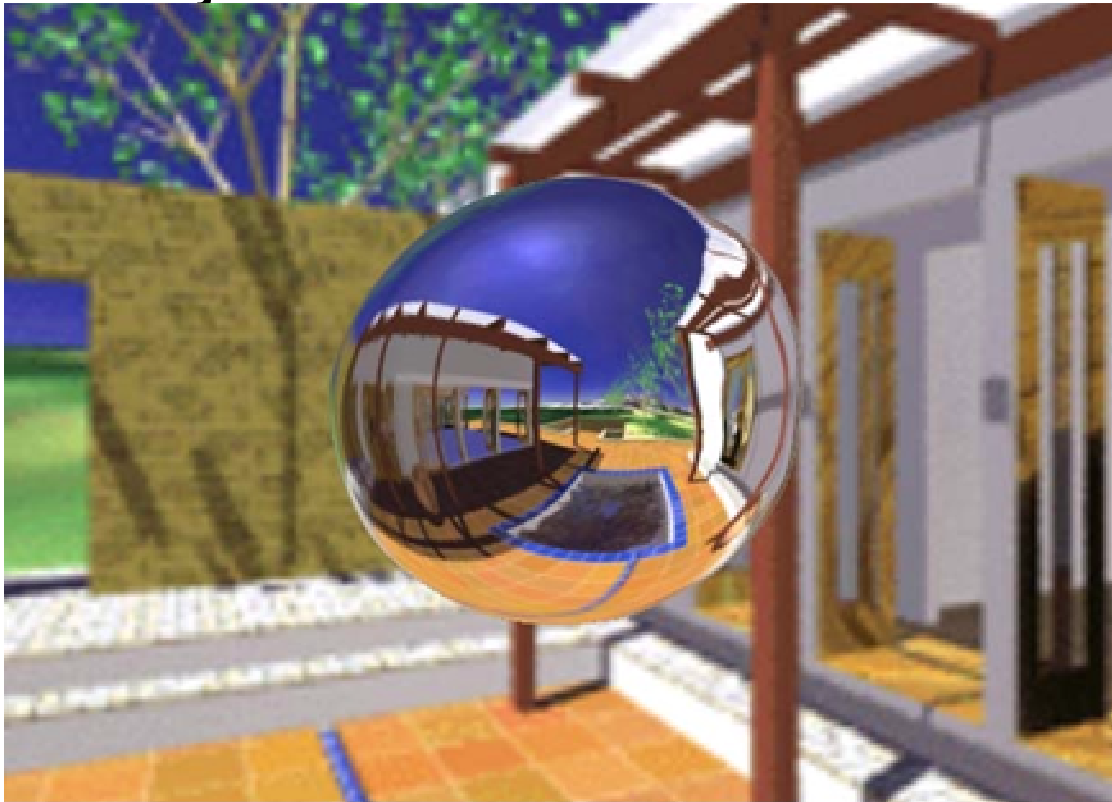
# Displacement Mapping

- bump mapping gets silhouettes wrong
  - shadows wrong too
- change surface geometry instead
  - only recently available with realtime graphics
  - need to subdivide surface



# Environment Mapping

- cheap way to achieve reflective effect
  - generate image of surrounding
  - map to object as texture

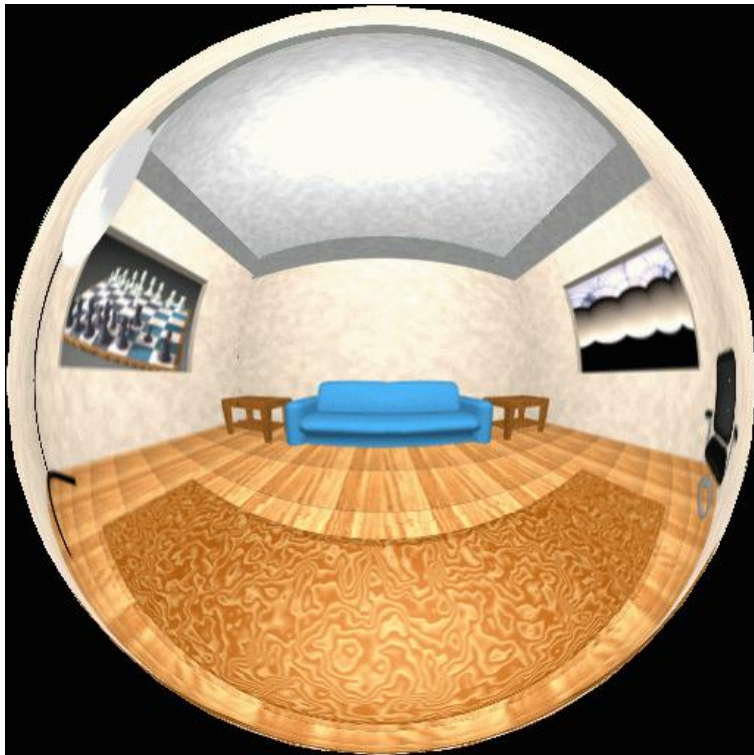


# Environment Mapping

- used to model object that reflects surrounding textures to the eye
  - movie example: cyborg in Terminator 2
- different approaches
  - sphere, cube most popular
    - OpenGL support
      - `GL_SPHERE_MAP`, `GL_CUBE_MAP`
  - others possible too

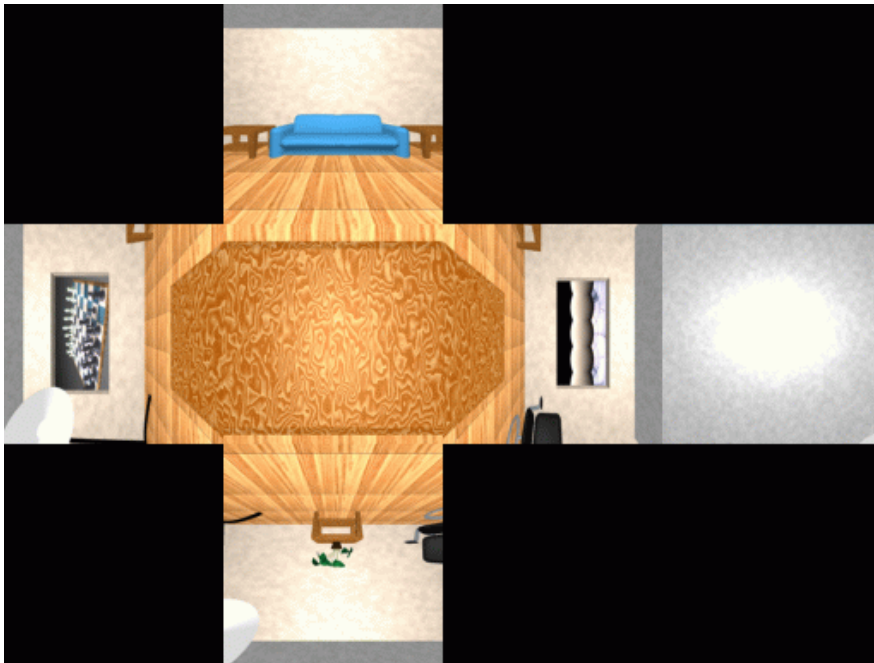
# Sphere Mapping

- texture is distorted fish-eye view
  - point camera at mirrored sphere
  - spherical texture mapping creates texture coordinates that correctly index into this texture map



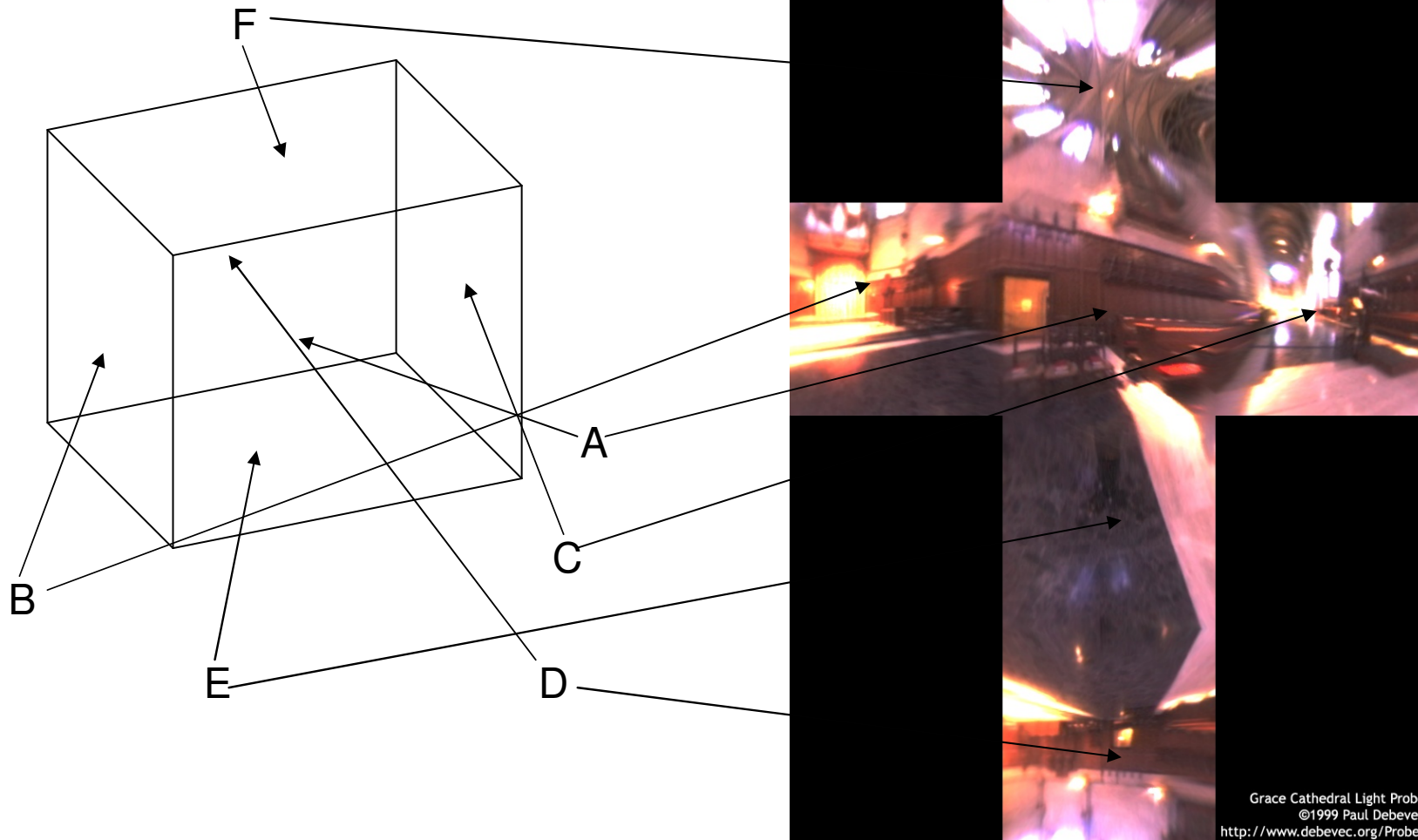
# Cube Mapping

- 6 planar textures, sides of cube
  - point camera in 6 different directions, facing out from origin





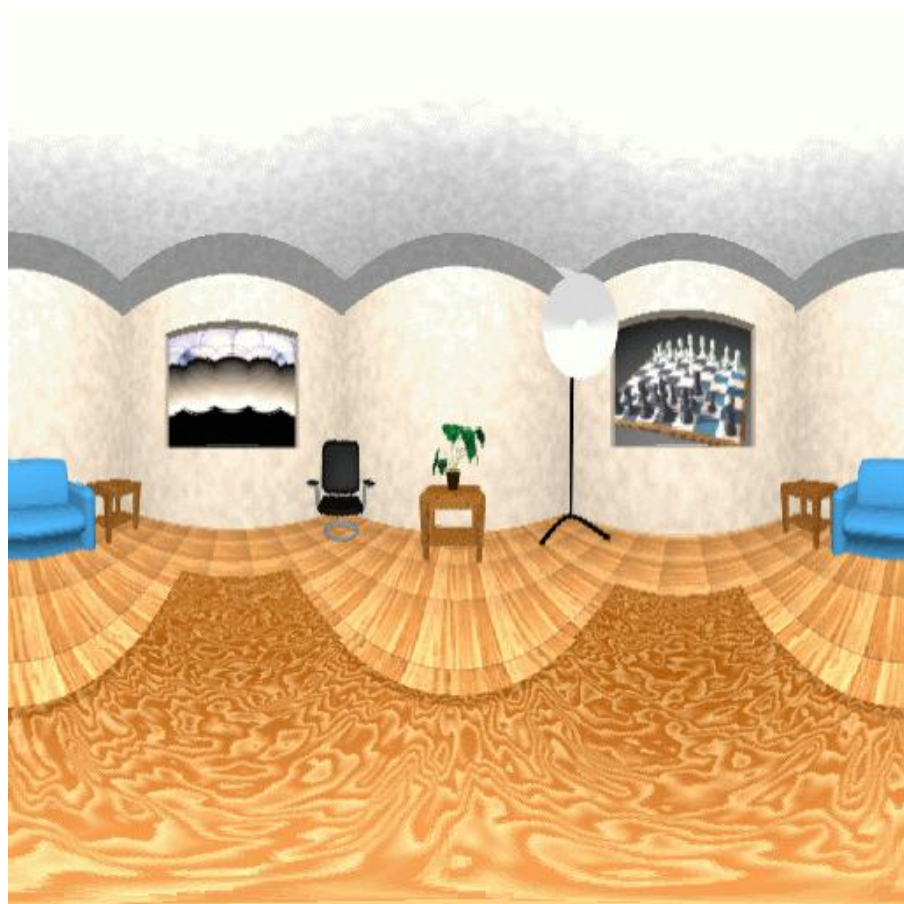
# Cube Mapping



# Cube Mapping

- direction of reflection vector  $r$  selects the face of the cube to be indexed
  - co-ordinate with largest magnitude
    - e.g., the vector  $(-0.2, 0.5, -0.84)$  selects the  $-Z$  face
  - remaining two coordinates (normalized by the 3<sup>rd</sup> coordinate) selects the pixel from the face.
    - e.g.,  $(-0.2, 0.5)$  gets mapped to  $(0.38, 0.80)$ .
- difficulty in interpolating across faces

# Blinn/Newell Latitude Mapping

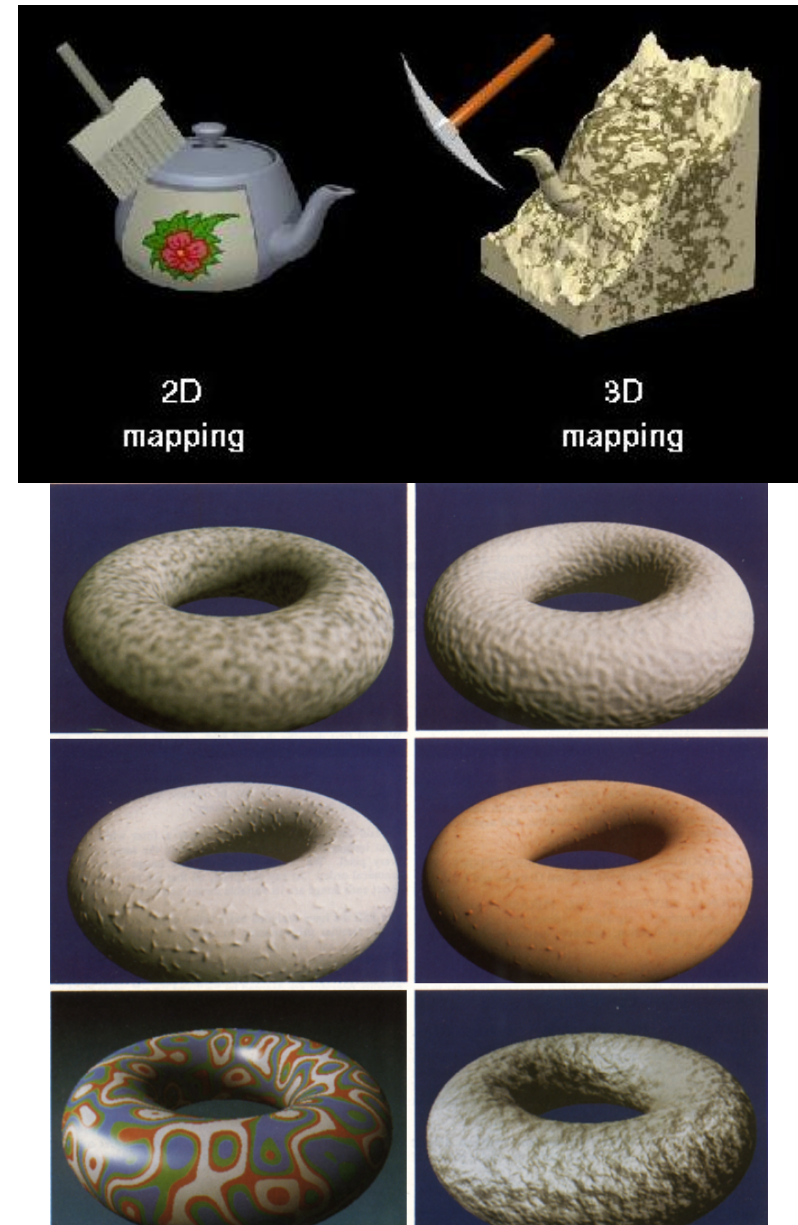


# Review: Texture Objects and Binding

- texture objects
  - texture management: switch with bind, not reloading
  - can prioritize textures to keep in memory
  - Q: what happens to textures kicked out of memory?
    - A: resident memory (on graphics card) vs. nonresident (on CPU)
    - details hidden from developers by OpenGL

# Volumetric Texture

- define texture pattern over 3D domain - 3D space containing the object
  - texture function can be digitized or **procedural**
  - for each point on object compute texture from point location in space
- common for natural material/irregular textures (stone, wood, etc...)



# Volumetric Bump Mapping

Marble



Bump



# Volumetric Texture Principles

- 3D function  $\rho$ 
  - $\rho = \rho(x, y, z)$
- texture space – 3D space that holds the texture (discrete or continuous)
- rendering: for each rendered point  $P(x, y, z)$  compute  $\rho(x, y, z)$
- volumetric texture mapping function/space transformed with objects

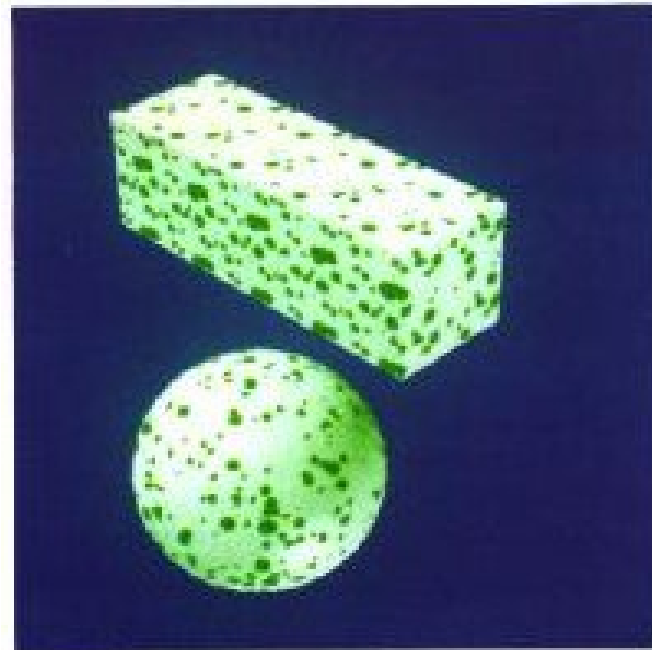
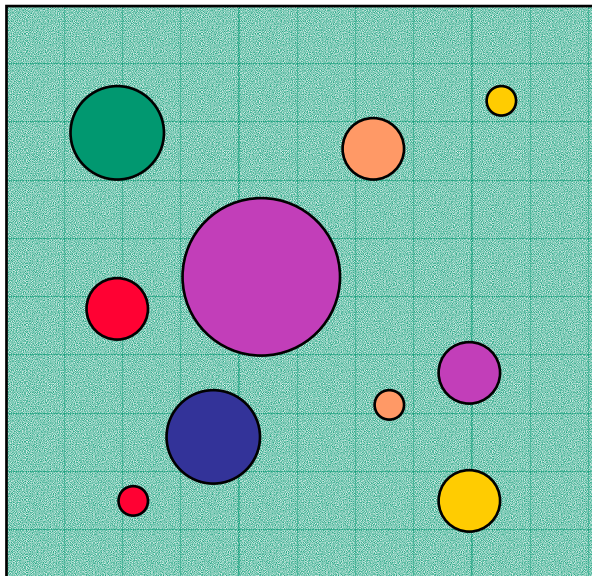
# Procedural Textures

- generate “image” on the fly, instead of loading from disk
  - often saves space
  - allows arbitrary level of detail



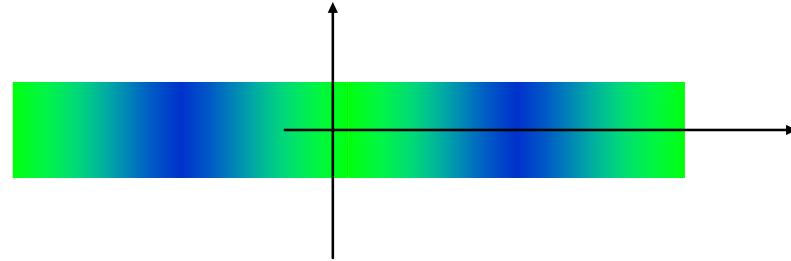
# Procedural Texture Effects: Bombing

- randomly drop bombs of various shapes, sizes and orientation into texture space (store data in table)
  - for point P search table and determine if inside shape
    - if so, color by shape
    - otherwise, color by objects color



# Procedural Texture Effects

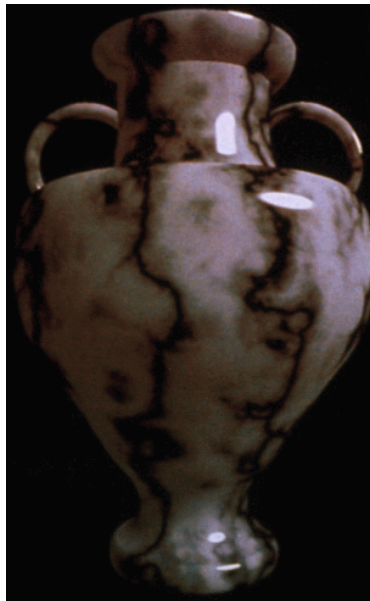
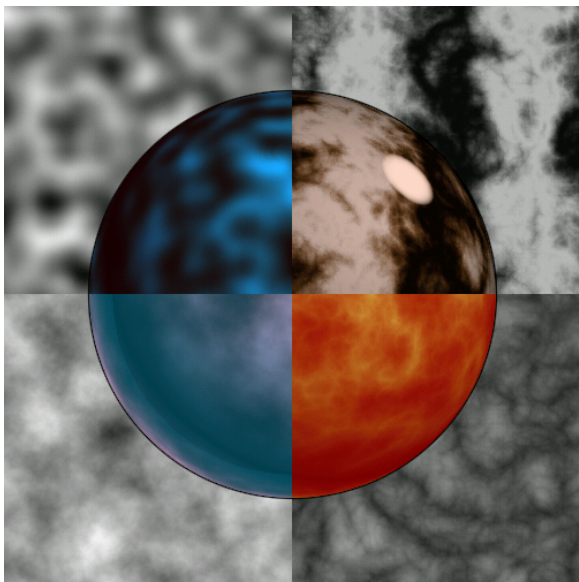
- simple marble



```
function boring_marble(point)
  x = point.x;
  return marble_color(sin(x));
// marble_color maps scalars to colors
```

# Perlin Noise: Procedural Textures

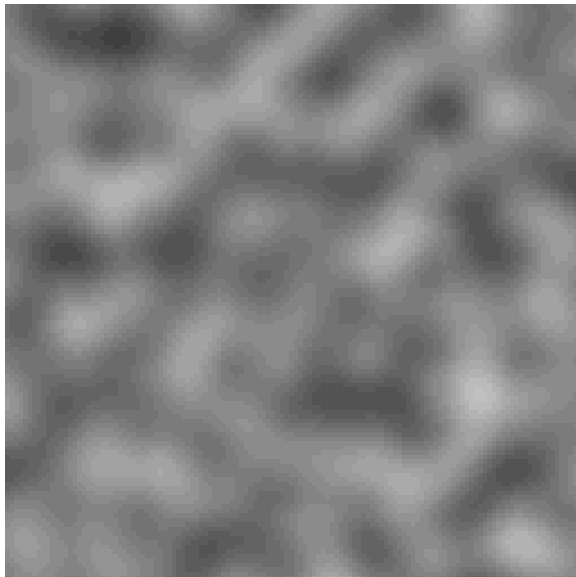
- several good explanations
  - FCG Section 10.1
  - <http://www.noisemachine.com/talk1>
  - [http://freespace.virgin.net/hugo.elias/models/m\\_perlin.htm](http://freespace.virgin.net/hugo.elias/models/m_perlin.htm)
  - <http://www.robo-murito.net/code/perlin-noise-math-faq.html>



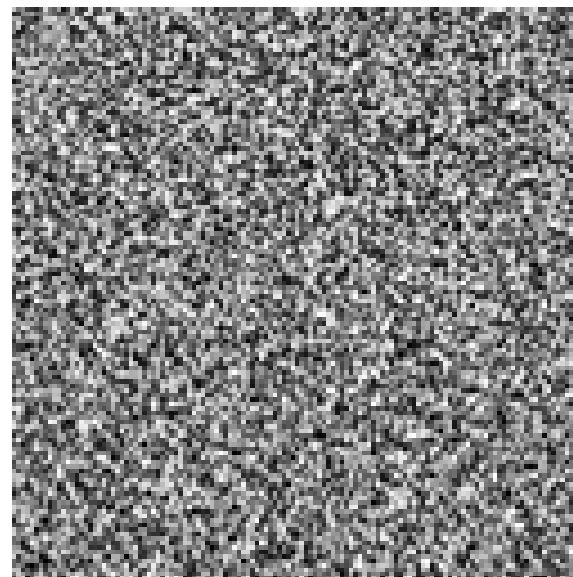
# Perlin Noise: Coherency

- smooth not abrupt changes

coherent



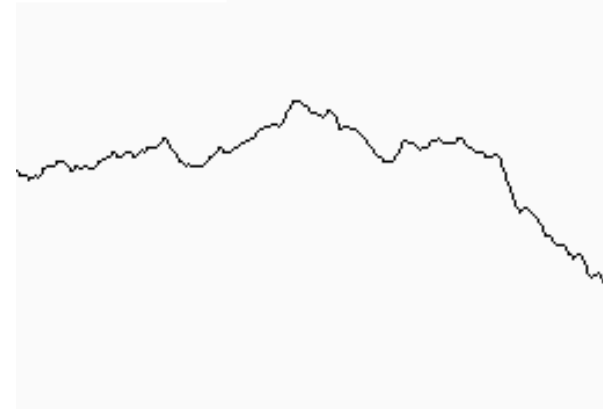
white noise



# Perlin Noise: Turbulence

- multiple feature sizes
  - add scaled copies of noise

Sum of Noise Functions = ( Perlin Noise )



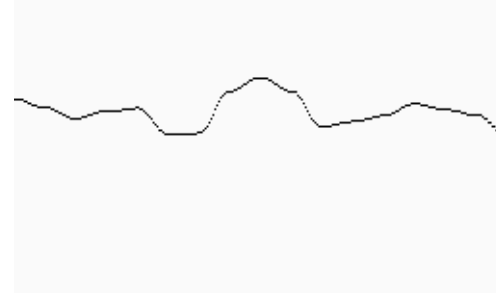
Amplitude : 128  
frequency : 4



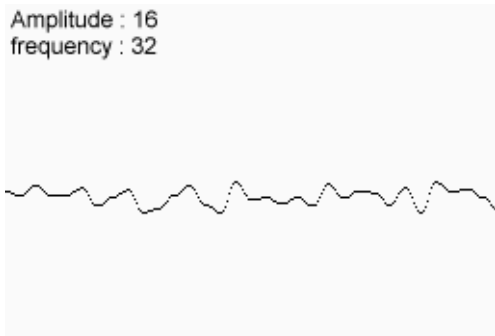
Amplitude : 64  
frequency : 8



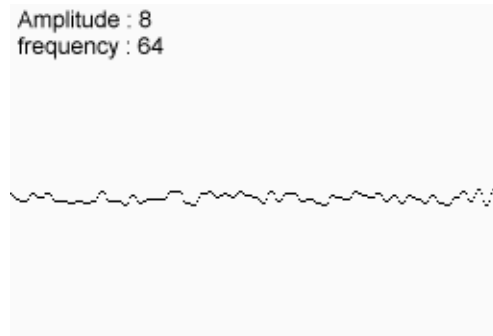
Amplitude : 32  
frequency : 16



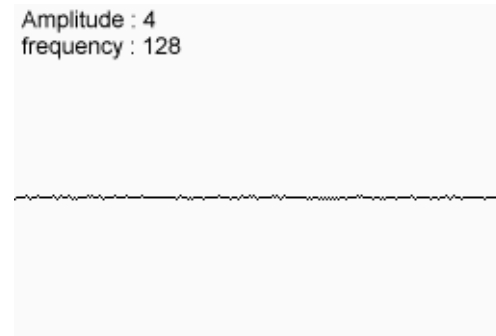
Amplitude : 16  
frequency : 32



Amplitude : 8  
frequency : 64

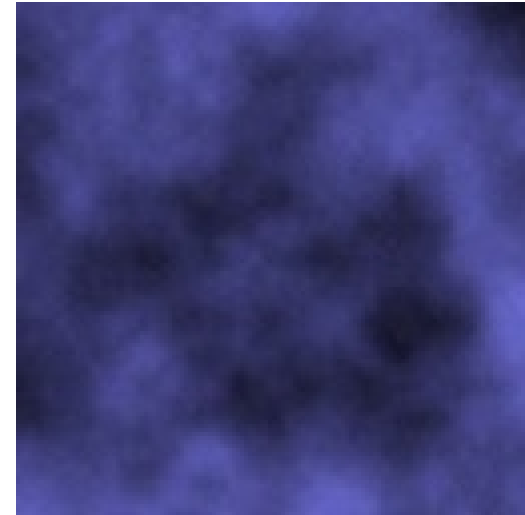
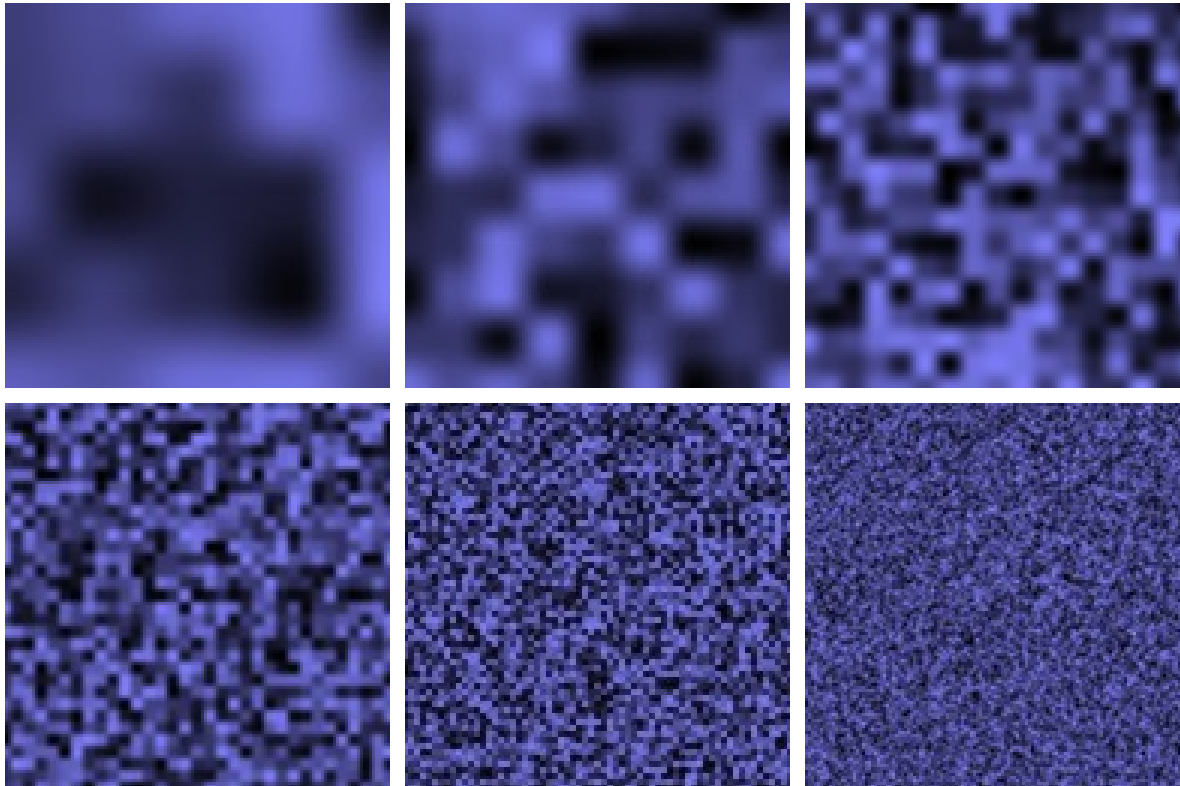


Amplitude : 4  
frequency : 128



# Perlin Noise: Turbulence

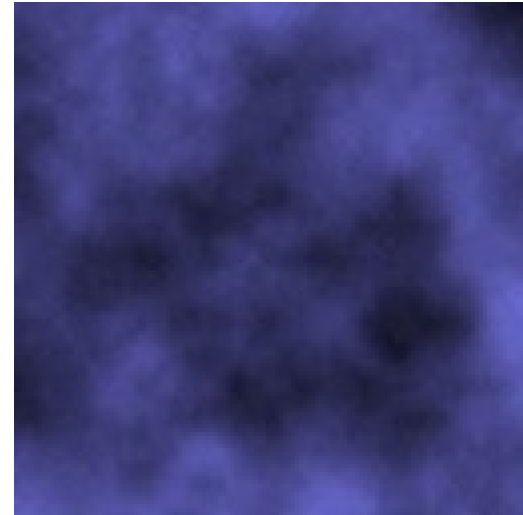
- multiple feature sizes
  - add scaled copies of noise



# Perlin Noise: Turbulence

- multiple feature sizes
  - add scaled copies of noise

```
function turbulence(p)
  t = 0; scale = 1;
  while (scale > pixelsize) {
    t +=
abs(Noise(p/scale)*scale);
    scale/=2;
  } return t;
```



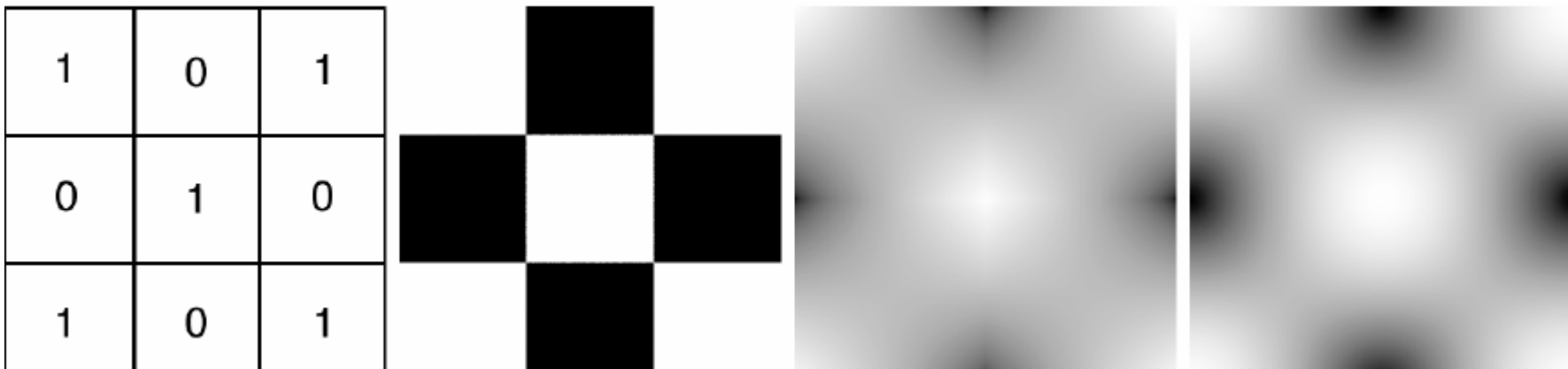
# Generating Coherent Noise

- just three main ideas
  - nice interpolation
  - use vector offsets to make grid irregular
  - optimization
    - sneaky use of 1D arrays instead of 2D/3D one



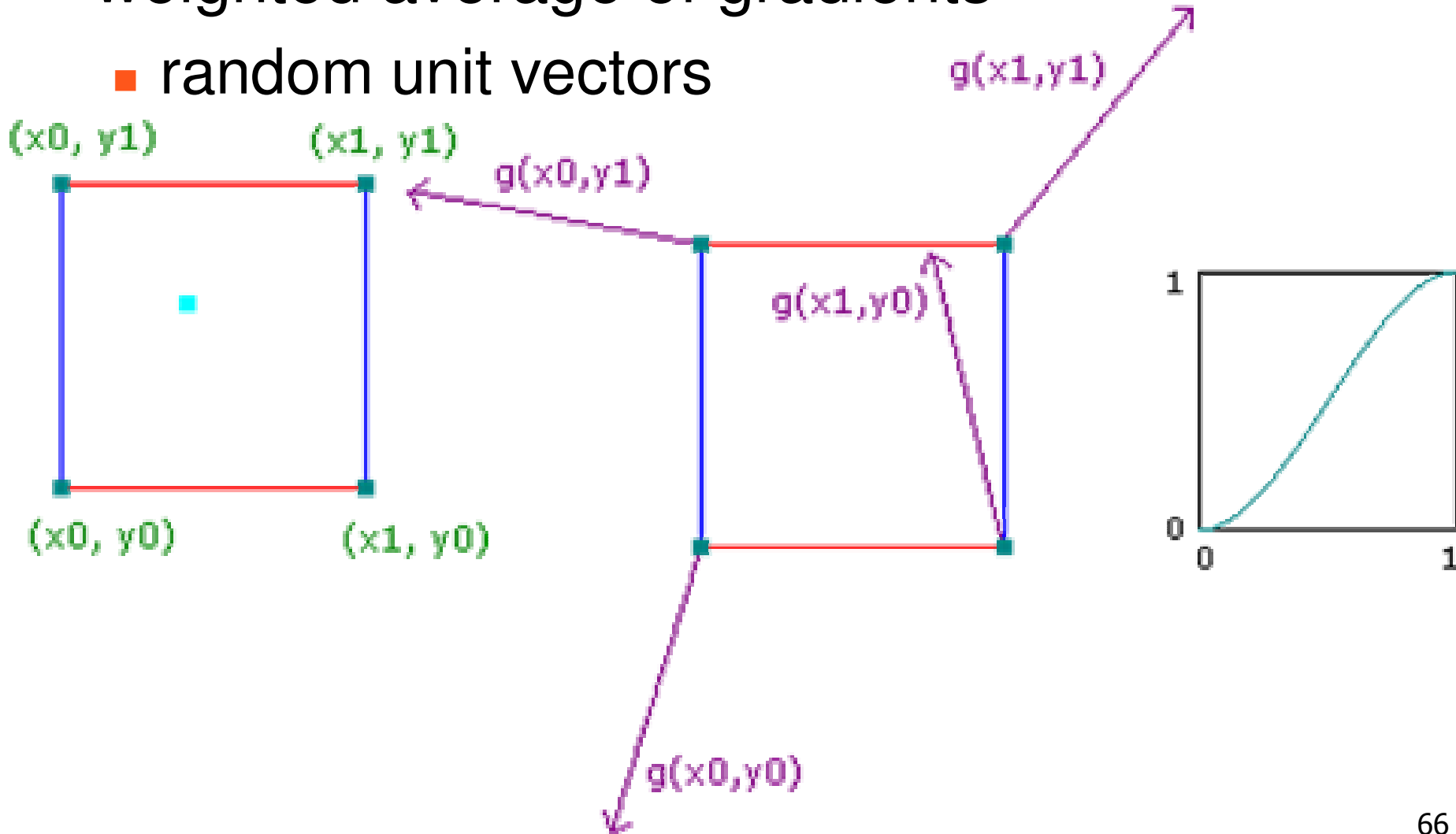
# Interpolating Textures

- nearest neighbor
- bilinear
- hermite



# Vector Offsets From Grid

- weighted average of gradients
  - random unit vectors



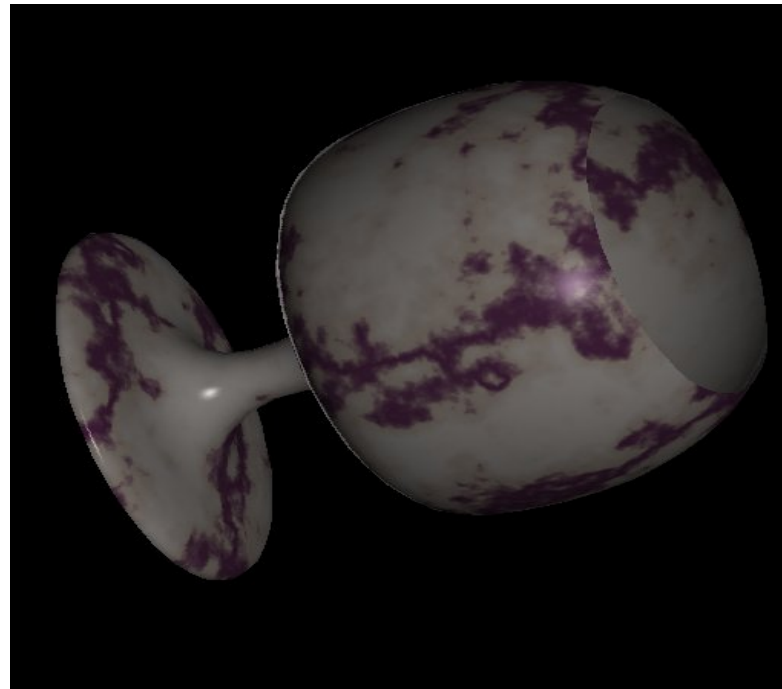
# Optimization

- save memory and time
- conceptually:
  - 2D or 3D grid
  - populate with random number generator
- actually:
  - precompute two 1D arrays of size n (typical size 256)
    - random unit vectors
    - permutation of integers 0 to n-1
  - lookup
    - $g(i, j, k) = G[ ( i + P[ ( j + P[k] ) \bmod n ] ) \bmod n ]$

# Perlin Marble

- use turbulence, which in turn uses noise:

```
function marble(point)
  x = point.x + turbulence(point);
  return marble_color(sin(x))
```



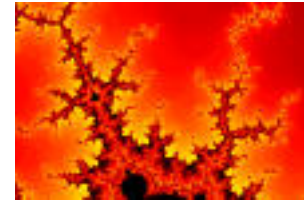
# Procedural Approaches

# Procedural Modeling

- textures, geometry
  - nonprocedural: explicitly stored in memory
- procedural approach
  - compute something on the fly
  - often less memory cost
  - visual richness
- fractals, particle systems, noise

# Fractal Landscapes

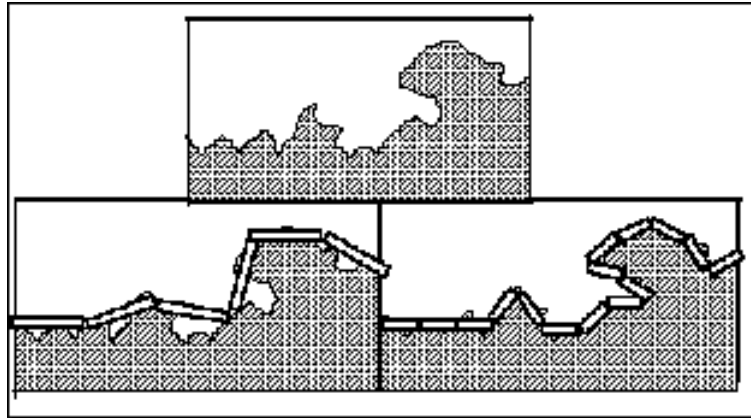
- fractals: not just for “showing math”
  - triangle subdivision
  - vertex displacement
  - recursive until termination condition



<http://www.fractal-landscapes.co.uk/images.html>

# Self-Similarity

- infinite nesting of structure on all scales

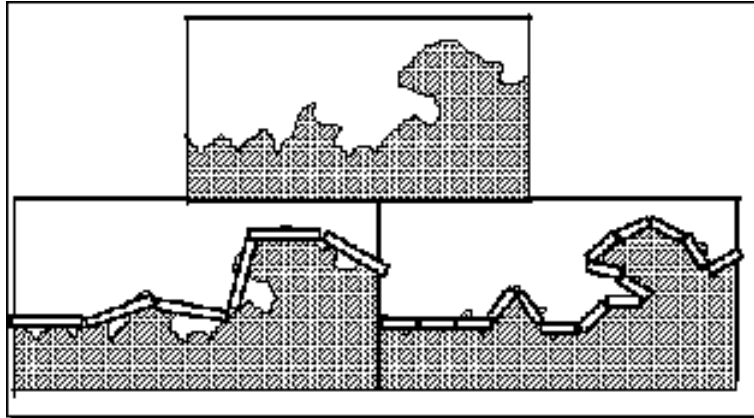




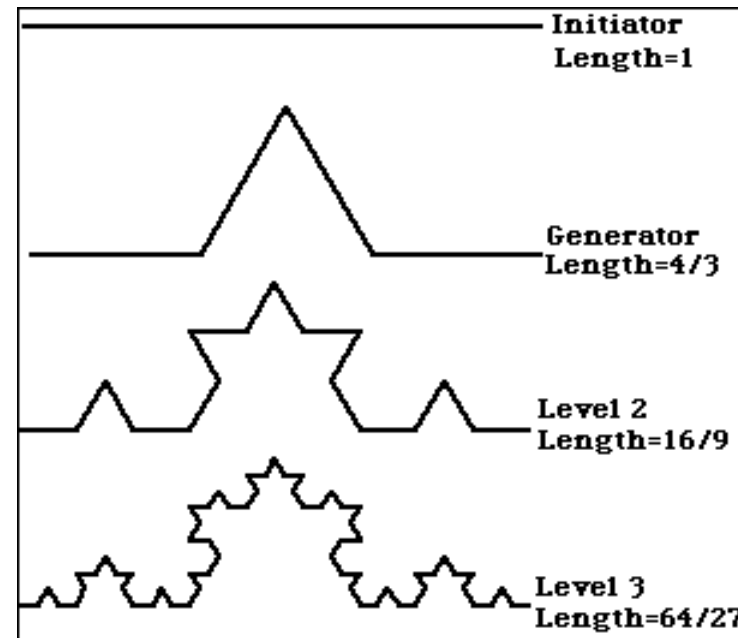
# Fractal Dimension

- $D = \log(N)/\log(r)$   
N = measure, r = subdivision scale
- Hausdorff dimension: noninteger

coastline of Britain



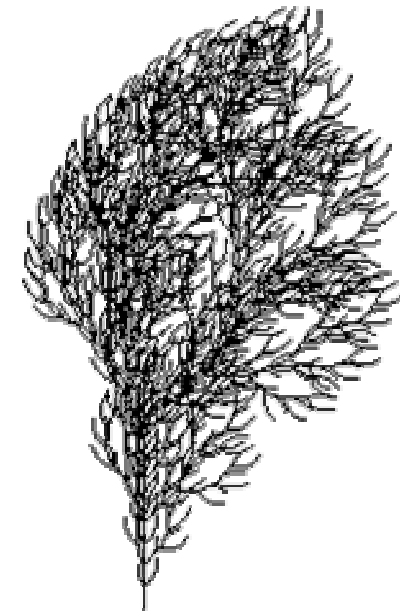
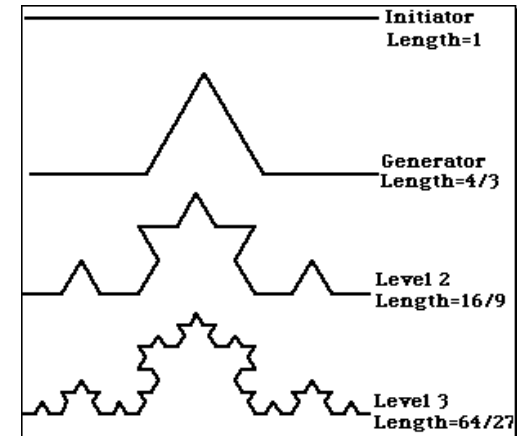
Koch snowflake



$$D = \log(N)/\log(r) \quad D = \log(4)/\log(3) = 1.26$$

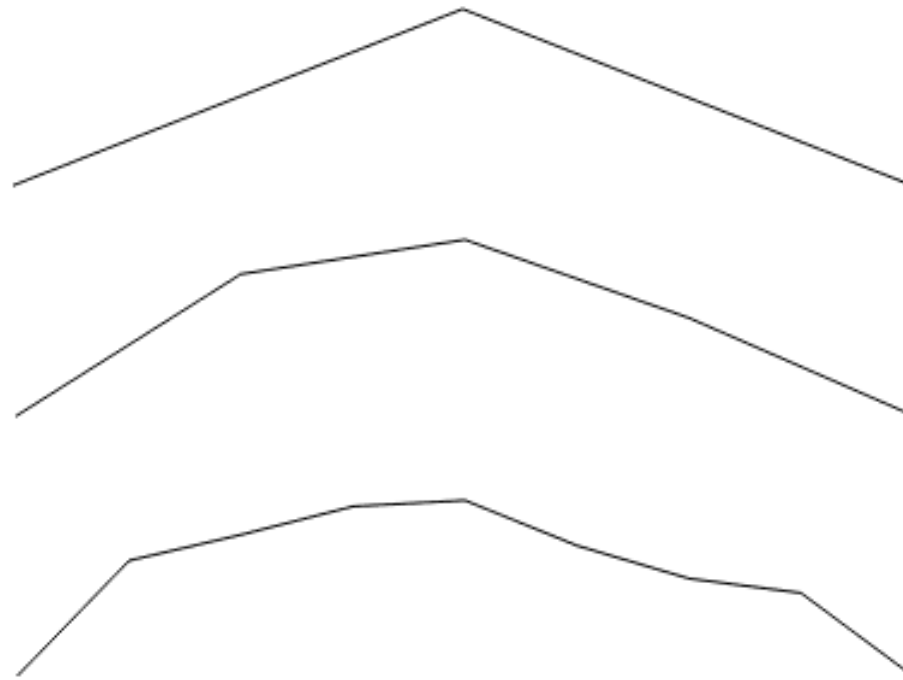
# Language-Based Generation

- L-Systems: after Lindenmayer
  - Koch snowflake:  $F :- FLFRRLFLF$ 
    - F: forward, R: right, L: left
  - Mariano's Bush:  
 $F = FF - [-F + F + F] + [+F - F - F] \}$ 
    - angle 16



# 1D: Midpoint Displacement

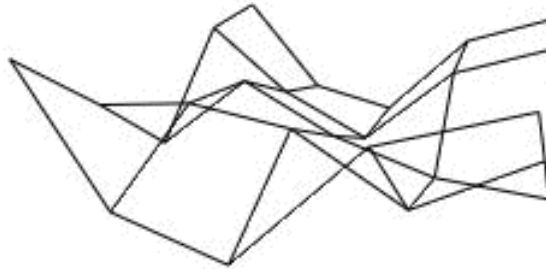
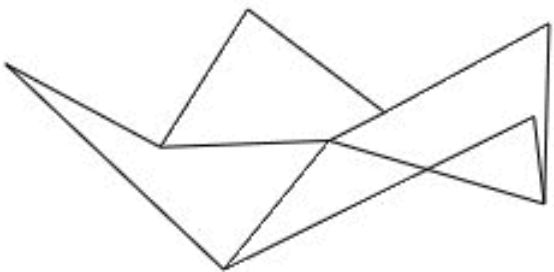
- divide in half
- randomly displace
- scale variance by half



<http://www.gameprogrammer.com/fractal.html>

# 2D: Diamond-Square

- diamond step
  - generate a new value at square midpoint
    - average corner values + random amount
    - gives diamonds when have multiple squares in grid
- square step
  - generate new value at diamond midpoint
    - average corner values + random amount
    - gives squares again in grid

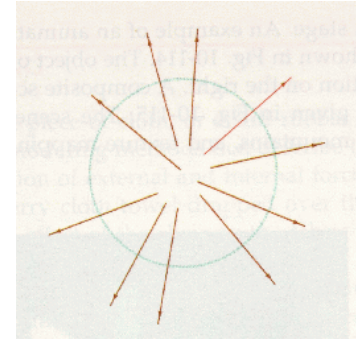
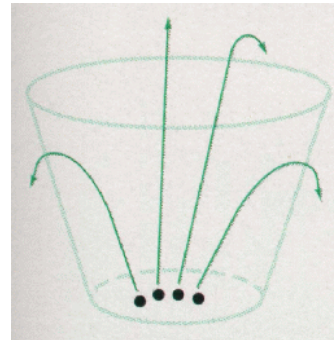


# Particle Systems

- loosely defined
  - modeling, or rendering, or animation
- key criteria
  - collection of particles
  - random element controls attributes
    - position, velocity (speed and direction), color, lifetime, age, shape, size, transparency
    - predefined stochastic limits: bounds, variance, type of distribution

# Particle System Examples

- objects changing fluidly over time
  - fire, steam, smoke, water
- objects fluid in form
  - grass, hair, dust
- physical processes
  - waterfalls, fireworks, explosions
- group dynamics: behavioral
  - birds/bats flock, fish school, human crowd, dinosaur/elephant stampede



# Particle Systems Demos

- general particle systems
  - <http://www.wondertouch.com>
- boids: bird-like objects
  - <http://www.red3d.com/cwr/boids/>

# Particle Life Cycle

- generation
  - randomly within “fuzzy” location
  - initial attribute values: random or fixed
- dynamics
  - attributes of each particle may vary over time
    - color darker as particle cools off after explosion
  - can also depend on other attributes
    - position: previous particle position + velocity + time
- death
  - age and lifetime for each particle (in frames)
  - or if out of bounds, too dark to see, etc



# Particle System Rendering

- expensive to render thousands of particles
- simplify: avoid hidden surface calculations
  - each particle has small graphical primitive (blob)
  - pixel color: sum of all particles mapping to it
- some effects easy
  - temporal anti-aliasing (motion blur)
    - normally expensive: supersampling over time
    - position, velocity known for each particle
    - just render as streak

# Procedural Approaches Summary

- Perlin noise
- fractals
- L-systems
- particle systems
  
- not at all a complete list!
  - big subject: entire classes on this alone

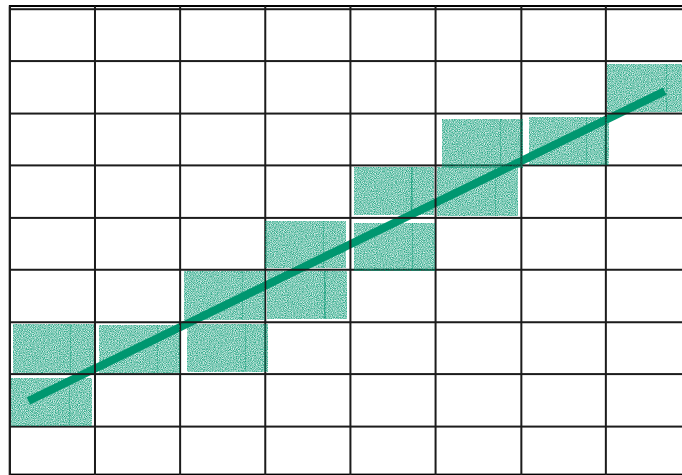
# Sampling

# Samples

- most things in the real world are **continuous**
- everything in a computer is **discrete**
- the process of mapping a continuous function to a discrete one is called **sampling**
- the process of mapping a discrete function to a continuous one is called **reconstruction**
- the process of mapping a continuous variable to a discrete one is called **quantization**
- rendering an image requires sampling and quantization
- displaying an image involves reconstruction

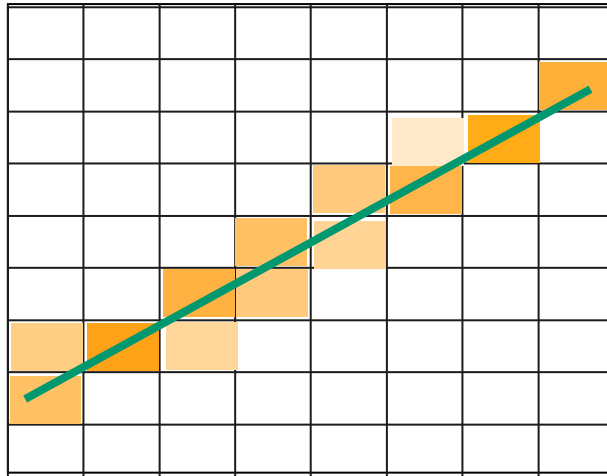
# Line Segments

- we tried to sample a line segment so it would map to a 2D raster display
- we quantized the pixel values to 0 or 1
- we saw stair steps, or jaggies



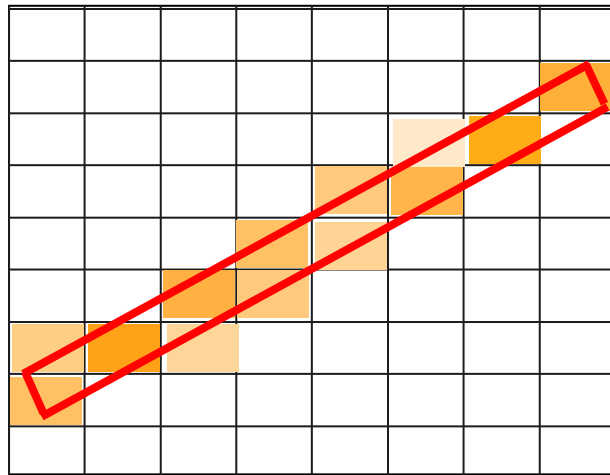
# Line Segments

- instead, quantize to many shades
- but what sampling algorithm is used?



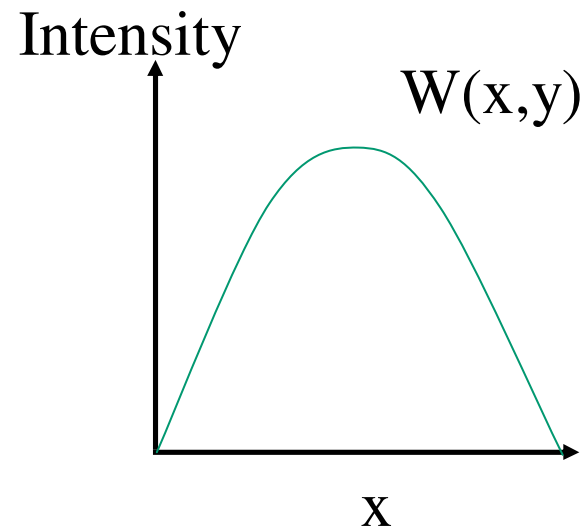
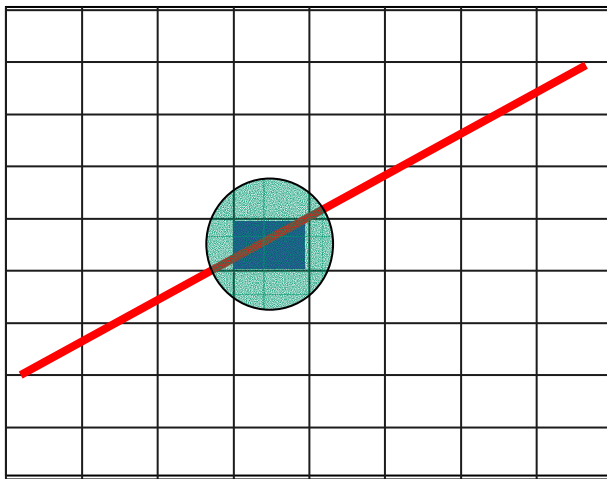
# Unweighted Area Sampling

- shade pixels wrt area covered by thickened line
- equal areas cause equal intensity, regardless of distance from pixel center to area
  - rough approximation formulated by dividing each pixel into a finer grid of pixels
- primitive cannot affect intensity of pixel if it does not intersect the pixel



# Weighted Area Sampling

- intuitively, pixel cut through the center should be more heavily weighted than one cut along corner
- weighting function,  $W(x,y)$ 
  - specifies the contribution of primitive passing through the point  $(x, y)$  from pixel center





# Images

- an image is a 2D function  $I(x, y)$  that specifies intensity for each point  $(x, y)$

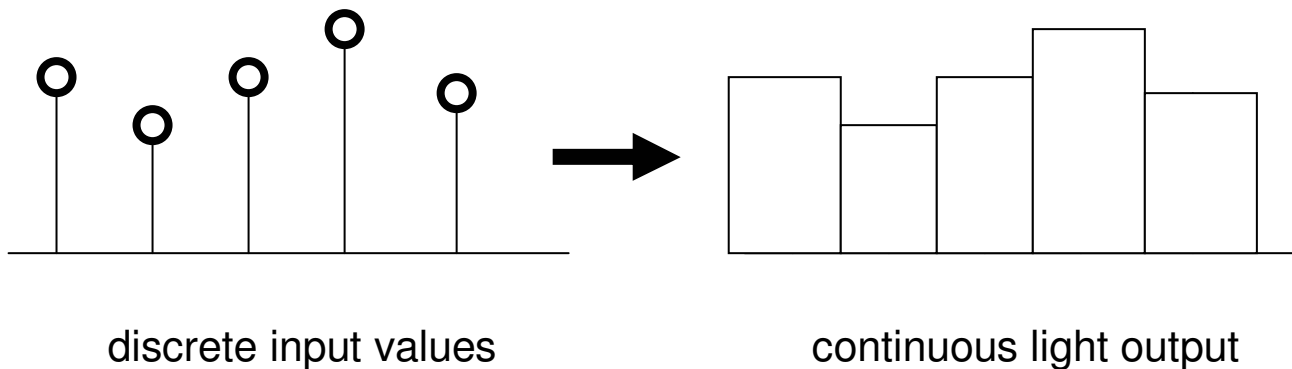


An image seen as a continuous 2D function



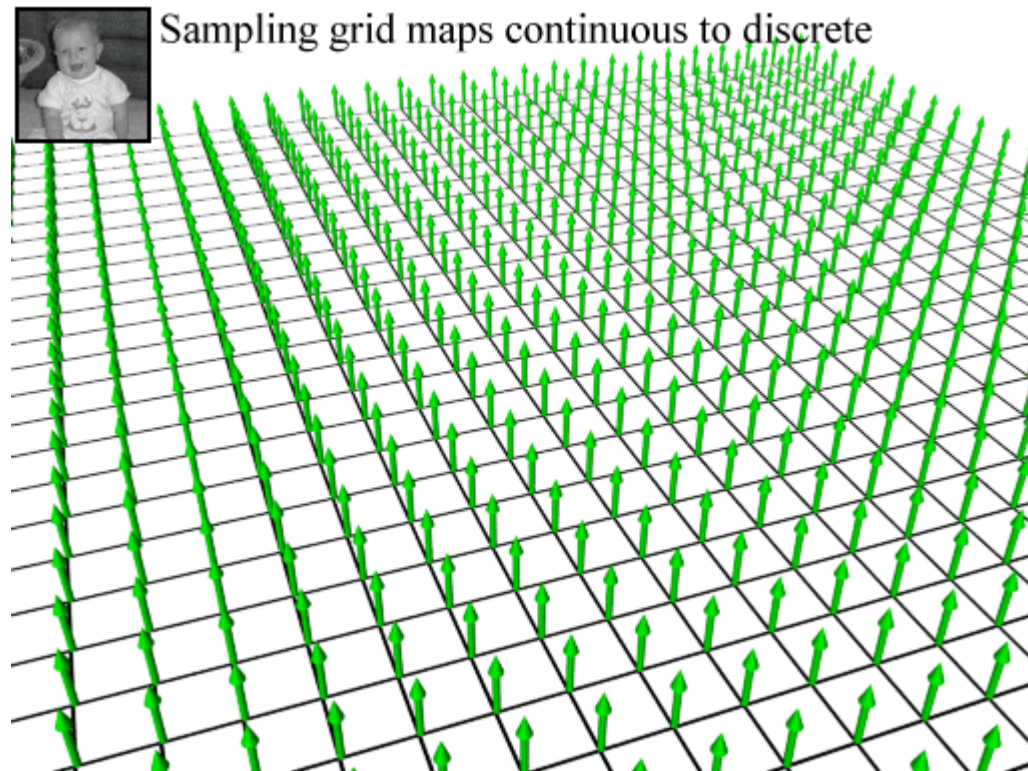
# Image Sampling and Reconstruction

- convert **continuous** image to **discrete** set of samples
- display hardware **reconstructs** samples into continuous image
  - finite sized source of light for each pixel



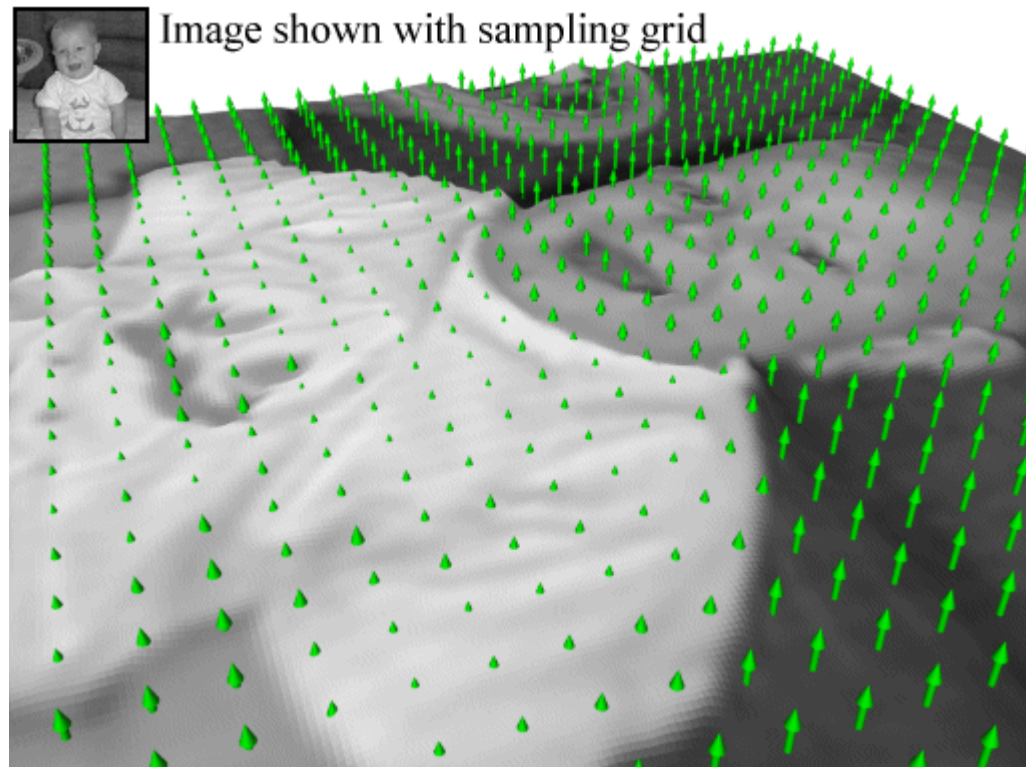
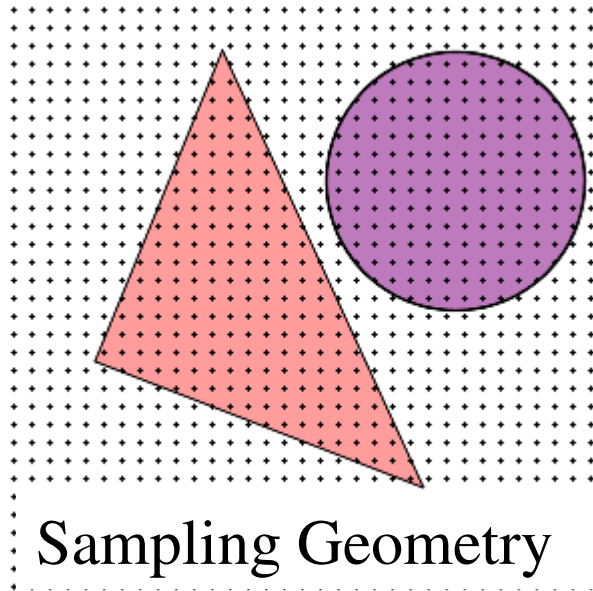
# Point Sampling an Image

- simplest sampling is on a grid
- sample depends solely on value at grid points



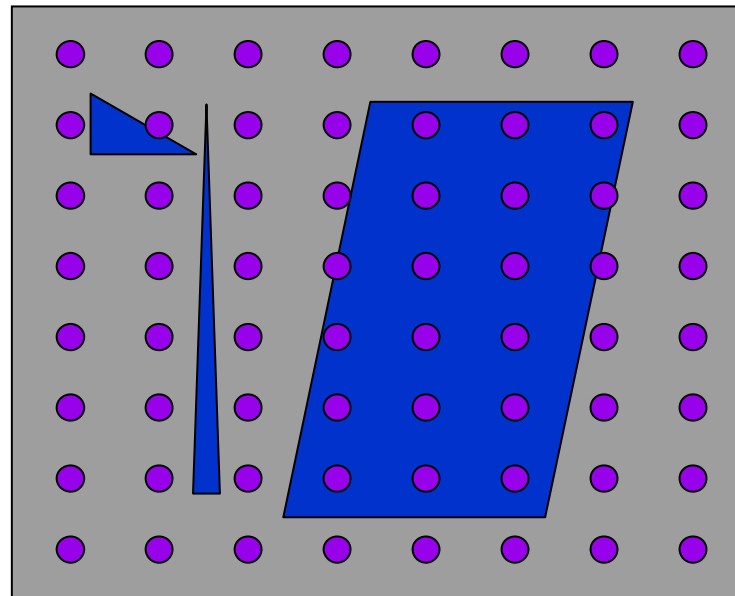
# Point Sampling

- multiply sample grid by image intensity to obtain a discrete set of points, or samples.



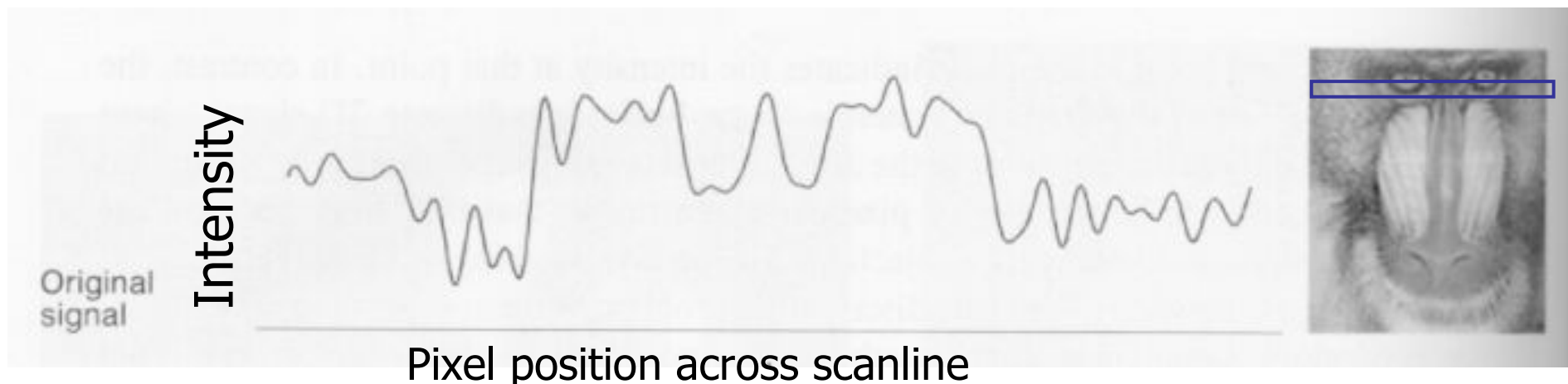
# Sampling Errors

- some objects missed entirely, others poorly sampled
  - could try unweighted or weighted area sampling
  - but how can we be sure we show everything?
- need to think about entire class of solutions!



# Image As Signal

- image as spatial signal
- 2D raster image
  - discrete sampling of 2D spatial signal
- 1D slice of raster image
  - discrete sampling of 1D spatial signal



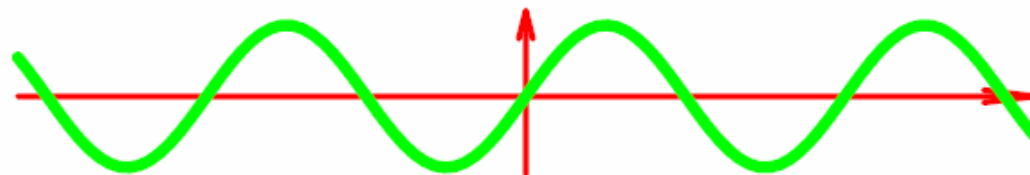
# Sampling Theory

- how would we generate a signal like this out of simple building blocks?
- theorem
  - any signal can be represented as an (infinite) sum of sine waves at different frequencies

# Sampling Theory in a Nutshell

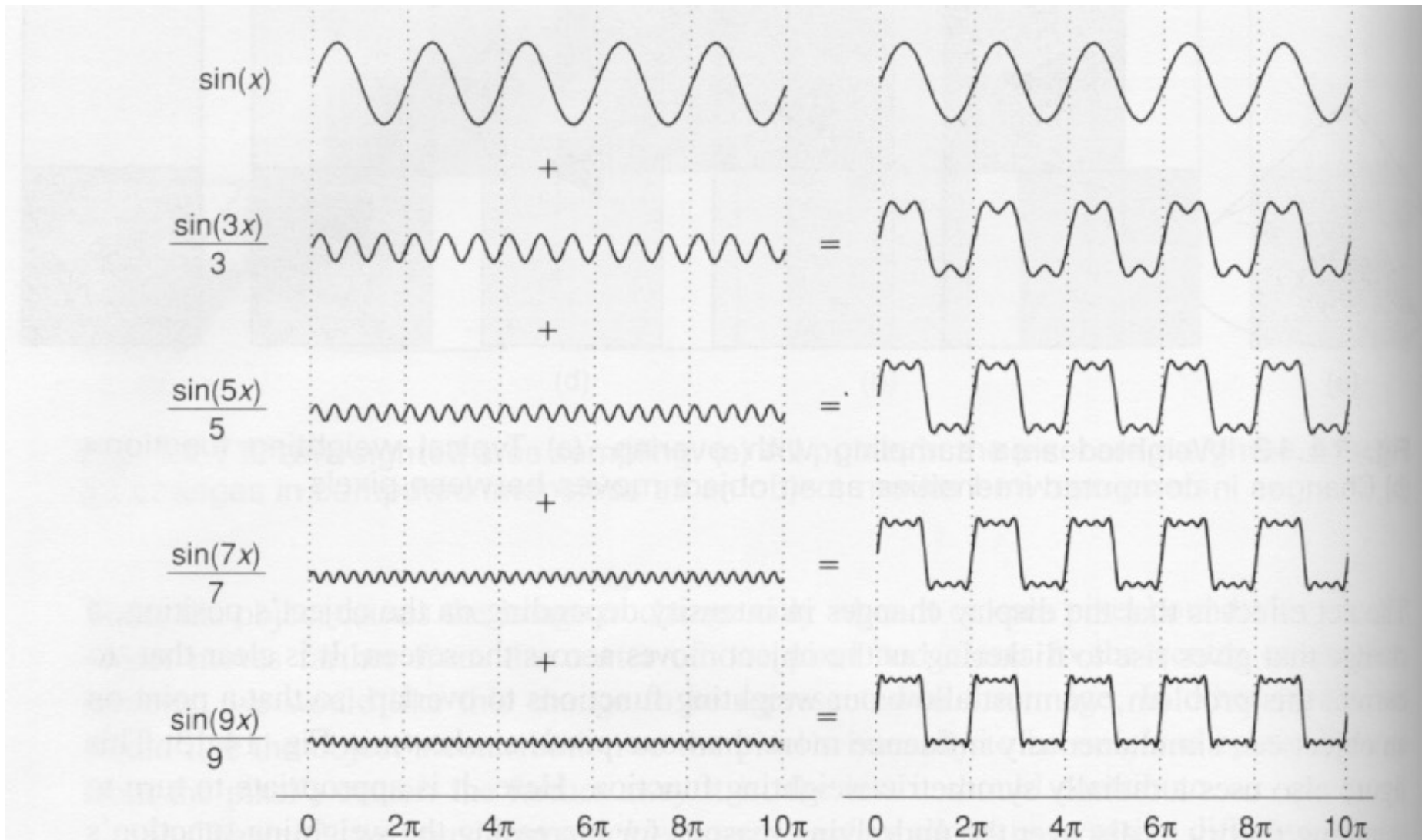
- terminology
  - bandwidth – length of repeated sequence on infinite signal
  - frequency –  $1/\text{bandwidth}$  (number of repeated sequences in unit length)
- example – sine wave
  - bandwidth =  $2\pi$
  - frequency =  $1/2\pi$

$\sin(t)$

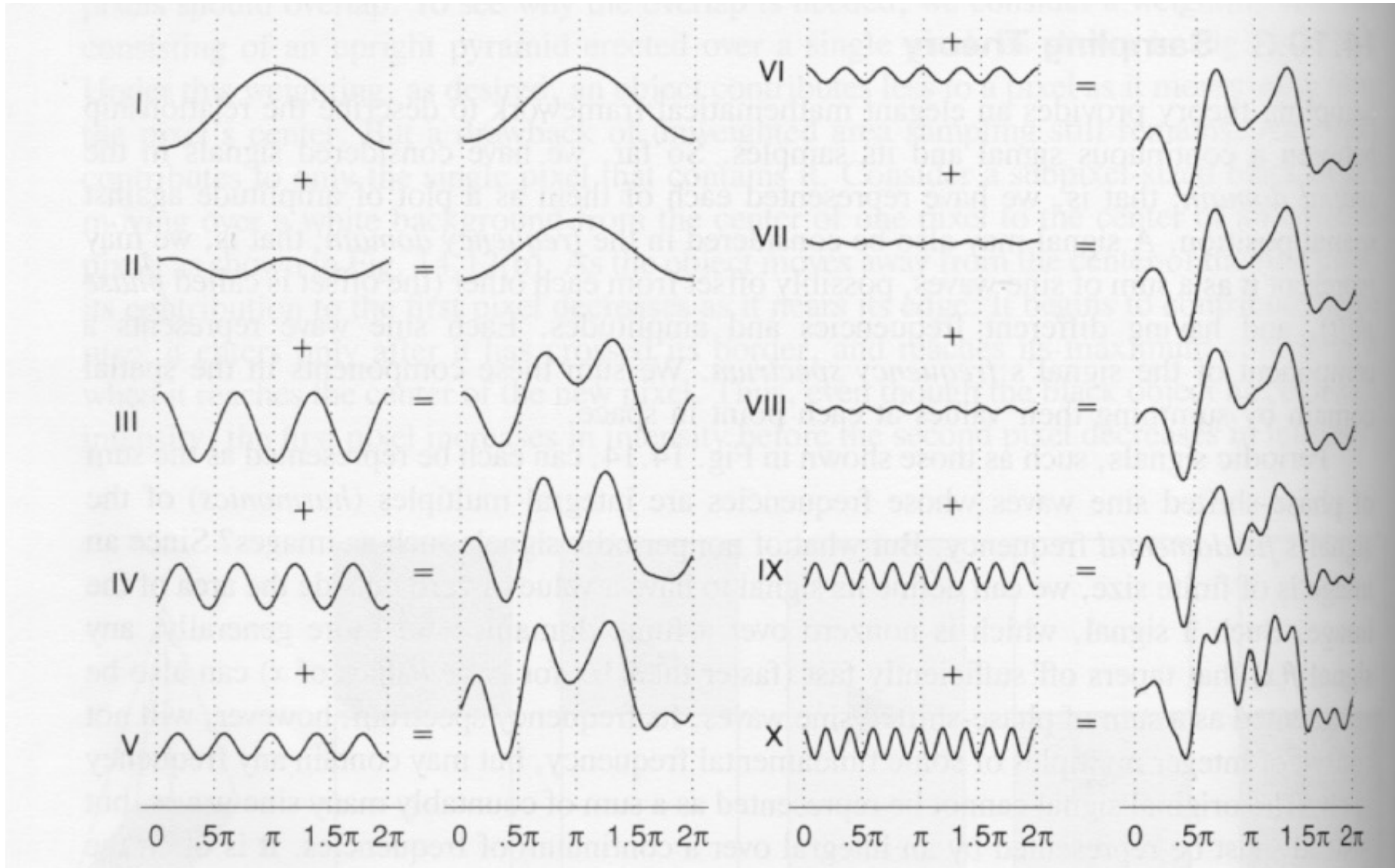




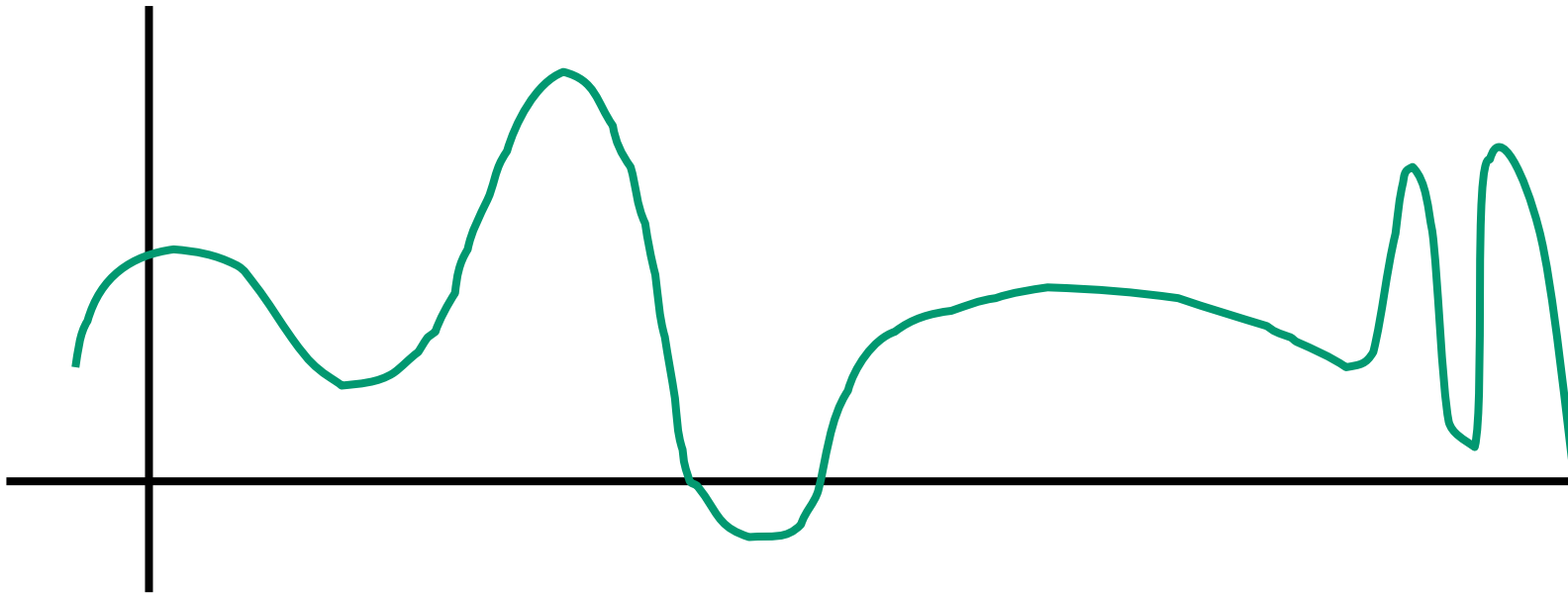
# Summing Waves I



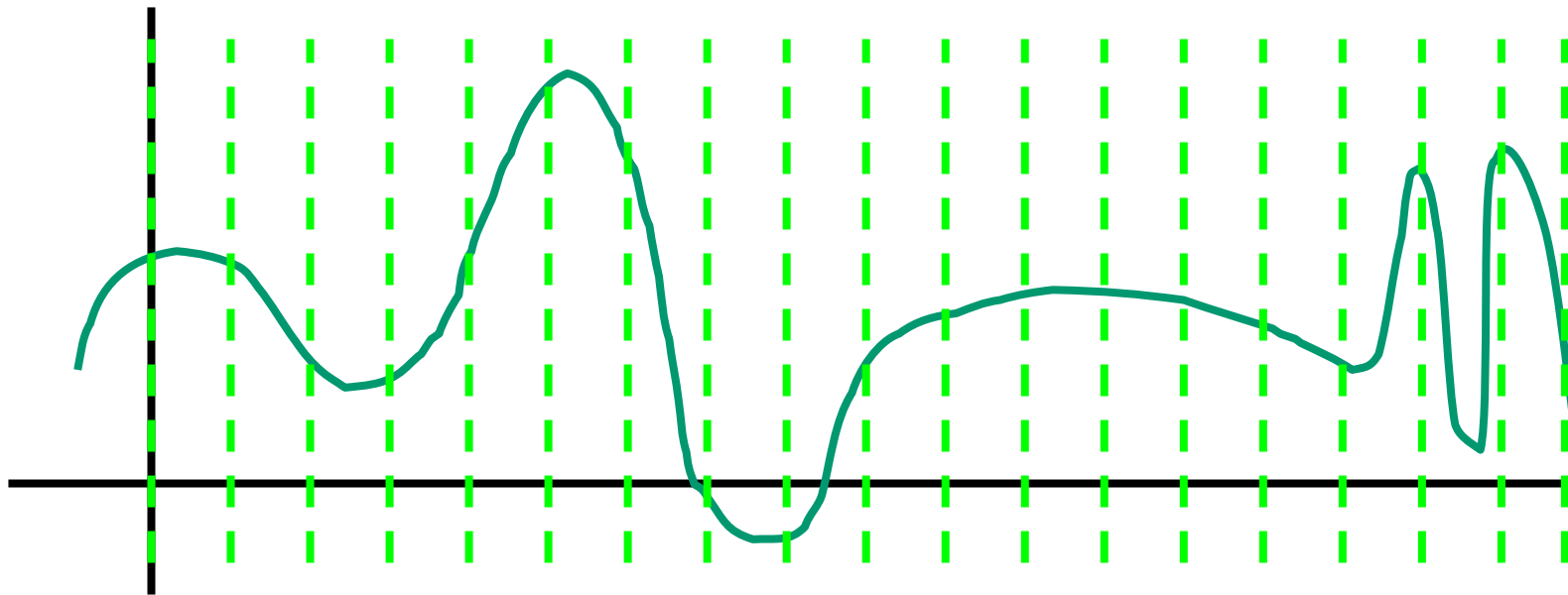
# Summing Waves II



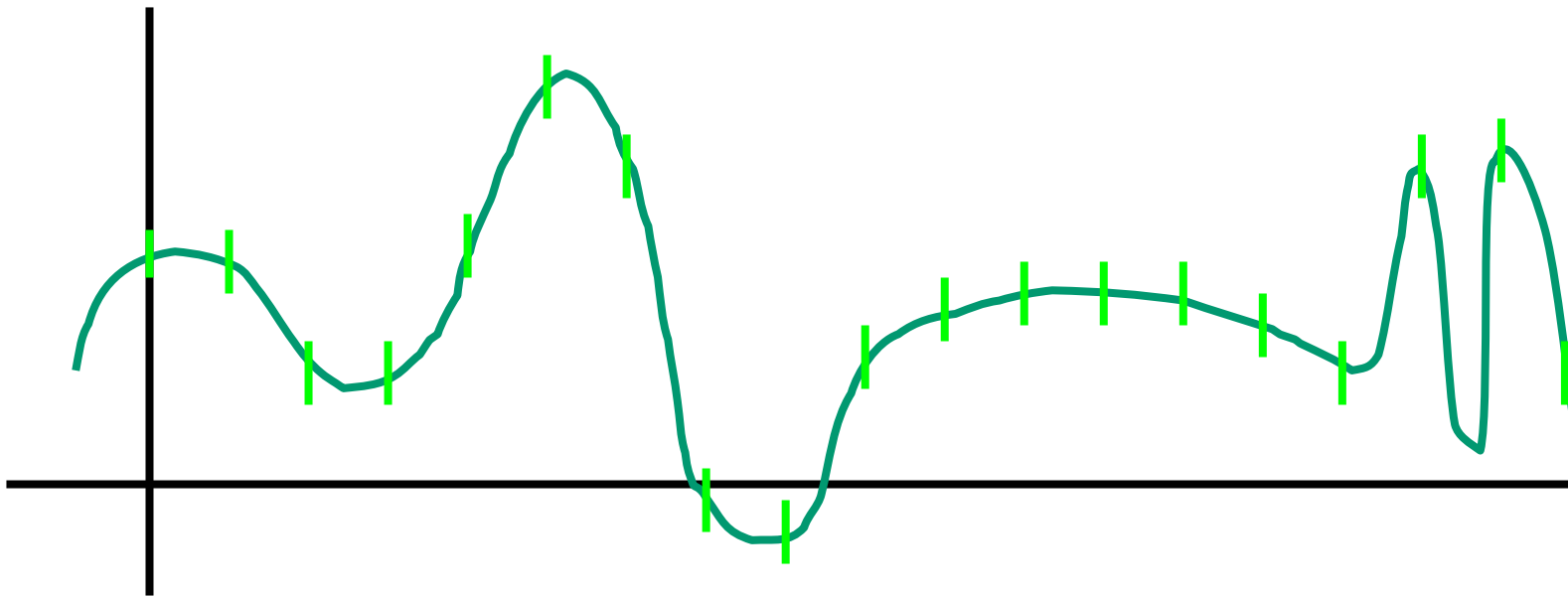
# 1D Sampling and Reconstruction



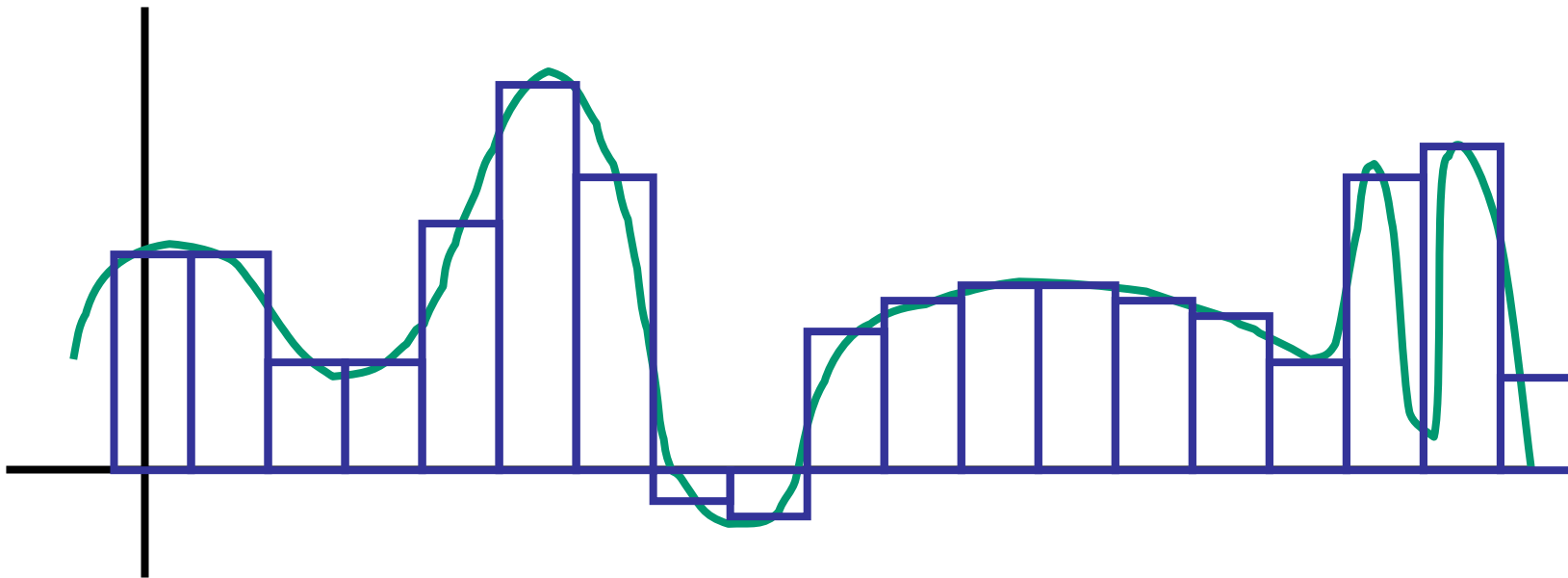
# 1D Sampling and Reconstruction



# 1D Sampling and Reconstruction

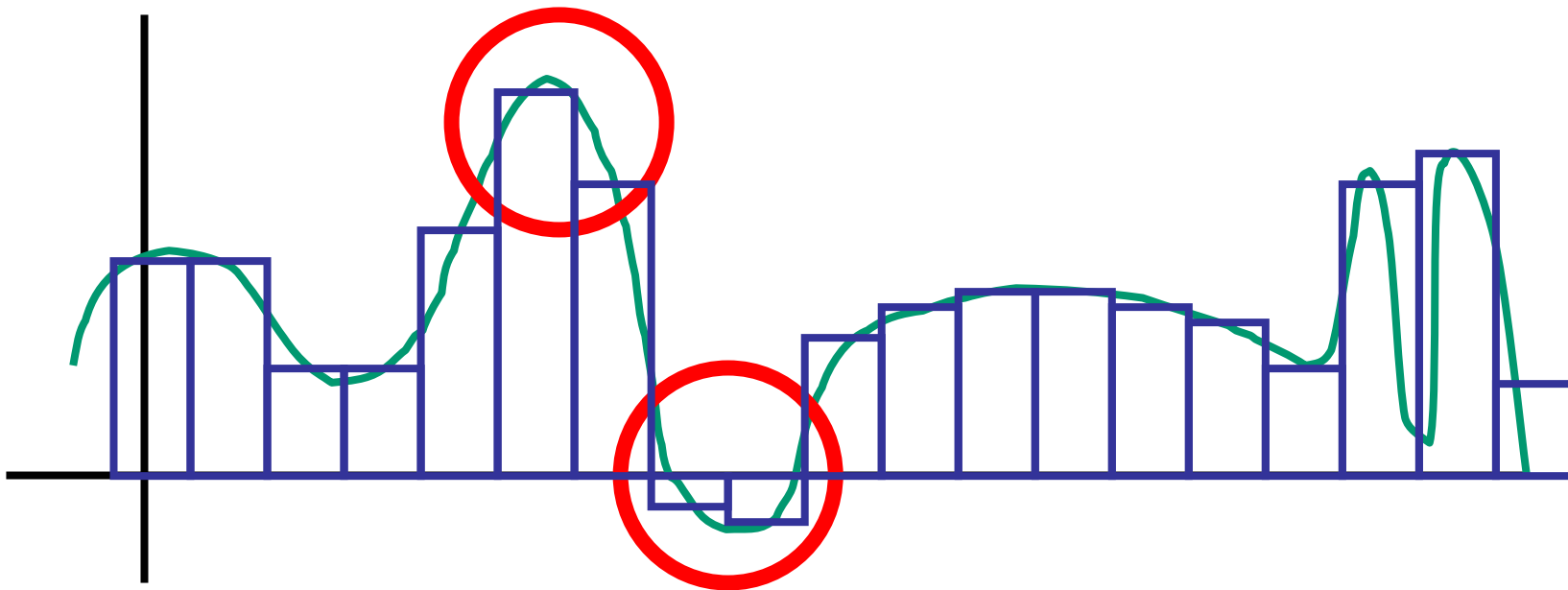


# 1D Sampling and Reconstruction



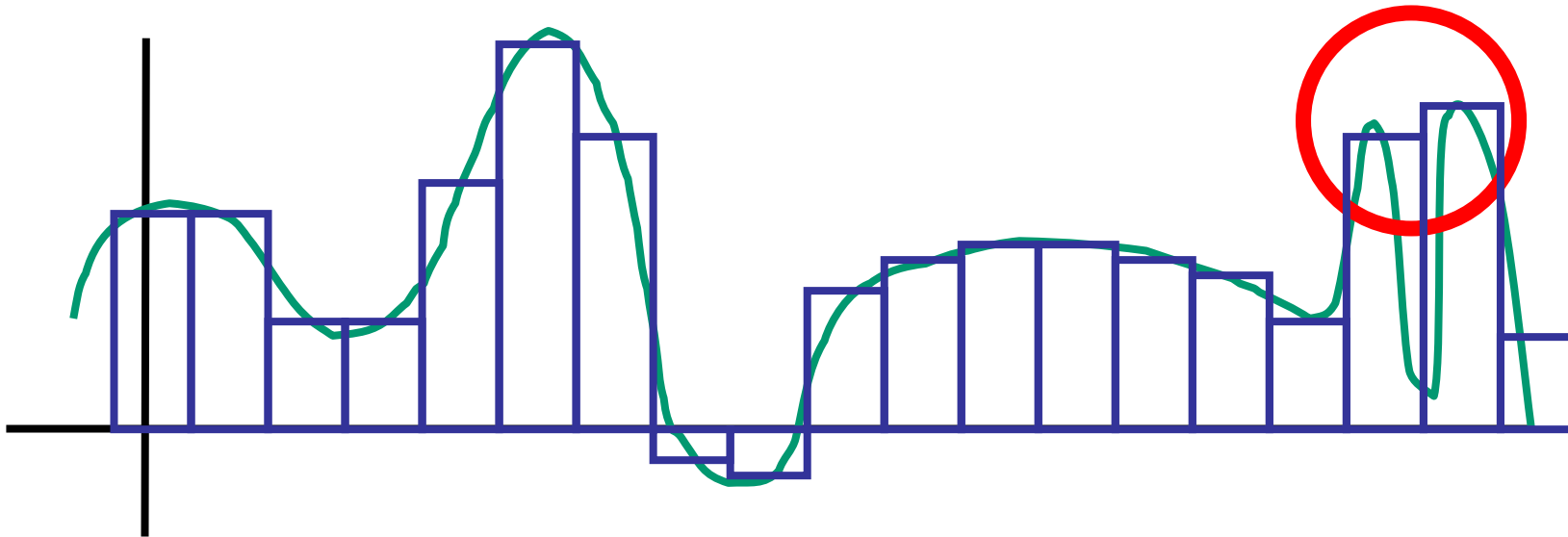
# 1D Sampling and Reconstruction

- problems
  - jaggies – abrupt changes



# 1D Sampling and Reconstruction

- problems
  - jaggies – abrupt changes
  - lose data





# Sampling Theorem

continuous signal can be completely recovered from its samples

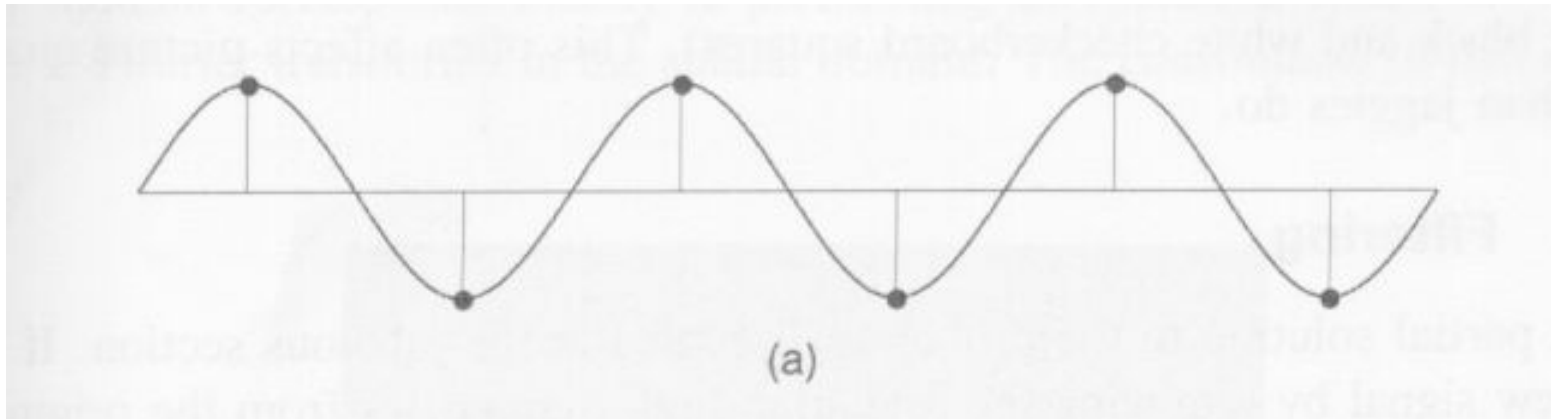
iff

sampling rate greater than twice maximum frequency present in signal

- Claude Shannon

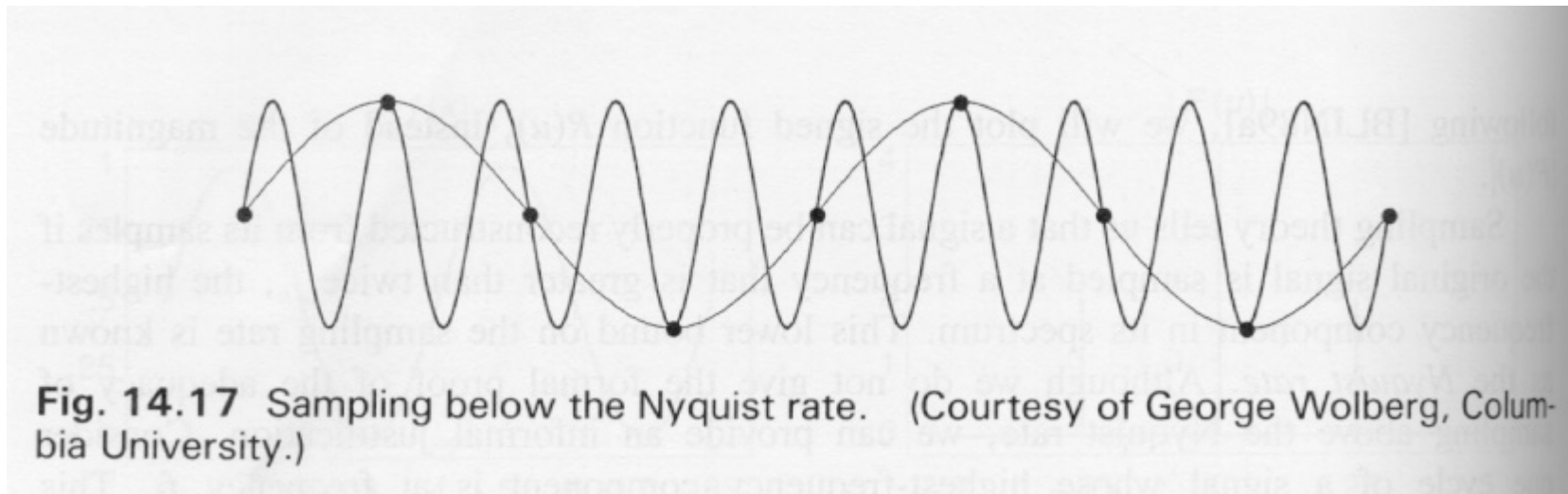
# Nyquist Rate

- lower bound on sampling rate
  - twice the highest frequency component in the image's spectrum

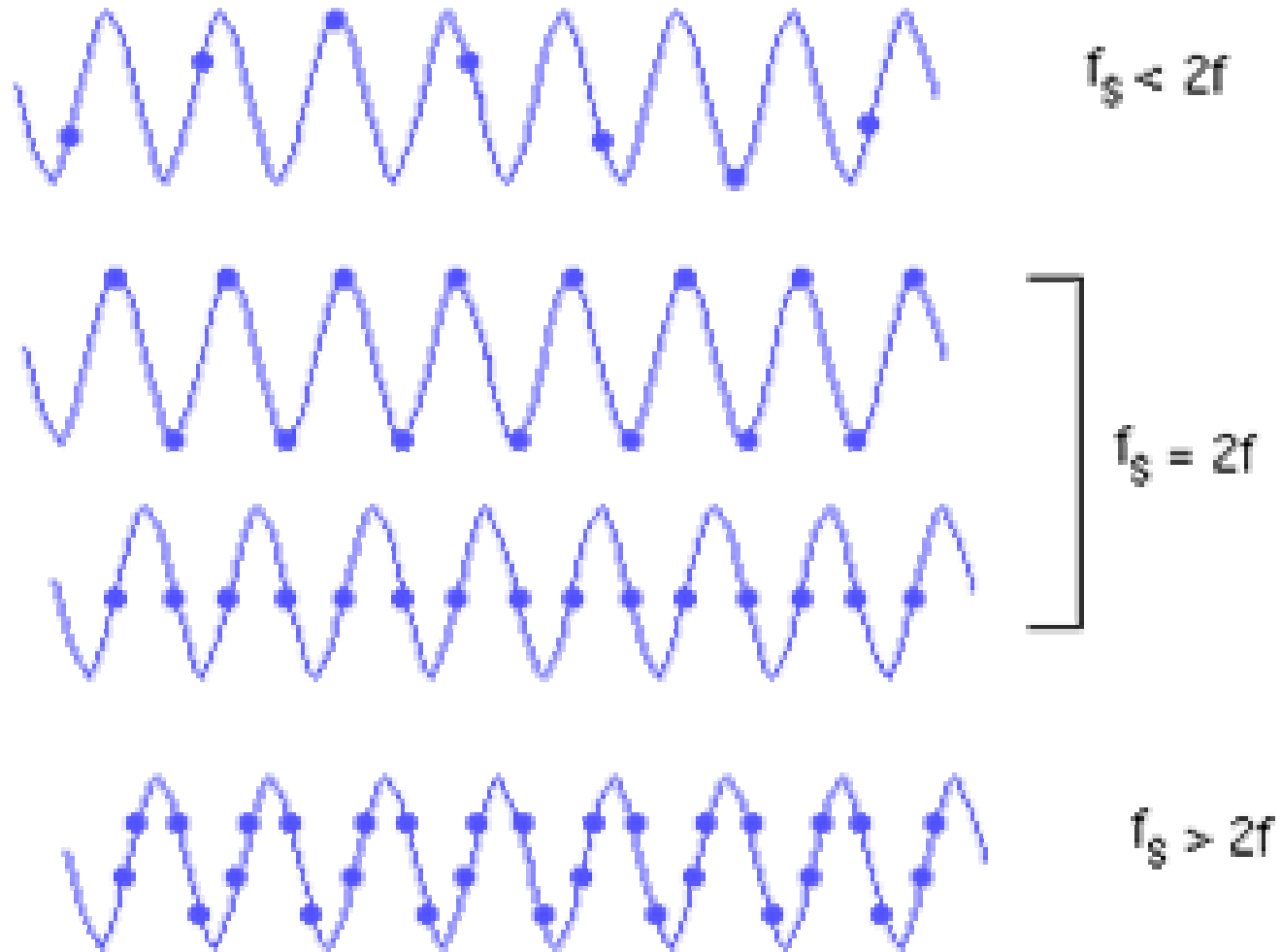


# Falling Below Nyquist Rate

- when sampling below Nyquist Rate, resulting signal looks like a lower-frequency one
  - this is **aliasing!**



# Nyquist Rate



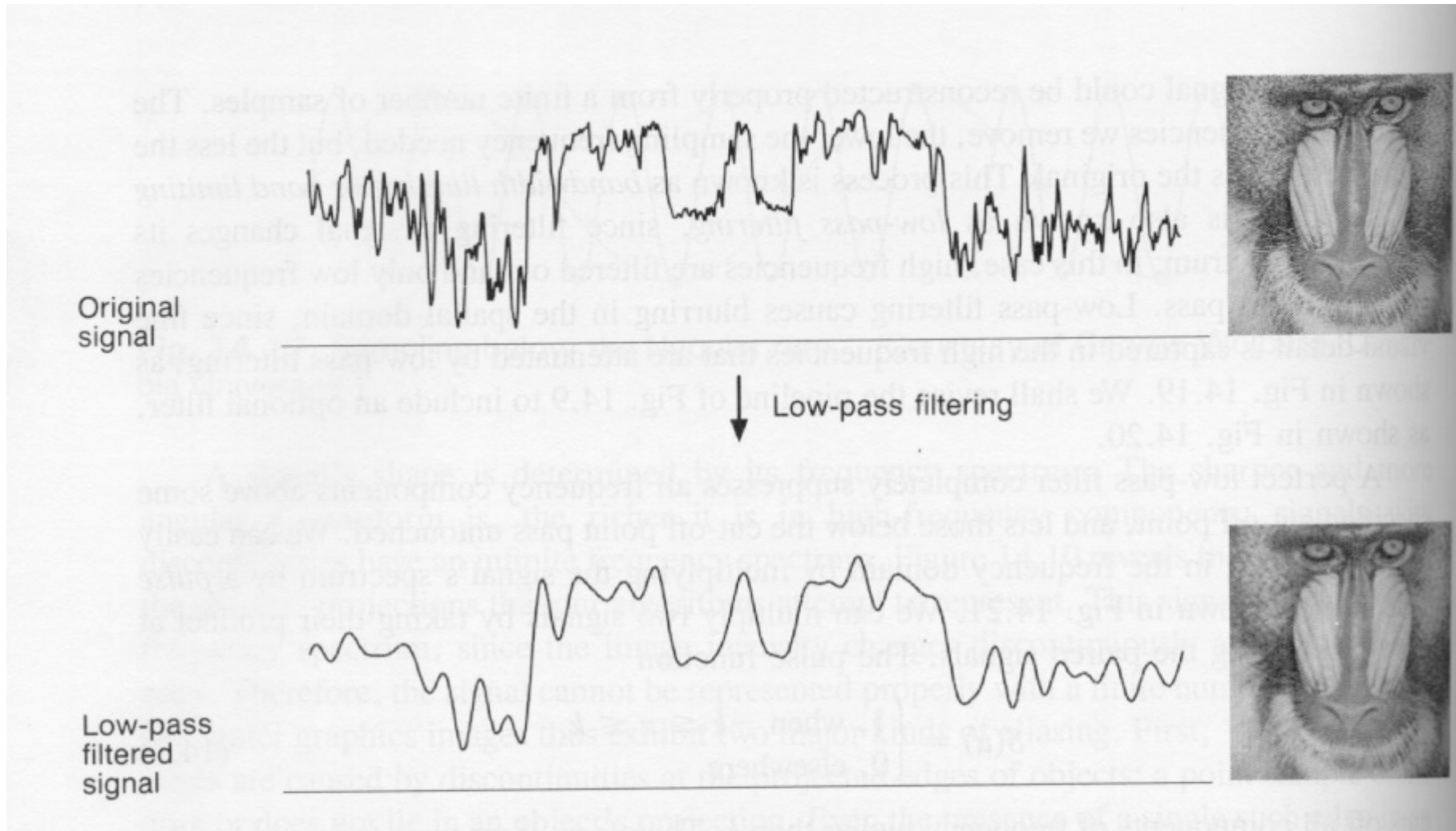
# Aliasing

- incorrect appearance of high frequencies as low frequencies
- to avoid: **antialiasing**
  - supersample
    - sample at higher frequency
  - low pass filtering
    - remove high frequency function parts
    - aka prefiltering, band-limiting

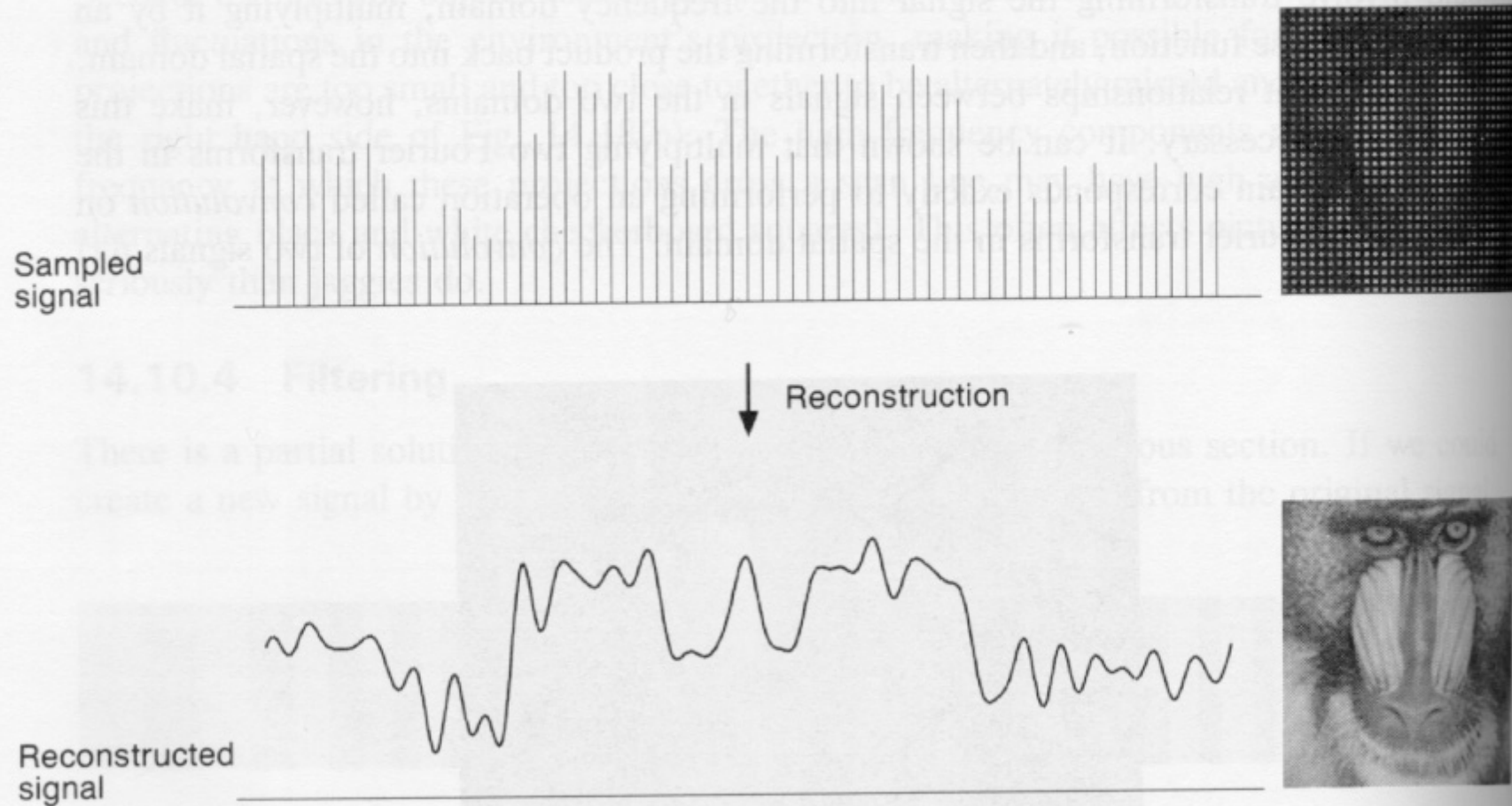
# Supersampling



# Low-Pass Filtering



# Low-Pass Filtering

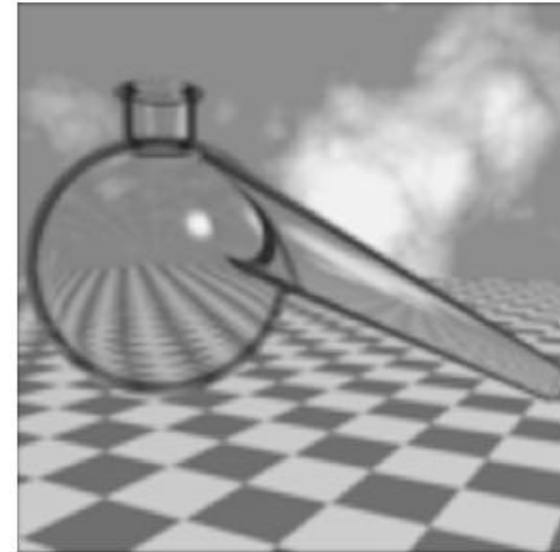
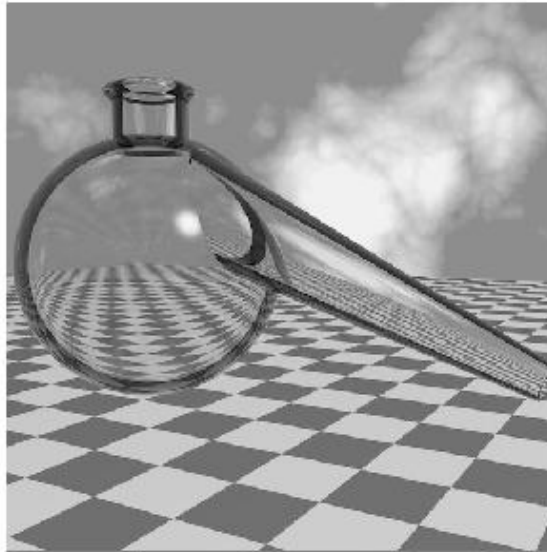


**Fig. 14.20** The sampling pipeline with filtering. (Courtesy of George Wolberg, Columbia University.)

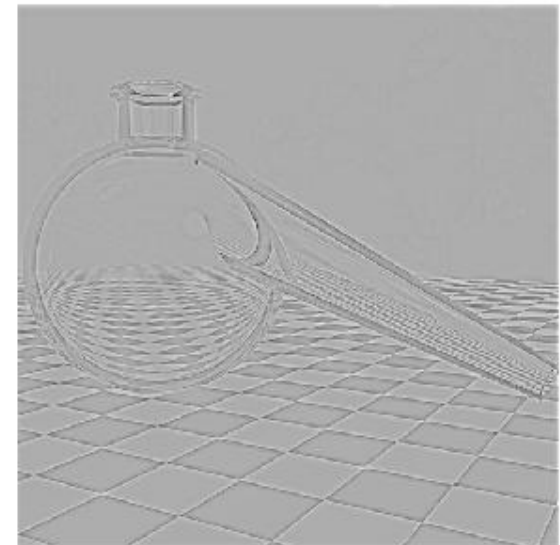
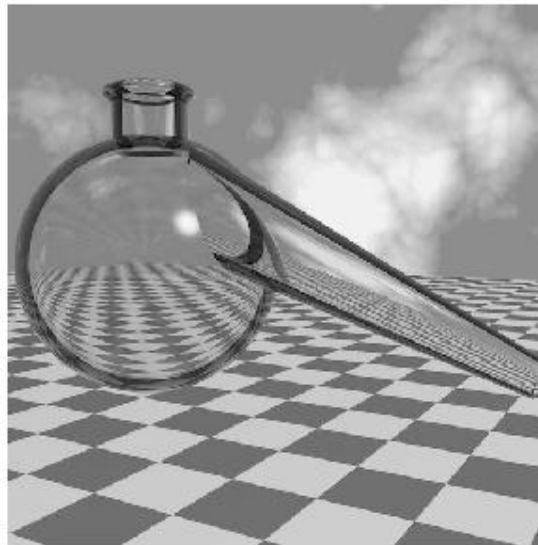


# Filtering

- low pass
  - blur



- high pass
  - edge finding

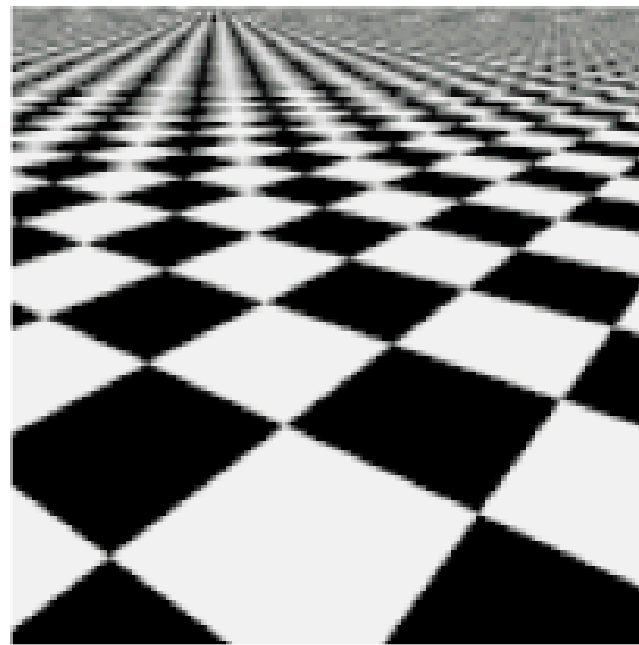


# Previous Antialiasing Example

- texture mipmapping: low pass filter



(a)



(b)