Marching Cubes

(Lorensen and Cline)

Overview

- Marching cubes: method for approximating surface defined by isovalue $\alpha$, given by grid data

- **Input:**
  - Grid data (set of 2D images)
  - Threshold value (isovalue) $\alpha$

- **Output:**
  - Triangulated surface that matches isovalue surface of $\alpha$
**Voxels**

- Voxel – cube with values at eight corners
  - Each value is above or below isovalue \( \alpha \)
  - Method processes one voxel at a time
- \( 2^8 = 256 \) possible configurations (per voxel)
  - Reduced to 15 (symmetry and rotations)
- Each voxel is either:
  - Entirely inside isosurface
  - Entirely outside isosurface
  - Intersected by isosurface

**Algorithm**

- First pass
  - Identify voxels which intersect isovalue
- Second pass
  - Examine those voxels
  - For each voxel produce set of triangles
    - Approximate surface inside voxel
Configurations

- For each configuration add 1-4 triangles to isosurface
- Isosurface vertices computed by:
  - Interpolation along edges (according to pixel values)
    - Better shading, smoother surfaces
  - Default – mid-edges
Example

Digital Geometry Processing
Marching cubes
**MC Problem**

- Marching Cubes method can produce erroneous results
  - E.g. isovalue surfaces with “holes”
- Example:
  - Voxel with configuration 6 that shares face with complement of configuration 3:

**Solution**

- Use different triangulations
- For each problematic configuration have more than one triangulation
- Distinguish different cases by choosing pairwise connections of four vertices on common face
Ambiguous Face

- **Ambiguous Face**: face containing two diagonally opposite marked grid points and two unmarked ones.

- Source of the problems in MC method.

Solution by Consistency

- **Problem**: Connection of isosurface points on common face done one way on one face & another way on the other.

- Need consistency $\rightarrow$ use different triangulations.

- If choices are consistent get topologically correct surface.
Asymptotic Decider

- **Asymptotic Decider**: technique for choosing which vertices to connect on ambiguous face
- Use bilinear interpolation over ambiguous face

Bilinear Interpolation

- Bilinear interpolation over face - natural extension of linear interpolation along an edge
- Consider face as unit square

\[
B(s,t) = (1-s)s \begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} \begin{pmatrix} 1-t \\ t \end{pmatrix}
\]

\[\{(s,t) : 0 \leq s \leq 1, \quad 0 \leq t \leq 1\}\]

- \(B_{ij}\) - values of four face corners
Bilinear Interpolation (cont.)

Contour curves of $B$ create hyperbolas

$\{(s, t): B(s, t) = \alpha\}$

Hyperbolas-Domain relation

Relation between hyperbolas and face
Asymptotic Decider Test

- Ambiguous case: both components of hyperbola intersect the domain

- Criterion for connecting the vertices based upon whether they are joined by a component of the hyperbolic arc

- Selection determined by comparing contour value $\alpha$

Asymptotic Decider Test (cont).

- If $\alpha > B(S_\alpha, T_\alpha)$
  - connect $(S_1, 1)-(1, T_1)$ & $(S_0, 0)-(0, T_0)$
  - else
  - connect $(S_1, 1)-(0, T_0)$ and $(S_0, 0)-(1, T_1)$
Separation

- Face is **separated** if asymptotic decider implies separation of 2 marked vertices by isovalue surface.

- Otherwise, face is said to be **not separated**.

Various Cases

- Configurations 0, 1, 2, 4, 5, 8, 9, 11 and 14 have no ambiguous faces → no modifications.

- Other configurations need modifications according to number of ambiguous faces.
Configuration 3+6
- Exactly one ambiguous face
- Two possible ways to connect vertices
  - two resulting triangulations
- Several different (valid) triangulations

Configuration 12
- Two ambiguous faces \(2^2 = 4\) boundary polygons
Configuration 10

- As in configuration 12 - two ambiguous faces

- When both faces are separated (10A) or not separated (10C) there are two components for the isovalue surface

Configuration 7

- Three ambiguous faces → $2^3 = 8$ possibilities

- Some are equivalent → only 4 triangulations
Remarks

- Modifications add considerable complexity to MC
- No significant impact on running time or total number of triangles produced
- New configurations occur in real data sets
  - But not very often
## Examples and Remarks (cont)

### Table 1: Frequency of Configurations

<table>
<thead>
<tr>
<th>Config</th>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>253,519</td>
<td>285,074</td>
<td>110,993</td>
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<td>1</td>
<td>7.705</td>
<td>1.912</td>
<td>1.672</td>
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<td>8.710</td>
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</tr>
<tr>
<td>3B</td>
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<td>0</td>
<td>0</td>
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</tr>
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<td>0</td>
</tr>
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<td>0</td>
</tr>
<tr>
<td>7A, C</td>
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<tr>
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</table>

*Table 1: Frequency of configurations*