

CPSC 314, Written Homework 1

Out: Sat 21 May 2005
Due: Wed 25 May 2005 4pm
Value: 5% of final grade
Total Points: 100

Note: solutions will be handed out Fri 27 Feb, so no late homeworks will be accepted after then.

1. (8 pts) Give the W2C, the camera/viewing transformation matrix for an eye position (-1,-1,5), a lookat point (1, 1, -10) and an up vector (1,-1,3).

Solution
$$\begin{bmatrix} -0.3880 & -0.9054 & -0.1725 & -0.4311 \\ 0.9688 & 0.4039 & 0.0678 & 0.1695 \\ -0.1310 & -0.1310 & 0.9827 & 5.1755 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{gluLookAt}(-1.0, -1.0, 5.0, 1.0, 1.0, -10.0, 1, -1, 3);$$

2. (8 pts) Give the C2P perspective projection matrix with a near plane of .1, far plane of 30, left plane -1, right plane .25, top plane 1, and bottom plane -2.

Solution
$$\begin{bmatrix} 0.16 & 0 & -0.6 & 0 \\ 0 & 0.066667 & -0.333333 & 0 \\ 0 & 0 & -1.006689 & -0.200669 \\ 0 & 0 & -1 & 0 \end{bmatrix} \text{glFrustum}(-1, 0.25, -2.0, 1.0, 0.1, 30.0);$$

3. (8 pts) Give the N2V, the NDC-to-viewport transformation matrix for a viewport 300 pixels wide and 400 pixels high, where that window's upper left corner is offset 200 pixels down and 600 pixels over from the origin of the display in the upper left corner.

Solution
$$\begin{bmatrix} 150 & 0 & 750 \\ 0 & -200 & 400 \\ 0 & 0 & 1 \end{bmatrix}$$
 Need to scale the NDC from a 2×2 square (**not** 1×1) to 300×400 screen, translate the centre from (0,0) to (600 + 150, 200 + 200). OpenGL commands are
`glTranslatef(750.0, 400, 0.0);`
`glScalef(300.0/2.0, -400.0/2.0, 0);`

4. (8 pts) A quadrilateral has points $A=(-6,5,1,1)$, $B=(-4,-4,0,1)$, $C=(3,-2,-1,1)$, $D=(7,5,2,1)$ in world coordinates. Give the coordinates of these four points in the camera coordinate system, after the viewing transformation from problem 2 above has been applied.

Solution $A = (-2.802, -7.256, -4.062, 1)$, $B = (4.742, -1.864, -4.127, 1)$, $C = (0.388, 3.646, -6.289, 1)$, $D = (-8.019, 4.672, -4.782, 1)$

5. (8 pts) Then give the coordinates of these points in the normalized device coordinate system, after the perspective transformation above in problem 3 has been applied to the answer from problem 4.

Solution $A = (0.489, 0.214, 0.957, 1)$, $B = (0.734, 0.303, 0.958, 1)$, $C = (0.610, 0.372, 0.975, 1)$, $D = (0.332, 0.399, 0.968, 1)$
Don't forget to homogenise the coordinates to get normalised device coordinates.

6. (8 pts) Finally, give the point coordinates in the display coordinate system described above, after the viewport transformation of problem 4 has been applied to the answer from problem 5.

Solution $A = (823, 357)$, $B = (867, 339)$, $C = (841, 326)$, $D = (800, 320)$ Do not homogenise/normalise these coordinates.

7. (20 pts) Show that the class of affine transformations is larger than the class of linear transformations.

Solution One way to show this is to show that every linear transformation is equivalent to an affine transformation.

The linear transformation $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is equivalent to the affine transformation $\begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

This shows that the class of affine transformations is at least as large as the class of linear transformations.

All linear transformations map the origin (0, 0, 0) to (0, 0, 0), whereas affine transformations can translate the origin.

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
 where $a \neq 0$ is an example of a translating affine transformation that cannot be reproduced by a linear transformation.

8. (10 pts) Let $P = (2, 0, -5)$ and $Q = (3, 3, 4)$. Suppose the projection matrix has been modified with

```
glLoadIdentity();
gluPerspective(90.0, 1.0, 2.0, 10.0);
```

and that the modelview matrix is unchanged from its default setting. To which points (in normalized device coordinates) are P and Q transformed? Show your work by first indicating the frustum produced by the `gluPerspective()` call.

Solution The frustum has a 90 degree viewing angle, 1:1 aspect ratio, near clipping plane at 2 and the far clipping plane at 10. Note that although x and y coordinates can be derived via similar triangles, the z coordinate generated by the perspective matrix is **not** the same as the one calculated by similar triangles.

The perspective matrix is
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{3}{2} & -5 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$
 (see slide 24 of Week 2 Day 2 - **not** slide 18 which describes only part of the transformation).

The points generated are $P' = (0.4, 0, 0.5)$ and $Q' = (-0.75, -0.75, 2.75)$. Note that you need to normalise the points after applying the perspective matrix to get normalised device coordinates.

9. (22 pts) We are designing a software module for computer-aided design in land surveying. The world-coordinate database contains objects in a two-dimensional world whose units are feet by feet. The display screen has size W inches wide by H inches high, and pixel resolution 1152×900 , and as usual, $(0,0)$ is the upper-left corner. The user window on the world is specified by a 3-tuple (xc, yc, S) where (xc,yc) = the world coordinate point of the window center, and S = the display "scale factor" in feet per inches, e.g. $S = 50$ means that one inch on the display represents 50 feet in the world. Write a function to map a world point to a device point, in your favorite programming language.

Solution To convert from world coordinates x_{world}, y_{world} we need to

- Shift by xc, yc (A point at (xc, yc) is translated to $(0, 0)$)
- Flip the y axis
- Convert from feet to inches (i.e. divide by S)
- Convert from inches to pixels (for the width multiply by $900/W$, for the height multiply by $1152/H$)
- Shift so that $(0, 0)$ to centre $(450, 576)$

So $x_{pixel} = [(x_{world} - xc)1152/(WS) + 576]$ and $y_{pixel} = [-(y_{world} - yc)900/(HS) + 450]$