

## CPSC 314, Midterm Exam

11 Feb 2005

Closed book, one single-sided sheet of handwritten notes allowed. Answer the questions in the space provided. If you run out of room for an answer, continue on the back.

Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

Question	Points Earned	Points Possible
1		2
2		5
3		5
4		5
5		5
6		16
7		10
8		2
9		8
10		8
11		6
12		8
13		20
Total		100

1. (2 pts) Write down the 4x4 matrix for rotating an object by 90 degrees around the y axis.

General rotation around the y axis:

$$\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\theta = 90$ , so  $\cos(\theta) = 0$  and  $\sin(\theta) = 1$ . Thus, the matrix is

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. (5 pts) Describe in words what this matrix does in two different ways: a) in terms of moving the object, b) in terms of changing the coordinate frame.

$$\begin{bmatrix} .5 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

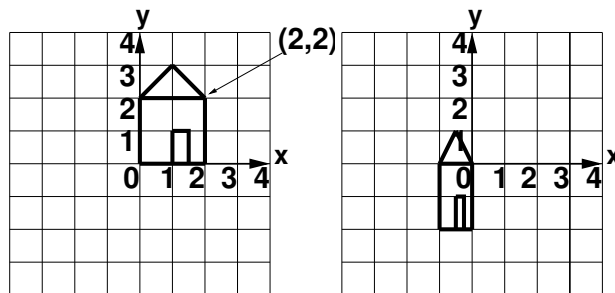
First, decompose the matrix into separate transformations such that  $p' = ABp$ :

$$A_{scale} = \begin{bmatrix} .5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B_{trans} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Another possibility is

$$A_{trans} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B_{scale} = \begin{bmatrix} .5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

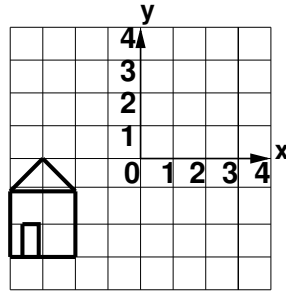
- a) When considering a moving object in a global coordinate system: translate object by  $x=-2$  and  $y=-2$  (down 2 and left 2), then scale by  $1/2$  in  $x$ .
- b) When considering changing the local coordinate frame: scale the coordinate frame by  $1/2$  in  $x$ , then translate the scaled coordinate frame by 2 units to the left in  $x$  (ends up being 1 unit in global coordinates, because of the previous scale) and 2 units down in  $y$ .
3. (5 pts) Sketch a picture of the house transformed by the above matrix



4. (5 pts) Give sequence of OpenGL commands necessary to implement the above transformation.

```
glScale(.5,1,1); glTranslate(-2,-2,0); drawHouse();
```

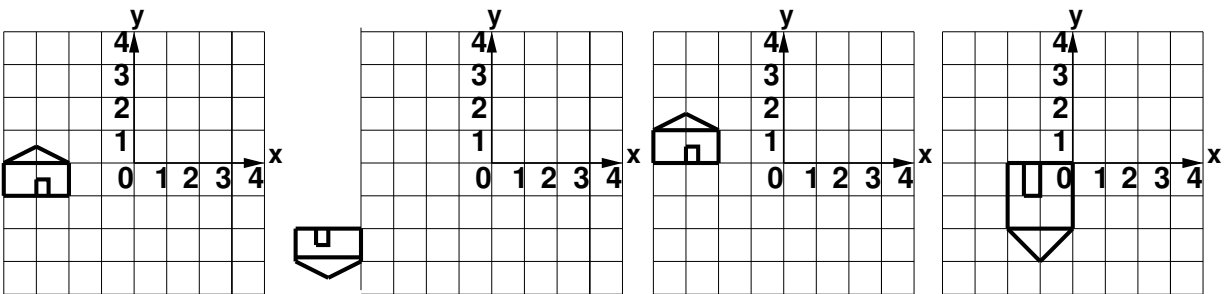
5. (5 pts) If  $p' = ABp$ , give the the  $4 \times 4$  matrices A and B needed to create the picture below, assuming the house started from the initial position shown in the above question.



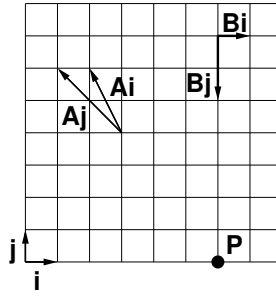
$$A_{\text{translate}} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad B_{\text{reflectY}} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. (16 pts) Draw houseA, houseB, houseC, and house D transformed by the appropriate OpenGL commands. The untransformed house is below.

```
glLoadIdentity();
glPushMatrix();
glScale(1, .5, 1);
glTranslate(-4, -2, 0);
drawHouseA();
glPushMatrix();
glRotate(180, 0, 0, 1);
glTranslate(0, 2, 0);
drawHouseB();
glPopMatrix();
glTranslate(0, 2, 0);
drawHouseC();
glPopMatrix();
glRotate(180, 0, 0, 1);
drawHouseD();
```



7. (10 pts) Specify the coordinates of point P with respect to coordinate frames A and B.



$$A = (-1, -1) \quad B = (0, 3.5)$$

8. (2 pts) Homogenize the point (9,6,3,3).

$$(3, 2, 1)$$

9. (8 pts) In normalized device coordinates, point A is  $(.9, .2, .3)^T$  and point B is  $(.4, .5, .8)^T$ . Compute the (x,y,z) coordinates of points A and B in display coordinates, given a viewport that's 200 pixels wide and 400 pixels tall. The upper left corner of the window is 500 pixels down and 100 pixels over from the origin of the display in the upper left corner.

We need to flip the y coordinate, scale, and translate. In NDC, the x, y, and z values all range from -1 to +1. In display coordinates, we want x to range from 500.5 to 700.5, y to range from 100.5 to 500.5, and z to range from 0 to 1. The w values will always be 1 in both of these coordinate systems, since the perspective divide has already taken place.

$$M = \begin{bmatrix} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 500 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & (200-1)/2 \\ 0 & 1 & 0 & (400-1)/2 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 200/2 & 0 & 0 & 0 \\ 0 & 400/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 100 & 0 & 0 & 199.5 \\ 0 & -200 & 0 & 699.5 \\ 0 & 0 & .5 & .5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A' = MA = [.9 * 100 + 199.5 * 1, .2 * (-200) + 699.5, .5 * .3 + .5]^T = [289.5, 659.5, .65]^T$$

$$B' = MB = [.4 * 100 + 199.5 * 1, .5 * (-200) + 699.5, .5 * .8 + .5]^T = [239.5, 599.5, .9]^T$$

10. (8 pts) Give an example of a transformation matrix that illustrates the problem with using the modelview matrix to transform normals to a surface.

Any matrix with nonuniform scaling or shearing would be correct. For example:

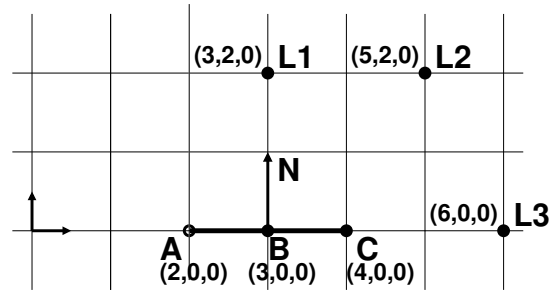
$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

11. (6 pts) If an object is transformed by the matrix you gave above, give the matrix that would correctly compute the normals.

The inverse transpose of the above example is:

$$\begin{bmatrix} -.5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12. (8 pts) Lights L1, L2, and L3 all full white lights: (1,1,1). The surface is red, with a diffuse coefficient  $k_d$  of (1,.5,.5). Compute the diffuse illumination for each of the three lights at point B.



Before doing any computations at all, notice that L1 is directly above point B, so the normal and the vector from the point to the light are the same, and the light will be at full intensity. The L2 light will partially illuminate point B, and the L3 light will not illuminate B at all since the angle between the normal and the light is 90 degrees.

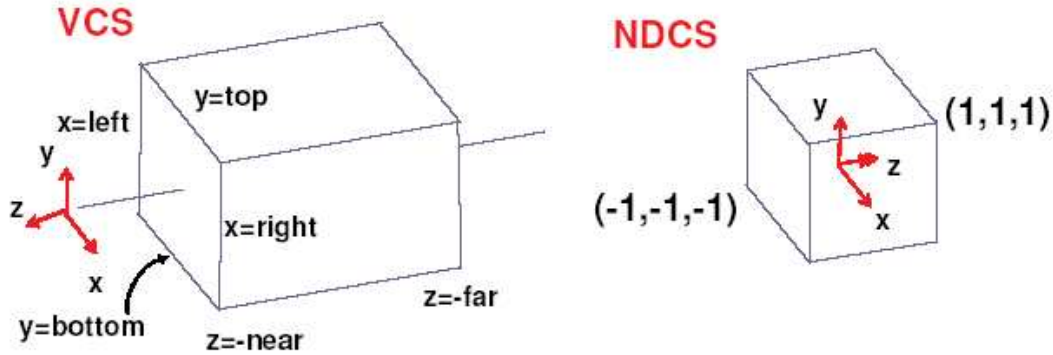
Computations shared for all three lights: The normal vector at point B is  $(3, 1, 0) - (3, 0, 0) = (0, 1, 0)$ . Using RGB triplets to compute colors by multiplying the R, G, and B components separately, we have  $((1, 1, 1) * (1, .5, .5) = (1 * 1, 1 * .5, 1 * .5) = (1, .5, .5)$ .

L1: We must normalize the light vectors to do our computations with unit-length vectors. The vector from point B to L1 is  $(3, 2, 0) - (3, 0, 0) = (0, 2, 0)$ , normalized is  $(0, 2/2, 0) = (0, 1, 0)$ . The intensity from L1 at B is  $(N \cdot L) = (0, 1, 0) \cdot (0, 1, 0) = 0 * 0 + 1 * 1 + 0 * 0 = 1$ . To finish, the surface color is scaled by the light intensity, for  $(1, .5, .5) * 1 = (1, .5, .5)$ .

L2: The vector from point B to L2 is  $(5, 2, 0) - (3, 0, 0) = (2, 2, 0)$ , normalized is  $(2/\sqrt{8}, 2/\sqrt{8}, 0) = (1/\sqrt{2}, 1/\sqrt{2}, 0) = (.707, .707, 0)$ . The intensity from L2 at B is  $(N \cdot L) = (0, 1, 0) \cdot (.707, .707, 0) = 0 * .707 + 1 * .707 + 0 * .707 = .707$ . To finish, the color is scaled by the intensity, for  $(1, .5, .5) * .707 = (.707, .3535, .3535)$ .

L3: The vector from point B to L3 is  $(6, 0, 0) - (3, 0, 0) = (3, 0, 0)$ , normalized is  $(3/3, 0, 0) = (1, 0, 0)$ . The intensity from L3 at B is  $(N \cdot L) = (0, 1, 0) \cdot (1, 0, 0) = 0 * 1 + 1 * 0 + 0 * 0 = 0$ . To finish, the surface color is scaled to nothing:  $(1, .5, .5) * 0 = (0, 0, 0)$ .

13. (20 pts) Derive the 4x4 orthographic projection matrix that transforms from the viewing coordinate system to normalized device coordinates using orthographic projection, as shown below. Show your work.



Except for the change from left-handed to right-handed coordinate system, the transformation from viewing to orthographic is just scaling and translating, an equation of the form  $y' = ay + b$ . We start with the top/bottom case, solving for two unknowns. We first substitute the  $y$  values at the top of the cube:  $y = T$  and  $y' = 1$ , so  $1 = aT + b$ . Solving for  $b$  gives  $b = 1 - aT$ . We create a second equation by substituting at the bottom of the cube:  $y = B$  and  $y' = -1$ , so  $-1 = aB + b$ . Using the value of  $b$  found above, we have

$$\begin{aligned} -1 &= aB + (1 - aT) \\ -1 - 1 &= aB - aT \\ -2 &= a(B - T) \\ 2 &= a(T - B) \\ a &= 2/(T - B) \end{aligned}$$

We can now find  $b$  in terms of just  $T$  and  $B$ :

$$b = 1 - aT = 1 - \left(\frac{2}{T - B}\right)T = 1 - \frac{2T}{T - B} = \frac{T - B}{T - B} - \frac{2T}{T - B} = \frac{-B - T}{T - B} = -\frac{B + T}{T - B}$$

The top/bottom, left/right, and near/far cases are all identical and can be computed separately:

$$S = \begin{bmatrix} \frac{2}{R-L} & 0 & 0 & -\frac{R+L}{R-L} \\ 0 & \frac{2}{T-B} & 0 & -\frac{T+B}{T-B} \\ 0 & 0 & \frac{2}{F-N} & -\frac{F+N}{F-N} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The RHS to LHS switch happens with a reflection in the  $z$  axis, which is simply the matrix

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We finish with  $M = RS$ :

$$\begin{bmatrix} \frac{2}{R-L} & 0 & 0 & -\frac{R+L}{R-L} \\ 0 & \frac{2}{T-B} & 0 & -\frac{T+B}{T-B} \\ 0 & 0 & -\frac{2}{F-N} & -\frac{F+N}{F-N} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$