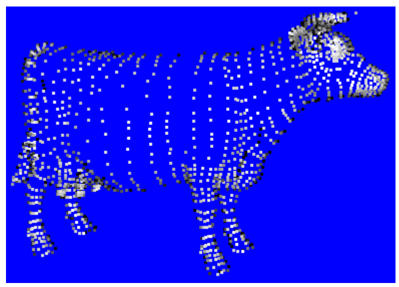
Mesh Geometry Coding



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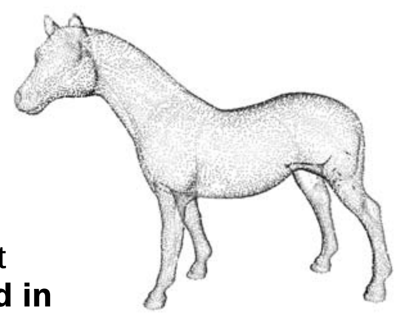
The Geometry

- Vertex coordinates (x, y, z) are
  - Floating point values
  - Almost unrestricted in:
    - range
    - precision
  - **Uniformly** spread in 3D



- Compression exploits input redundancy **- hard to find in raw geometry data**

- ***Lossy* compression is OK !!**

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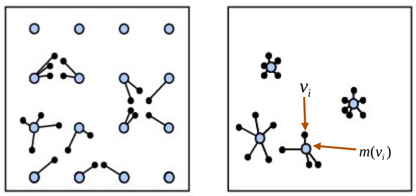
Quantization

- Map n values v_i to k<<n values $m(v_i)$, without losing **too much** information



Uniform Non-uniform

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Quantization

- Applications:
  - Image and voice compression
  - Voice recognition
  - Color display
  - Geometric compression

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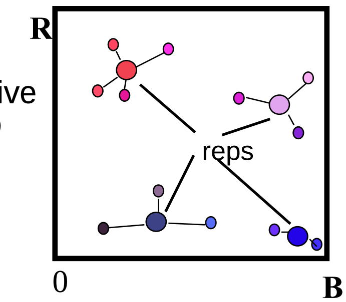
Example: color quantization

- Used for limited dynamic-range displays (e.g. an 8 bit display can display only 256 different colors)
- Reducing number of colors
  - Choosing set of representative colors (*colormap* or *palette)*
  - Map rest of colors to them
- Usually uses 256 colors

quantization to 4 colors



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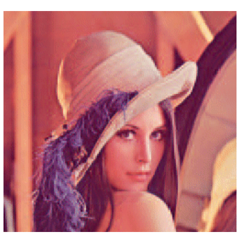
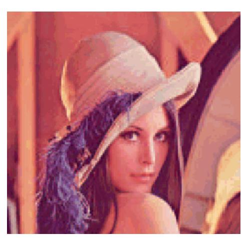
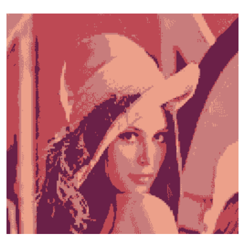
Color quantization examples


256 colors


64 colors


16 colors


8 colors


4 colors


2 colors

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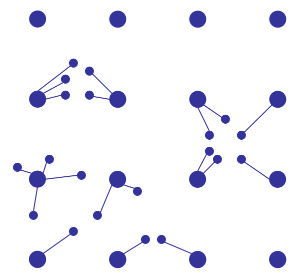
Uniform quantization

- Quantization space partitioned into equal sized regions (e.g. grid) – colors in each region mapped to its center
- Input independent
- Some representatives may be *wasted*
- Common way for 24->8 bit color quantization: retain 3+3+2 most significant bits of R, G & B components



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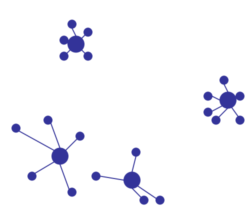
Non-uniform quantization

- Quantization space partitioned according to input data
- Goal: choosing "best" representatives
  - Minimal distance error (if "distance" is defined)



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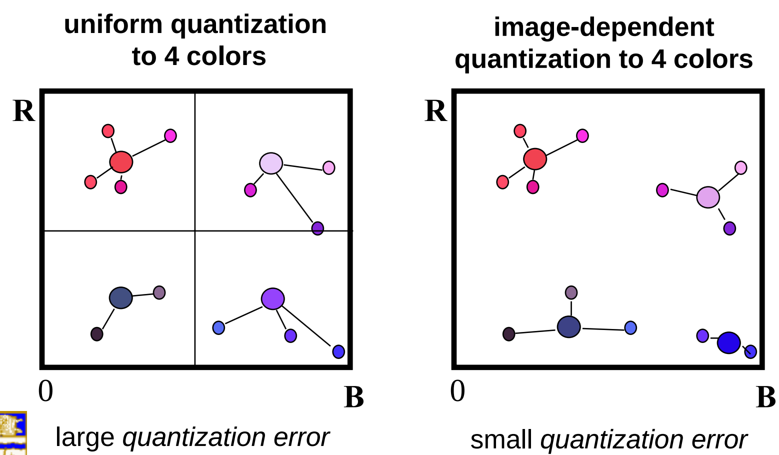
Examples



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Quantization & *Lossy* Coding

- Quantization used as lossy coding method when there is notion of distance between symbols to be coded
  - Coordinates
  - Colors
  - Not good for characters

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Uniform geometry quantization

- Coordinates can be considered integers in a finite range after quantization
- Quantization is done on the data bounding box/cube
- Geometry quantization to n bits:
  - All integer values in $[0, 2^n\text{-}1]$ can be used
  - Scale/transform coordinates to be maximal over given range
  - Quantize each coordinate (rounding to nearest integer)

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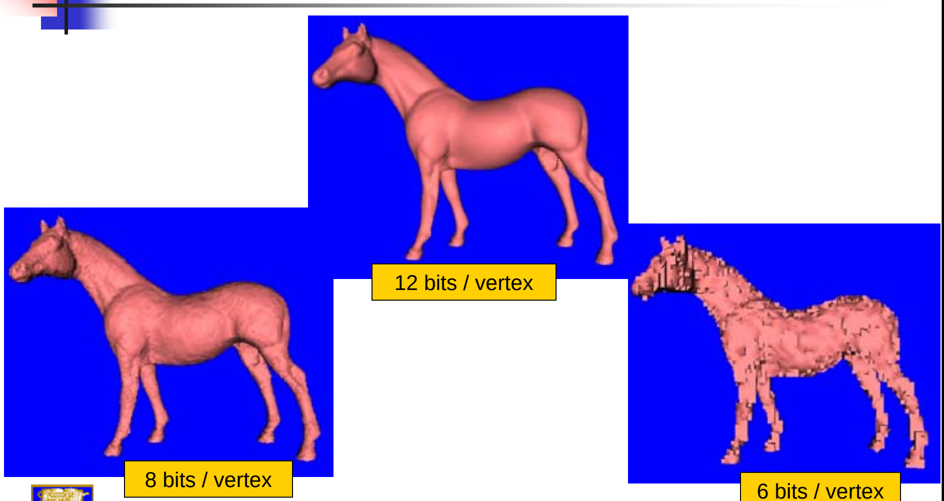
Uniform geometry quantization - results



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Prediction

- Quantization alone gives poor compression – vetex positions evenly (more or less) distributed in range
- To improve use *prediction*
  - Assume vertex positions can be estimated based on neighbours
  - Store & encode (quantize) prediction error
- Good prediction
  - small error range
  - uneven error distribution

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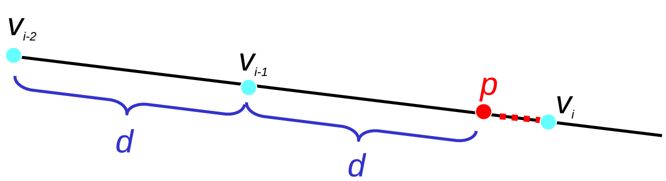
Prediction – History Repeats Itself

- Linear 2D predictor:



- Prediction rule: $v_i-1 - v_i-2 = p - v_i-1$
  or: $p = 2 v_i-1 - v_i-2$
- Prediction error: $e_i = v_i - p$

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Using Predicted Geometry

- $(v_1\ v_2\ v_3\ \ldots)$ - vertex coordinates
  $(e_3\ e_4\ e_5 \ldots)$ - prediction errors
- Naive geometry coding: $v_1\ v_2\ v_3\ \ldots$
- Coding using prediction: $v_1\ v_2\ e_3\ e_4\ e_5\ \ldots$
- Decoding: $v_1\ v_2$

$$v_i = 2\ v_i\text{-}1 - v_i\text{-}2 + e_i \qquad i>2$$

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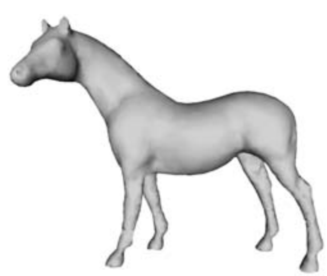
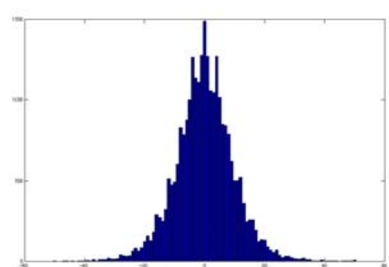
Good Prediction Reduces Entropy





0
Distribution of prediction errors

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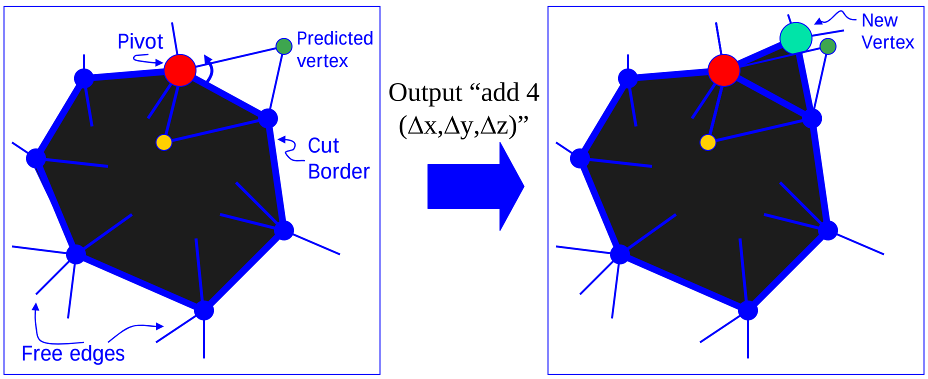
Surface-Based Prediction - TG



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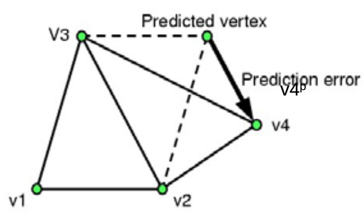
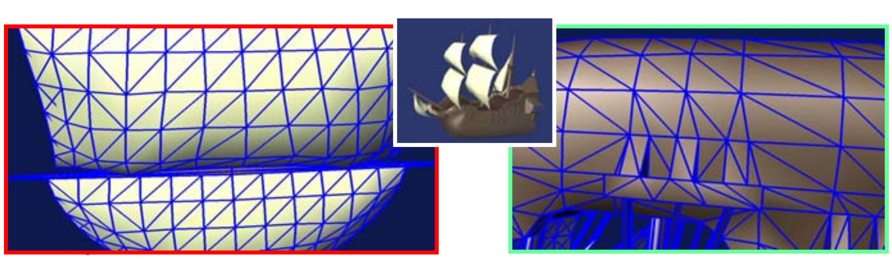
Parallelogram Prediction



- Use connectivity to predict geometry: $V_{4p} = v_2 + v_3 - v_1$
- (-1, 1, 1) in *barycentric coords*
- **Can** be applied to integers



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Some Results

Raw quantized data = 10 bits/coord = 30 bits/vertex

Model	vertices	line predictor	parallelogram	ratio
Eight	766	18.8	14.0	1.3
Triceratops	3100	18.4	14.1	1.3
Cow	3078	18.9	14.6	1.3
Beethoven	2847	22.7	17.3	1.3
Dodge	10466	19.8	12.4	1.6
Starship	4468	19.2	13.2	1.5
Average		19.6	14.3	1.4

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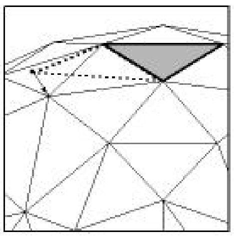
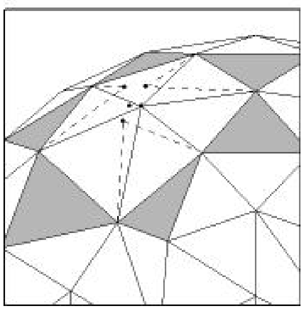
K-way prediction

predict that vertex is average of its neighbors





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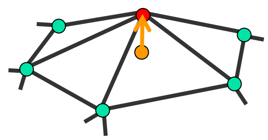
Correction

- Detail = surface – *smooth*(surface) = surface – predicted(surface)



$$\boldsymbol{\delta}_i = \mathbf{v}_i - \frac{1}{d_i}\sum_{j\in N(i)} \mathbf{v}_j$$

$$\boldsymbol{\delta}_i = \sum_{j\in N(i)} \frac{1}{d_i}\left(\mathbf{v}_i - \mathbf{v}_j\right)$$

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Laplacian matrix

- Transition between δ & xyz is linear:

$$\begin{pmatrix} \mathrm{L} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \delta_1^{(x)} \\ \delta_2^{(x)} \\ \vdots \\ \vdots \\ \delta_n^{(x)} \end{pmatrix}$$

$$A_{ij} = \begin{cases} 1 & i \in N(j) \\ 0 & otherwise \end{cases}$$

$$D_{ij} = \begin{cases} d_i & i = j \\ 0 & otherwise \end{cases}$$

$$\mathrm{L} = \mathrm{I} - \mathrm{D}^{-1}\mathrm{A}$$

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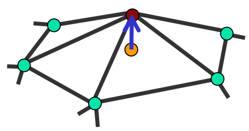
Laplacian matrix

- Transition between δ & xyz is linear:



$$\boldsymbol{\delta}_i = \sum_{j \in N(i)} w_{ij} \left(\mathbf{v}_i - \mathbf{v}_j \right)$$

$$L \, \mathbf{v}_x = \delta_x$$

$$L \, \mathbf{v}_y = \delta_y$$

$$L \, \mathbf{v}_z = \delta_z$$

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Basic properties

- Rank(L) = n-c (n-1 for connected meshes)
- Can reconstruct *xyz* geometry from delta up to translation

$$L\mathbf{x} = \boldsymbol{\delta}$$

$$\mathbf{x} = L^{-1}\boldsymbol{\delta}$$ (almost....)

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Quantizing differential coordinates

- Note: even if xyz are integer δ won't be
- Quantize δ-coordinates?
  - Can we still go back to xyz ?
  - How does reconstruction error behave?

$$L\mathbf{x} = \boldsymbol{\delta}$$

$$\boldsymbol{\delta} \rightarrow \boldsymbol{\delta}' = \boldsymbol{\delta} + \varepsilon$$

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Quantizing differential coordinates

How does reconstruction error behave?

$$\mathbf{x}' = L^{-1}\boldsymbol{\delta}' = L^{-1}(\boldsymbol{\delta} + \varepsilon)$$

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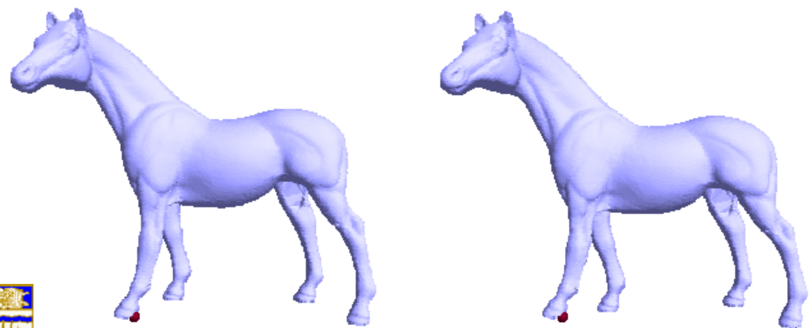
Quantizing differential coordinates

- Find the differences between the horses…



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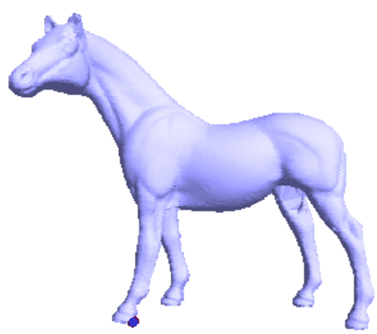
Quantizing differential coordinates

- Original horse model



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Quantizing differential coordinates

- Quantizing δ to 8 bits/coordinate
- One anchor point (front left leg)

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Quantizing differential coordinates

- Original model

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Quantizing differential coordinates

- Quantizing δ to 7 bits/coordinate, one anchor

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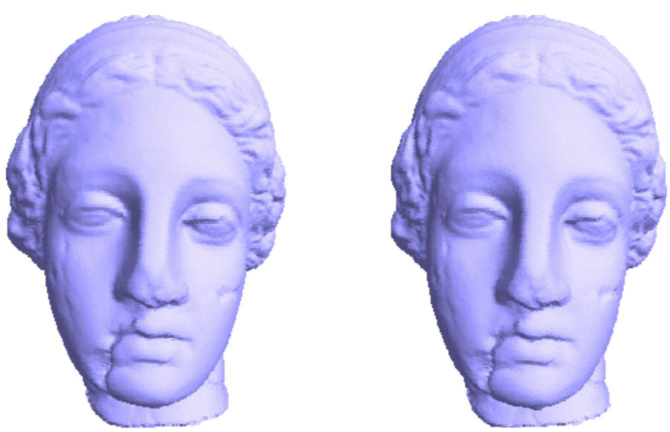
Quantizing differential coordinates

- Can reduce error by adding more anchors



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