Perspective Projection
- project all geometry
- through common center of projection (eye point)
- onto an image plane

Simple Perspective Projection Matrix
\[
\begin{bmatrix}
  x \\
  y \\
  z / d \\
  d
\end{bmatrix}
\]
is homogenized version of where \( w = z/d \)

News: Reminder
- extra TA office hours in lab 005
- Tue 2-5 (Kai)
- Wed 2-5 (Garrett)
- Thu 1-3 (Garrett), Thu 3-5 (Kai)
- Fri 2-4 (Garrett)
- Tamara’s usual office hours in lab
- Fri 4-5

Review: W2V vs. V2W
- MW2V = TR
- we derived position of camera in world
- invert for world with respect to camera
- use previously untouched bottom row

Review: Graphics Cameras
- real pinhole camera: image inverted
- computer graphics camera: convenient equivalent

Review: Projective Transformations
- planar geometric projections
- planar: onto a plane
- geometric: using straight lines
- projections: 3D -> 2D
- aka projective mappings
- counterexamples?

Review: Convenient Camera Motion
- rotate/translate/scale difficult to control
- arbitrary viewing position
- eye point, gaze/lookat direction, up vector

Review: World to View Coordinates
- translate eye to origin
- rotate view vector (lookat – eye) to w axis
- rotate around w to bring up into vw-plane

Basic Perspective Projection
- desired result for a point \([x, y, z, 1]^T\) projected onto the view plane:
- nonuniform foreshortening
- not affine

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Perspective Projection
- expressible with 4x4 homogeneous matrix
- use previously untouched bottom row
- perspective projection is irreversible
- many 3D points can be mapped to same (x, y, d) on the projection plane
- no way to retrieve the unique z values
Moving COP to Infinity
- as COP moves away, lines approach parallel
- when COP at infinity, orthographic view

Orthographic Camera Projection
- camera’s back plane parallel to lens
- infinite focal length
- no perspective convergence
- just throw away z values

Perspective to Orthographic
- transformation of space
- center of projection moves to infinity
- view volume transformed
- from frustum (truncated pyramid) to parallelepiped (box)

View Volumes
- specifies field-of-view, used for clipping
- restricts domain of z stored for visibility test

Orthographic Derivation
- scale, translate, reflect for new coord sys

Canonical View Volumes
- standardized viewing volume representation
- perspective
- orthographic orthogonal parallel

Why Canonical View Volumes?
- permits standardization
- clipping easier to determine if an arbitrary point is enclosed in volume with canonical view volume vs. clipping to six arbitrary planes
- rendering projection and rasterization algorithms can be reused

Normalized Device Coordinates
- convention
- viewing frustum mapped to specific parallelepiped
- Normalized Device Coordinates (NDC)
- same as clipping coords
- only objects inside the parallelepiped get rendered
- which parallelepiped depends on rendering system

Understanding Z
- z axis flip changes coord system handedness
- RHS before projection (eye/view coords)
- LHS after projection (clip, norm device coords)

Orthographic Derivation
- scale, translate, reflect for new coord sys

Normalized Device Coordinates
- left/right x = +/- 1, top/bottom y = +/- 1, near/far z = +/- 1

Perspective to Orthographic
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Understanding Z
- why near and far plane?
- near plane:
  - avoid singularity (division by zero, or very small numbers)
- far plane:
  - store depth in fixed-point representation (integer), thus have to have fixed range of values (0...1)
  - avoid/reduce numerical precision artifacts for distant objects

Orthographic Derivation
- scale, translate, reflect for new coord sys
Orthographic Derivation
- scale, translate, reflect for new coord sys

Projective Rendering Pipeline
- OCS - object/model coordinate system
- WCS - world coordinate system
- VCS - viewing/camera/eye coordinate system
- CCS - clipping coordinate system
- NDCS - normalized device coordinate system
- DCS - device/display/screen coordinate system

Perspective OpenGL
- glMatrixMode(GL_PROJECTION);
- glLoadIdentity();
- glFrustum(left,right,bot,top,near,far);
- orglPerspective(fovy,aspect,near,far);

Demo
- Brown applets: viewing techniques
  - parallel/orthographic cameras
  - projection cameras

Asymmetric Frusta
- our formulation allows asymmetry
- why bother?
  - binocular stereo
  - view vector not perpendicular to view plane

Perspective Warp
- warp perspective view volume to orthogonal view volume
  - render all scenes with orthographic projection!
  - aka perspective normalization

Predistortion
- perspective viewing frustum transformed to cube
- orthographic rendering of warped objects in cube produces same image as perspective rendering of original frustum
Projective Rendering Pipeline

Separate Warp From Homogenization
• matrix formulation
• warp and homogenization both preserve relative depth (z coordinate)

Perspective Divide Example
• specific example
• assume image plane at \( z = -1 \)
• a point \( [x,y,z,1]^T \) projects to \( [-x/z,-y/z,-z/z,1]^T \)

Demo
• Brown applets: viewing techniques
• parallel/orthographic cameras
• projection cameras

http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/viewing_techniques.html