

Relational Logistic Regression: the Directed Analog of Markov Logic Networks Seyed Mehran Kazemi¹, David Buchman¹, Kristian Kersting², Sriraam Natarajan³, David Poole¹

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LR vs. RLR vs. MLN

	Logistic Regression (LR)	Relational Logistic Regress
Model:	Directed	Directed
	Non-relational	Relational
Defines:	Conditional prob. dist. (CPD)	CPD (aggregator)
Using:	Pairwise terms	Weighted logic formulae
Definition:	$P(var parents) \propto \prod_i e^{w_i parent_i}$	$P(\text{ground var} \text{ground parents}) \propto all \text{ true } p$ $P(\text{ground var} \text{ground parents}) = sigma$ $sigmoid(x) = \frac{1}{1} + \frac{1}{1}$

- RLR = relational extension of logistic regression (LR)
- RLR = directed analog of MLNs
 - When all parents observed:
- MLN's CPD = RLR's CPD
- When using RLR for all PRVs: MLN's joint distribution \neq RLRs' joint distribution

Weighted Formulae (WFs)

	RLR [1]		MLN
Notation:	$\langle L, F, w \rangle$ F = logic formula w = weight $free_vars(F) \subseteq L = \text{set of logical variables}$		$\langle F, w \rangle$ F = logic formula w = weight
F instantiated for:	all bindings of individuals for <i>L</i>		all bindings of individuals for <i>free_vars</i> (<i>F</i>)
Example: R	LR WF	Equivalent MLN WF	#Instantiations
$\langle \{x\}, R(x) /$	$\langle Q, w \rangle$	$\langle R(x) \land Q, w \rangle$	x = population(x)
$\langle \{x, x'\}, R(x)$	$\land Q, w \rangle$	$\langle R(x) \land Q \land true(x'), w \rangle$ or $\langle R(x) \land Q \land (R(x') \lor \neg R(x')), w \rangle$	<i>x</i> ²

sion (RLR) [1]	MLN
	Undirected
	Relational
	Joint probability
	Weighted logic formulae
$ \prod_{formula instances} e^{w_{formula}} $ $ oid(\sum_{all true f. ins.} w_{formula}) $ $ \frac{1}{+e^{-x}} $	$P(all ground vars) \propto \prod_{\substack{all true formula \\ instances}} e^{w_{formula}}$

Unquantified RLRs: Canonical Forms

• Unquantified RLR formulae may use: $\{\neg, \land, \lor, \bigoplus (XOR), \ldots\}$

- Prop 3: RLRs using only $\{\Lambda\}$ are equally expressive.
 - Corresponds to [2]'s "canonical parametrization with reference state 'true'"
- Prop 4: RLRs using only {V} are equally expressive.
 - Related to [2]'s "canonical parametrization with reference state 'false'"
- RLRs using only $\{\bigoplus\}$ (XOR) are equally expressive. Note:
 - Corresponds to [2]'s "spectral parametrization"

RLR: Characterization

- Props 6,7 (if and only if): •
 - CPD representable by RLR \Leftrightarrow CPD has the form:
 - P(ground var = true | ground parents) =

sigmoid(polynomial of counts)

count = #tuples (of individuals), for which a formula of the parents is true.

RLR: Representing Common Aggregators

Unquantified RLRs can represent common aggregators.

E.g.: AND, OR, noisy-AND, noisy-OR, majority, #trues > constant, %trues > constant

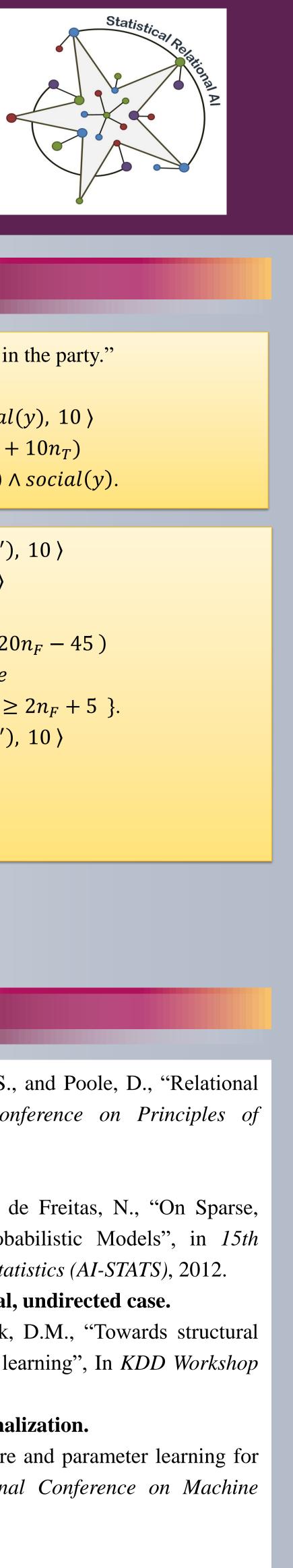
• Example
$$-Q \equiv OR(R(*))$$
: $\langle \{ \}, Q, -M \rangle$

$$\langle \{x\}, Q \land R(x), 2M \rangle$$

$$ror_{M\to\infty} \to 0.$$

eri

With non-binary values: mean > constant, max > constant, max, mode=constant, mode



Examples

"A party is usually fun if you know at least one social person in the party." As RLR: $\langle \{x\}, funFor(x), -5 \rangle$ $\langle \{x, y\}, funFor(x) \land knows(x, y) \land social(y), 10 \rangle$ $\Rightarrow P(funFor(x) | knows(x,*), social(*)) = sigmoid(-5 + 10n_T)$ n_T = #individuals y for which $knows(x, y) \land social(y)$.

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Can be considered as an approach for learning RLR.