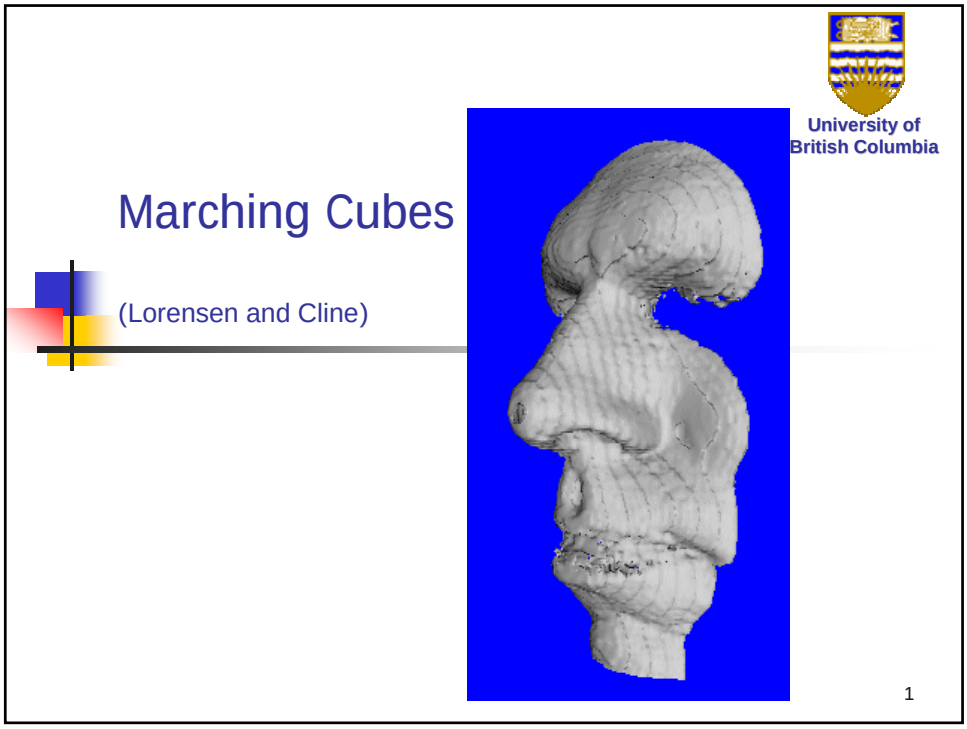
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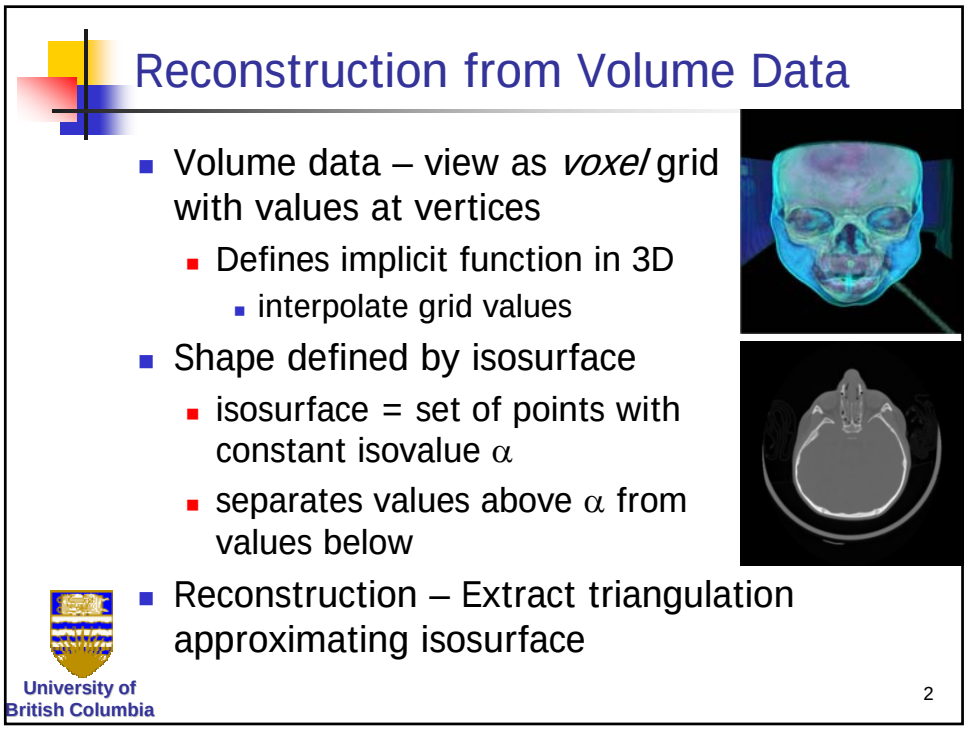
Marching Cubes
(Lorensen and Cline)
1


Reconstruction from Volume Data
Volume data – view as voxel grid with values at vertices
Defines implicit function in 3D
interpolate grid values
Shape defined by isosurface
isosurface = set of points with constant isovalue α
separates values above α from values below
Reconstruction – Extract triangulation approximating isosurface
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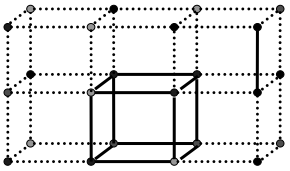
Voxels

- Voxel – cube with values at eight corners
  - Each value is above or below isovalue α
- 2^8=256 possible configurations (per voxel)
  - reduced to 15 (symmetry and rotations)
- Each voxel is either:
  - Entirely inside isosurface
  - Entirely outside isosurface
  - Intersected by isosurface



- MC main observation: Can extract triangulation independently per voxel

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Basic MC Algorithm

- For each voxel produce set of triangles
  - Based on above/below corner configuration
  - Empty for non-intersecting voxels
  - Approximate surface inside voxel

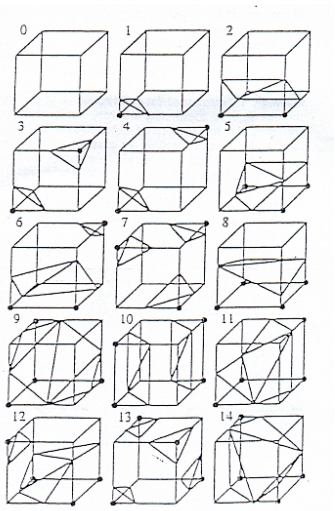

Figure 2. Configurations.

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Configurations

- For each configuration add 1-4 triangles to isosurface
- Isosurface vertices computed by:
  - Interpolation along edges (according to grid values)
    - better shading, smoother surfaces
  - Default – mid-edges

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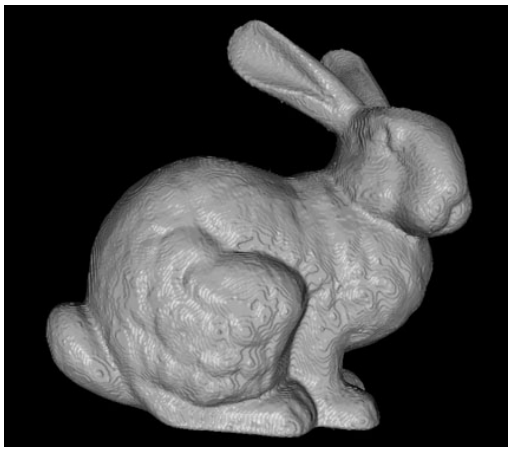
Example



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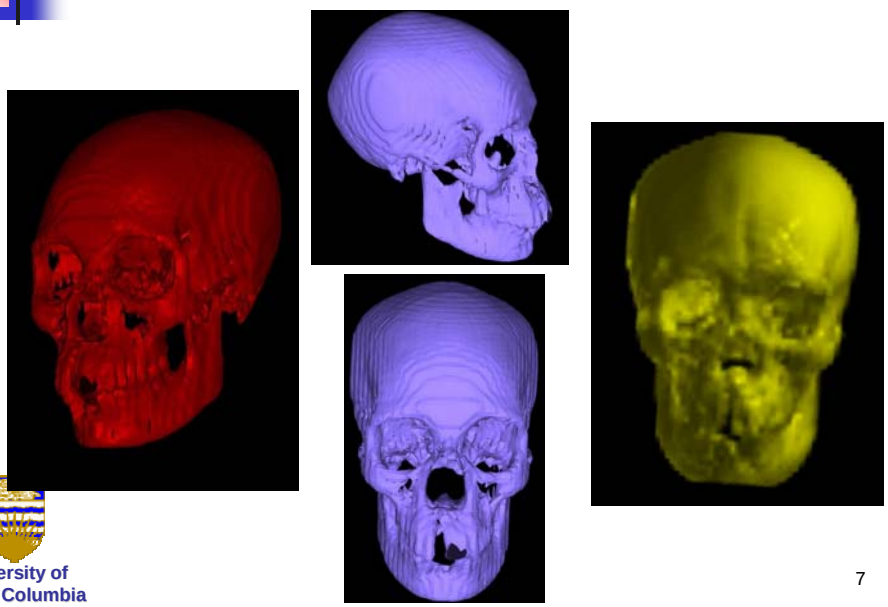
Example



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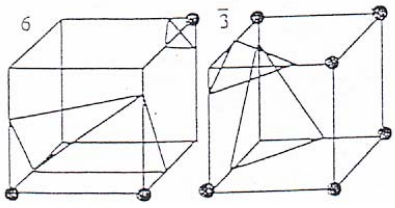
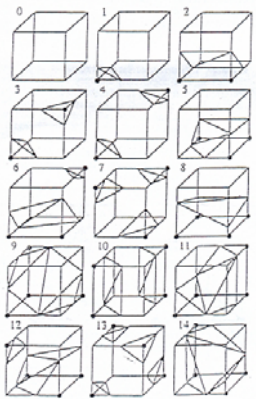
Consistency Problem

- Can produce non-manifold results
  - Isovalue surfaces with “holes”
- Example:





  - Voxel with configuration 6 sharing face with complement of configuration 3

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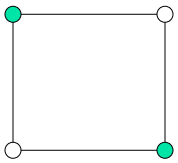
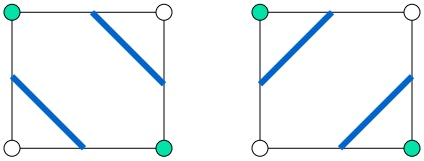
Ambiguous Faces

- Face containing two diagonally opposite marked grid points and two unmarked ones



- Two locally valid interpretations



- Source of MC consistency problem

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Consistency

- Problem:
  - Connection of isosurface points on shared face done one way on one face & another way on the other
- Need consistency → use different triangulations
- If choices are consistent get topologically correct surface

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Solution

- For each problematic configuration have more than one triangulation
- *Distinguish different cases* by choosing pairwise connections of four vertices on common face

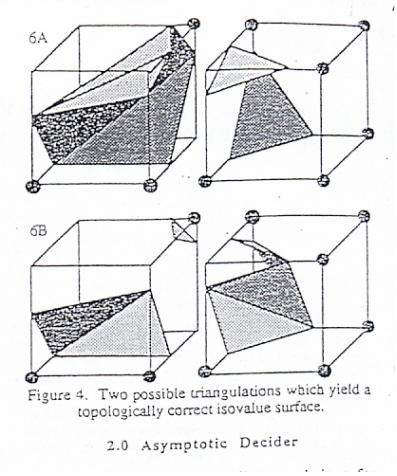

Figure 4. Two possible triangulations which yield a topologically correct isovalue surface.

2.0 Asymptotic Decider

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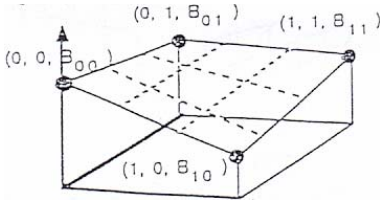
Asymptotic Decider

- Select connectivity that better fits implicit function
- Use *bilinear interpolation* to approximate function
  - 2D extension of linear interpolation



$$B(s,t) = \begin{pmatrix} 1-s & s \end{pmatrix} \begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} \begin{pmatrix} 1-t \\ t \end{pmatrix}$$

$$\{(s,t): 0 \le s \le 1, \quad 0 \le t \le 1\}$$

- B_{ij} - isovalues at face corners

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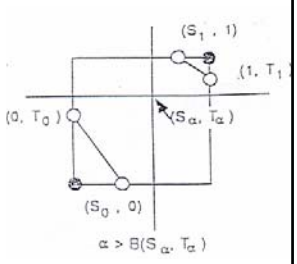
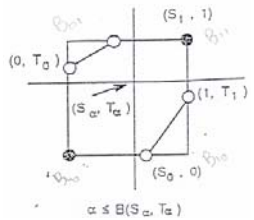
Asymptotic Decider

- E.g. B_{00} & B_{11} **above** α
- Test value at face "center" (S_α, T_α)
  - If $\alpha > B(S_\alpha, T_\alpha)$
    - connect $(S_1,1)$-$(1,T_1)$ & $(S_0,0)$-$(0,T_0)$
  - else
    - connect $(S_1,1)$-$(0,T_0)$ & $(S_0,0)$-$(1,T_1)$





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Asymptotic Decider

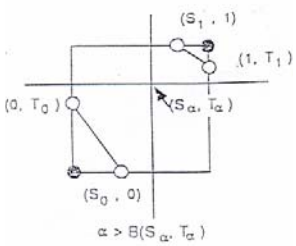
- Choice of "center":

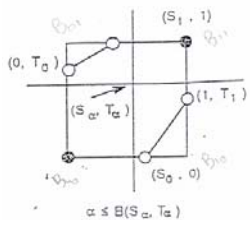
$$S_\alpha = \frac{B_{00} - B_{01}}{B_{00} + B_{11} - B_{01} - B_{10}}$$

$$T_\alpha = \frac{B_{00} - B_{10}}{B_{00} + B_{11} - B_{01} - B_{10}}$$

$$B(S_\alpha,T_\alpha) = \frac{B_{00}\,B_{11} + B_{10}\,B_{01}}{B_{00} + B_{11} - B_{01} - B_{10}}.$$

  - Related to contour curves asymptotic behaviour





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Various Cases

- Some configurations have no ambiguous faces → no modifications
- Other configurations need modifications according to number of ambiguous faces
  - Apply decoder to each face to decide on triangulation template

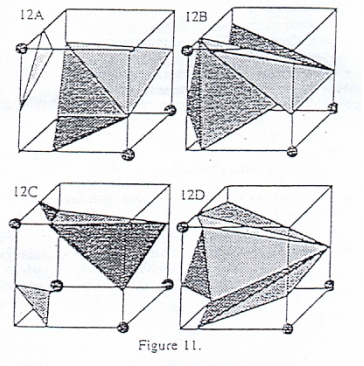

Figure 11.

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Remarks

- Add considerable complexity to MC
- No significant impact on running time or total number of triangles produced
- New configurations occur in real data sets
  - But not very often

Config.	Example 1	Example 2	Example 3
0	263,519	285,074	110,993
1	7,705	1,912	1,673
2	8,710	2,065	2,421
3A	60	0	6
3B	46	0	6
4	28	0	0
5	5,611	1,228	1,143
6A	20	0	0
6B	47	0	0
7A	3	0	0
7B,D	3	0	0
7C	3	0	0
8	4,637	906	1,146
9	1,003	304	261
10A,C	13	0	0
10B,D	1	0	0
11	36	0	0
12A,C	7	0	0
12B,D	4	0	0
13	0	0	0
14	69	0	0

Table 1. Frequency of configurations

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