Domain Recursion for Lifted Inference with Existential Quantifiers

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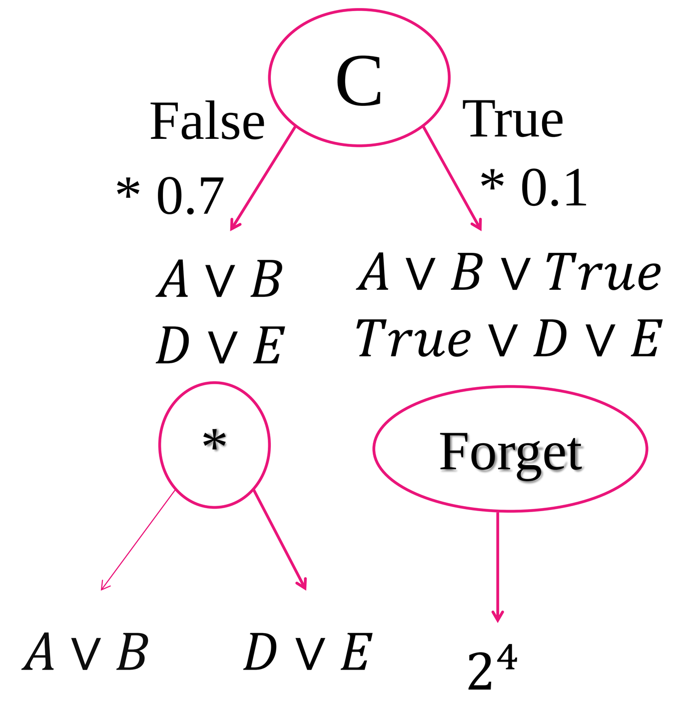
Weighted Model Counting (WMC)

Rules $\boldsymbol{R}$ for efficient WMC:

$A \vee B \vee C$
$C \vee D \vee E$

w(A=True) = 0.2
w(A=False) = 0.6
w(B=True) = 1.1
w(B=False) = 1
w(C=True) = 0.1
w(C=False) = 0.7
w(D=True) = 2.2
w(D=False) = 0.5
w(E=True) = 1.5
w(E=False) = 2



Cache the results

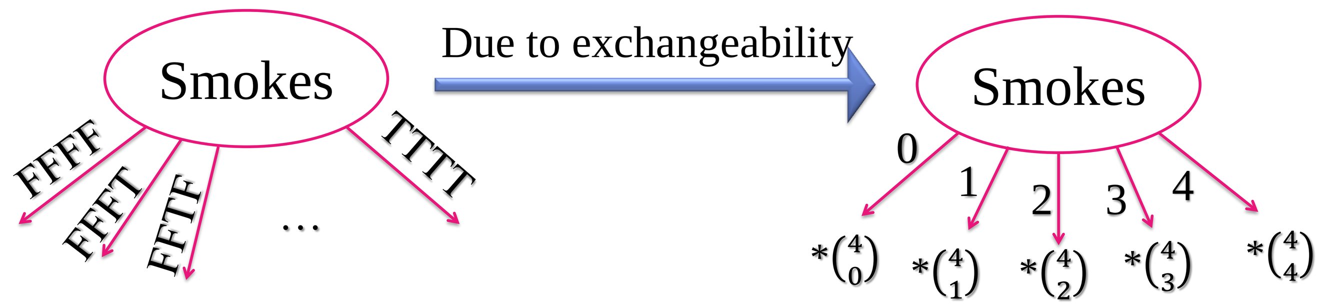
$\forall x: Smokes(x) \Rightarrow Cancer(x) \quad x \in \{A, B, C, D\}$

- $\boxed{Smokes(A) \Rightarrow Cancer(A)}$
- …
- $\boxed{Smokes(D) \Rightarrow Cancer(D)}$

$*$

Due to exchangeability: $\left[\boxed{Smokes(A) \Rightarrow Cancer(A)}\right]^4$

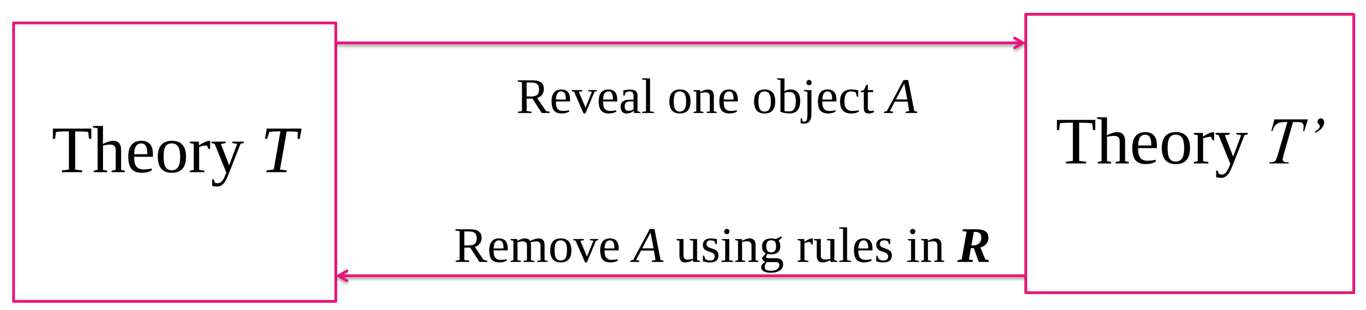
$\forall x, y: Smokes(x) \wedge Friend(x, y) \Rightarrow Smokes(y)$



$*\binom{4}{0} \quad *\binom{4}{1} \quad *\binom{4}{2} \quad *\binom{4}{3} \quad *\binom{4}{4}$

Domain Recursion Rule

- Domain recursion (DR) for a theory *T*:
  - Reveal/separate one object *A* from a population and get *T'*
  - Apply rules in $\boldsymbol{R}$ to *T'* until *A* is entirely removed and get *T''*
  - Call domain recursion for *T''*
- DR is **bounded** if *T''* is identical to *T*, but over a reduced population
- DR is bounded for *T* ⇒ Compute *WMC(T)* using dynamic prog.



$\forall x, y \neq x: Fr(x, y) \Rightarrow Fr(y, x)$
x, y ∈ {A,B,…,Z}

Reveal/Separate one person (e.g., A)

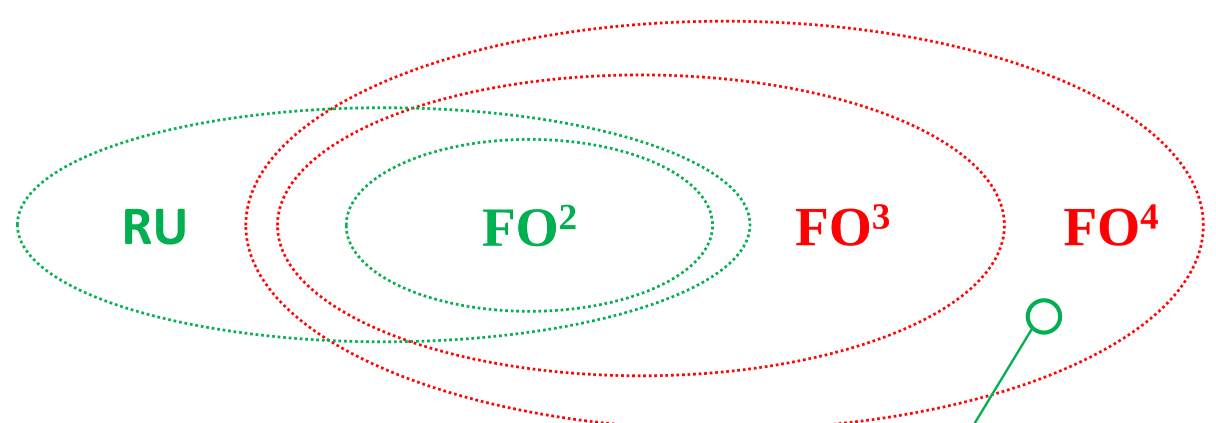
$\forall y': Fr(A, y') \Rightarrow Fr(y', A)$
$\forall x': Fr(x', A) \Rightarrow Fr(A, x')$
$\forall x', y' \neq x': Fr(x', y') \Rightarrow Fr(y', x')$
$x', y' \in \{B, \dots, Z\}$

Apply the rules in $\boldsymbol{R}$ and remove *A*

$\forall x', y' \neq x': Fr(x', y') \Rightarrow Fr(y', x')$
$x', y' \in \{B, \dots, Z\}$

Previously Known Results

- **Previously proved (without domain recursion)**
  - $\boldsymbol{FO^2}$ is liftable.
  - ***RU*** is liftable.
  - Not every $T \in \boldsymbol{FO^3}$ is liftable.
  - ***S4*** is liftable, but the rules in $\boldsymbol{R}$ fail on it.

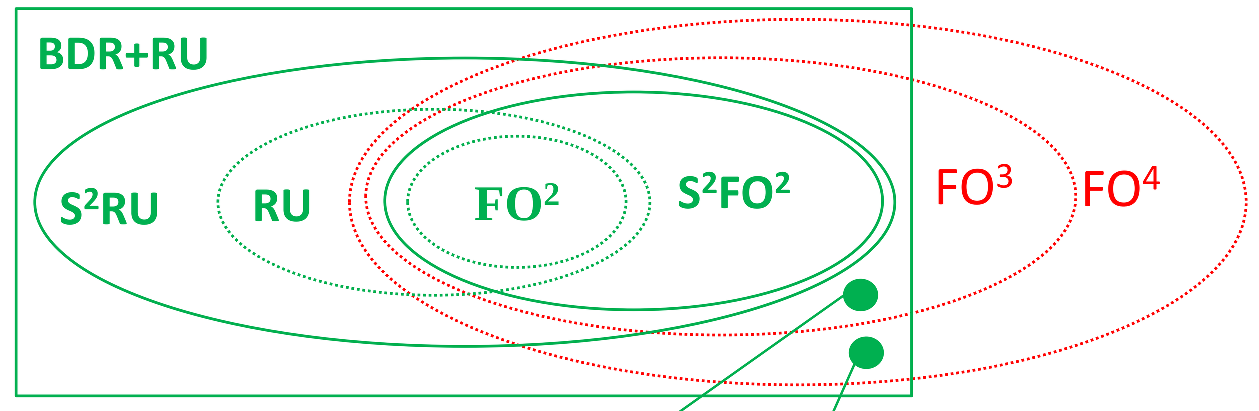


- **Definitions:** A theory is **liftable** if calculating its WMC is polynomial in sizes of the populations. A class *C* is **liftable** if every $T \in C$ is liftable. A theory *T* is in $\boldsymbol{FO^i}$ class if its sentences have up to *i* variables. A theory *T* is ***recursively unary (RU)*** if for every theory *T'* resulting from exhaustively applying the rules in $\boldsymbol{R}$ except atom counting, either *T'* is empty or exists atom $\boldsymbol{S}$ in *T'* with only one logvar, and the theory resulting from applying (symbolic) atom counting on $\boldsymbol{S}$ is RU.

S4 Clause
$\forall x_1, x_2, y_1, y_2: S(x_1, y_1) \vee \neg S(x_1, y_2) \vee S(x_2, y_2) \vee \neg S(x_2, y_1)$

Kazemi et al. 2016 Results + Two New Conjectures

- Let $\boldsymbol{R^D} = \boldsymbol{R}$*+BDR*
  - $\boldsymbol{S^2FO^2}$ and $\boldsymbol{S^2RU}$ are liftable using $\boldsymbol{R^D}$.
  - $\boldsymbol{FO^2} \subset \boldsymbol{S^2FO^2}$ and $\boldsymbol{RU} \subset \boldsymbol{S^2RU}$.
  - Symmetric transitivity is liftable using $\boldsymbol{R^D}$.
  - The rules in $\boldsymbol{R^D}$ are sufficient for S4 clause.
- Let *BDR+RU* be a class of theories identical to *RU* except that $\boldsymbol{R}$ is replaced with $\boldsymbol{R^D}$.
- ***Conjecture 1:*** *BDR+RU* is the largest possible liftable class.
- ***Conjecture 2:*** $\boldsymbol{R^D}$ is a complete set of rules for lifted inference.



Symmetric Transitivity
$\forall x, y, z: F(x, y) \wedge F(y, z) \Rightarrow F(x, z)$
$\forall x, y: F(x, y) \Rightarrow F(y, x)$

S4 Clause
$\forall x_1, x_2, y_1, y_2: S(x_1, y_1) \vee \neg S(x_1, y_2) \vee S(x_2, y_2) \vee \neg S(x_2, y_1)$

S^2FO^2: Definition and an Example

- Let $T \in FO^2$, $S(x, m) \in T$, and for any sentence $c \in T$, if $S(x, m) \in c$, all other atoms in c have at most one variable.
- Add any sentence $\alpha(S)$ to T having exactly 2 S atoms, e.g.:
  - $\forall x, m_1, m_2: S(x, m_1) \vee S(x, m_2)$
  - $\forall x, m_1, m_2: \neg S(x, m_1) \vee S(x, m_2)$
  - $\forall x_1, x_2, m: S(x_1, m) \vee S(x_2, m)$
  - $\forall x_1, x_2, m_1, m_2: S(x_1, m_1) \vee S(x_2, m_2)$
  - …
- T is in S^2FO^2

Example: Volunteers (v) & Jobs (j)

$$\left.\begin{array}{r} \forall j, v: InvolvesGas(j) \wedge Smokes(v) \Rightarrow \neg Assigned(j, v) \\ \forall v1, v2: AUX(v1, v2) \Leftrightarrow Smokes(v1) \wedge Friends(v1, v2) \Rightarrow Smokes(v2) \end{array}\right\} \in FO^2$$

$$\left.\begin{array}{r} \forall v1, v2 \neq v1, j: \neg Assigned(j, v1) \vee \neg Assigned(j, v2) \\ \forall v, j1, j2 \neq j1: \neg Assigned(j1, v) \vee \neg Assgined(j2, v) \end{array}\right\} \in \alpha(Assgined)$$

Clause1: Jobs involving gas are not assigned to smokers
Clause2: Smokers are mostly friends with each other
Clause3: Each volunteer is assigned to at most one job
Clause4: At most one volunteer is assigned to any job

Existential Quantifiers

- Consider theory *T1* where |c|=|p|=n
  $\forall c, \exists p: S(c, p)$
  $\forall p, \exists c: S(c, p)$
  $\forall p, c1, c2 \neq c1: \neg S(c1, p) \vee \neg S(c2, p)$
- **Current approach** (for WMC):
  - Remove existentials through Skolemization:
    $\forall c, \exists p: \neg S(c, p) \vee A(c)$
    $\forall p, \exists c: \neg S(c, p) \vee B(p)$
    $\forall p, c1, c2 \neq c1: \neg S(c1, p) \vee \neg S(c2, p)$
  - Find WMC using the rules $\boldsymbol{R}$ in $O(n^2)$
  - **Problem:** *A* and *B* have negative weights that are potentially inconvenient for log-space computations
- **Alternative approach:** Bounded domain recursion
  - **Advantage:** No negative weights.

Proposition 1: WMC(T1) can be computed in time $O(n^2)$ using the rules in $\boldsymbol{R^D}$ without Skolemization.

- Consider theory *T2* where |c|=|p|=n
  $\forall c, \exists p: S(c, p)$
  $\forall p, \exists c: S(c, p)$
  $\forall p, c1, c2 \neq c1: \neg S(c1, p) \vee \neg S(c2, p)$
  $\forall c, p1, p2 \neq p1: \neg S(c, p1) \vee \neg S(c, p2)$
- **Current approach** (for WMC):
  - Remove existentials through Skolemization:
    $\forall c, \exists p: \neg S(c, p) \vee A(c)$
    $\forall p, \exists c: \neg S(c, p) \vee B(p)$
    $\forall p, c1, c2 \neq c1: \neg S(c1, p) \vee \neg S(c2, p)$
    $\forall c, p1, p2 \neq p1: \neg S(c, p1) \vee \neg S(c, p2)$
  - Find WMC using the rules $\boldsymbol{R}$ in time $O(n^3)$
- **Alternative approach:** Bounded domain recursion
  - **Advantages:** No negative weights, lower time complexity.

Proposition 2: WMC(T2) can be computed in time $O(n)$ using the rules in $\boldsymbol{R^D}$ without Skolemization.

Take Away Messages

- Skolemization may increase the complexity of WMC.
- Skolemization introduces negative weights that are potentially inconvenient.
- Domain recursion can potentially replace Skolemization.
- Better Skolemization techniques may exist.

Future Work

- Characterizing a class of models with existential quantifiers for which domain recursion is bounded.
- Finding the properties that make theories amenable to BDR.
- Prove/disprove conjectures.

- Kazemi, S.M., Kimmig, A., Van den Broeck, G., Poole, D., "New Liftable Classes for First-Order Probabilistic Inference", In *NIPS-2016*.