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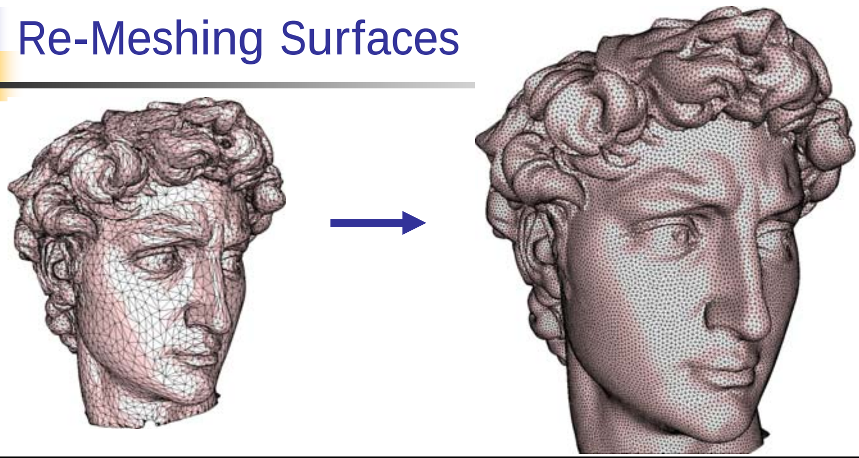
Re-Meshing Surfaces



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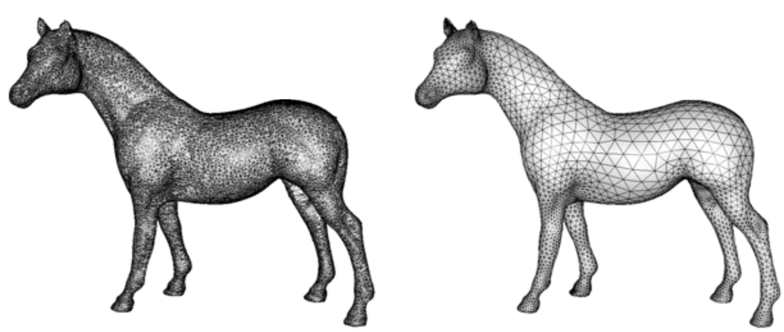
Re-Meshing Surfaces

- Given input mesh generate new mesh which is “better”
  - Element sizing
  - Element shape
- BUT is near (geometrically) to original surface



Hausdorff Metric

- Given two sets (surfaces) P and Q

  $H_P(Q)=\max_p \min_q ||p,q||$
  $H(P,Q)=\max(H_q(P), H_P(Q))$

  - point to point
- On mesh approximate by
  - measuring vertex to surface distance
  - measuring vertex to vertex distance
- Expensive to compute
  - Public Domain Code: Metro

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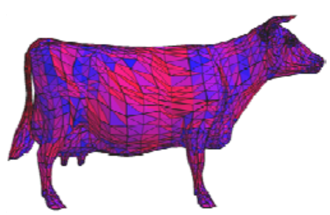
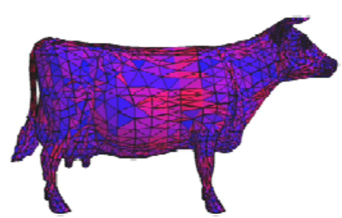
How to (re)mesh surfaces?

- Can we apply Delaunay triangulation?
  - What is Delaunay criterion on surface?
    - Option 1: Use sphere instead of circle
      - Works for volumetric meshes (tets)
    - Option 2: Use pairwise test only
    - Theoretical Delaunay properties do not hold
  - Boundary recovery = Approximation quality

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Approaches

- Mesh adaptation/Local Remeshing
  - Modify existing mesh using sequence of local operations
    - Evaluate approximation quality at every step
- Reduction to 2D/Global Remeshing
  - Segment surface into parameterizable pieces
  - Parameterize in 2D
  - Mesh in 2D (Delaunay)
  - Project back

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Mesh adaptation

- Store original for approximation evaluation
- Connectivity modification
  - Edge flips
  - Refinement/Coarsening
- Geometry modification - mesh smoothing
- Typical Sequence:
  - Refine/Coarsen to satisfy sizing
  - Smooth mesh (sizing + quality)
  - Perform flips after every other operation
  - Repeat entire sequence several times

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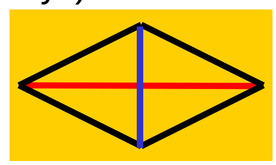
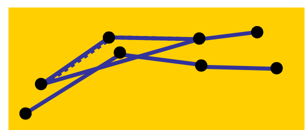
Mesh adaptation : Edge Flip

- Given 2 triangles flip one diagonal if longer than the other
- 3D equivalent of Delaunay test in 2D (why?)
- Test impact on approximation (why?)
  - Approximate Hausdorff metric
    - Normal error
    - Smoothness
  - Test self-intersection
    - Distance based
    - Very expensive test – often ignored





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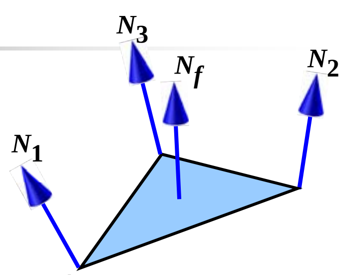
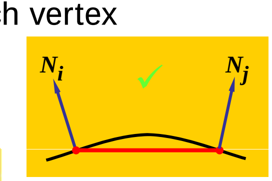
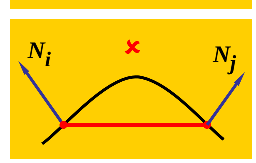
Normal Error



- Based on normals only
- Defined for a face
- Normals N_1, N_2 , N_3 - from **original** mesh
  - Store original locations for each vertex
- N_f - current face normal
- Distance:

$$E_{gap}(f) = \max_{i\in\{1,2,3\}} \langle N_{v_i}, N_{v_{i+1}} \rangle < \cos\theta_{gap}$$





- Why?

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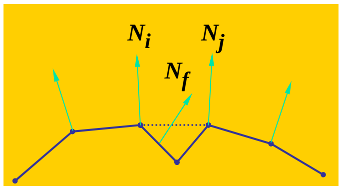
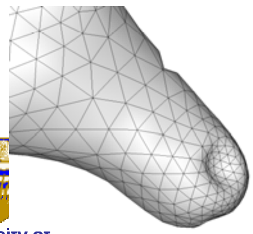
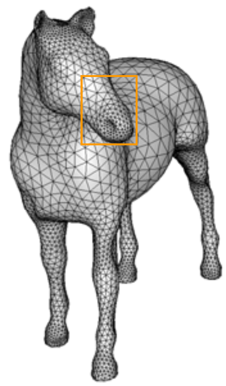
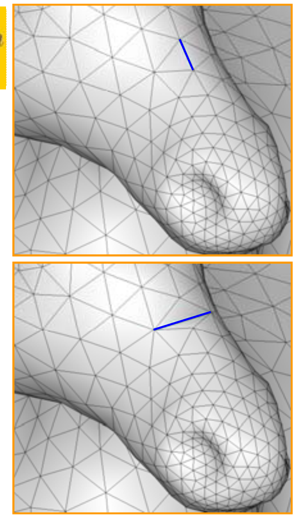
Smoothness

$$E_{smth}(f) = \max_{i\in\{1,2,3\}} \langle N_f, N_{v_i}\rangle < \cos\theta_{smth}$$









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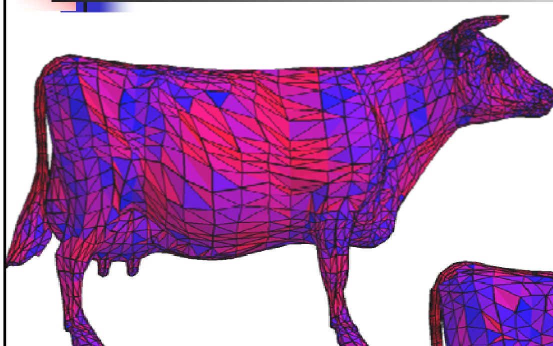
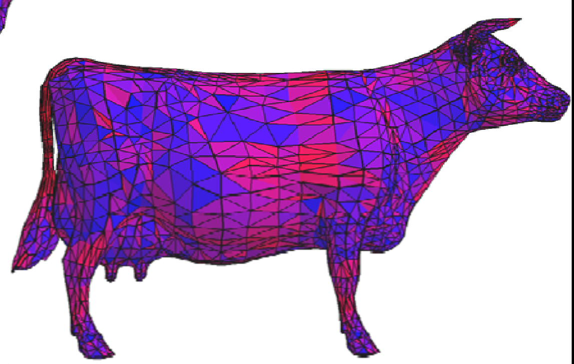
Example



Before (avg min 30)



After (avg min 33)

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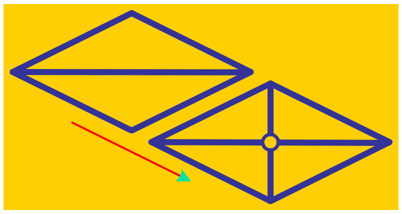
Mesh Adaptation: Refinement

- Add vertices – reach desired sizing or element count
- Strategy:
  - Split long edges – insert mid-points
- Vertex positioning
  - *Project to **original** mesh*
- Hard to achieve good spacing
  - Improve by smoothing



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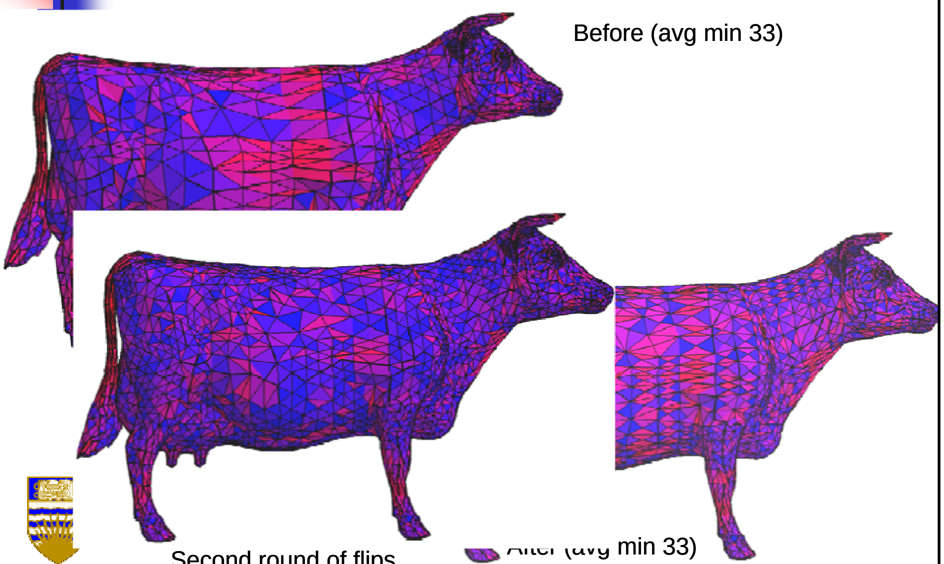
Example



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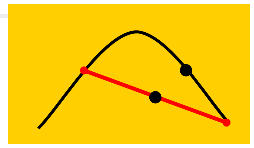
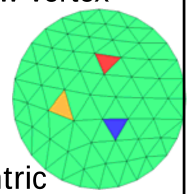
Projection to Original Mesh



- Nearest point
  - Requires search
    - Find original face closest to (estimated) new vertex
    - Expensive
  - Unlimited Hausdorff error



- Local parameterization
  - Compute new location in terms of barycentric coordinates of new face vertices
  - Locally parameterize old mesh to get corresponding location
    - Use sophisticated data structures for efficiency
  - Better approximation

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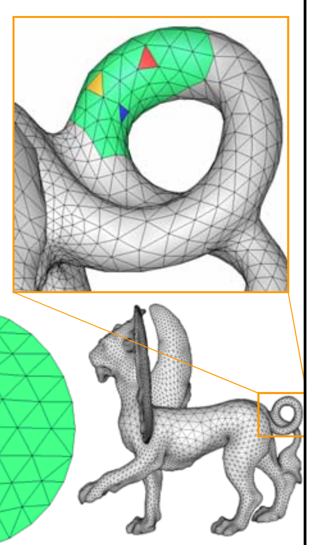
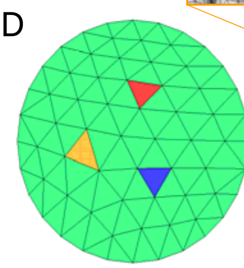
Local Parameterization

- Input:
  - New mesh face: 3 vertices
    - known locations (face+ coords) on original mesh
  - New vertex: barycentric coordinates on new face
- Compute small patch of old mesh containing all 3 vertices



- Parameterize patch in 2D
- Compute new location on parameterized patch
- Project to 3D



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Mesh Adaptation: Coarsening

- Similar to simplification
- Operations:
  - Vertex removal
  - Edge collapse
    - Project new vertex to original surface as in refinement
- Approximation Error
  - Quadrics
  - Normal based

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Mesh Adaptation: Smoothing

- Move vertices ON surface to improve sizing/quality
- Moving One Vertex:
  - Compute vertex location as function of neighbors in new mesh
    - E.g. convex combination
    - Use local parameterization
  - Project to original mesh
  - Check approximation error
    - If too large, keep previous location

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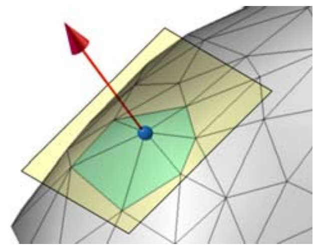
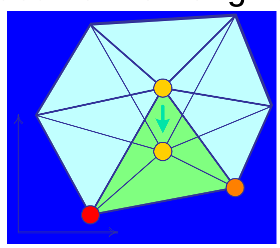
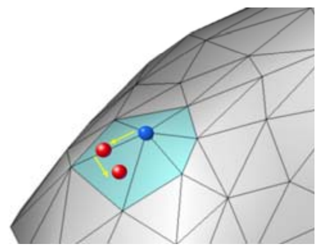
Local Parameterization

- Project vertex + neighbors to current normal plane
- Relocate vertex in plane
- Find new triangle in which vertex is located
- Compute barycentric coordinates in this triangle
- Use for placement on original mesh







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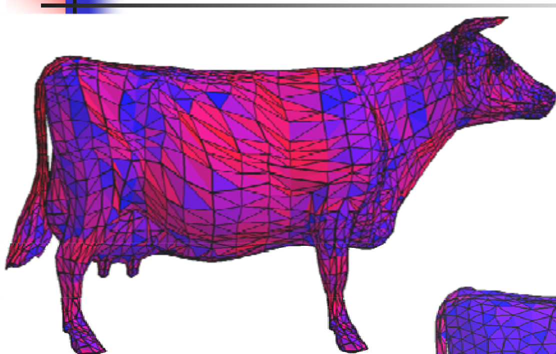
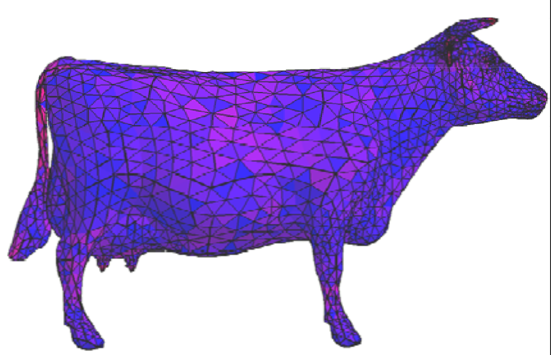
Example



Before (avg min 30)



Smoothing + Flips
(avg min 45)

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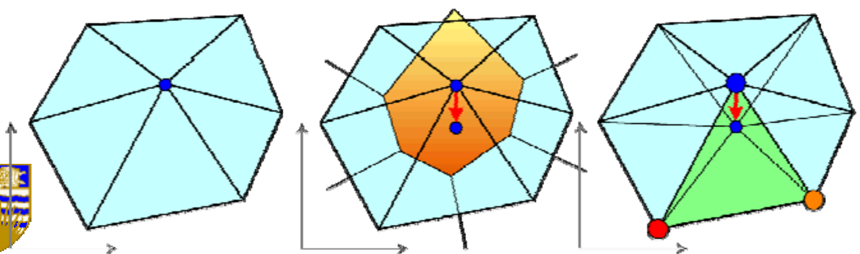
Smoothing:Centroidal Voronoi Diagram

- Relocate vertices (smoothing) to control sizing (sampling)
- Lloyd algorithm on surface mesh
  - On 2D umbrella compute VD of vertex + neighbors
  - Place vertex at center of mass of it's cell
  - Repeat



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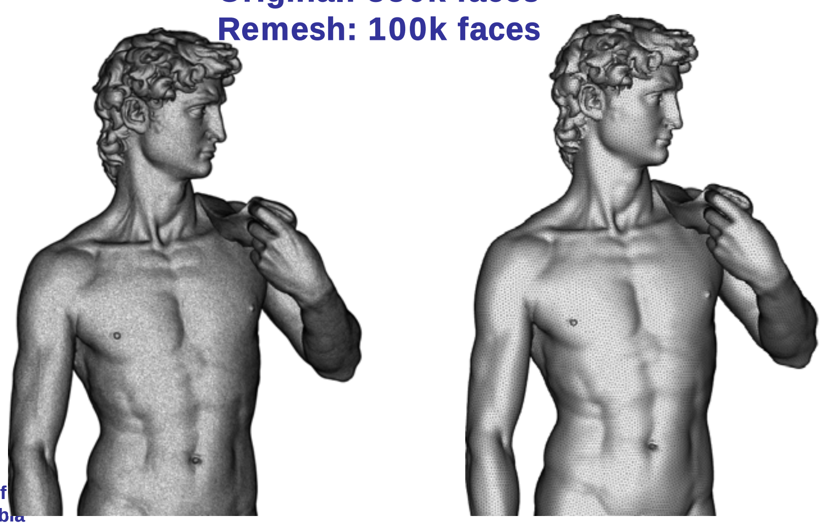
Michelangelo's David

Original: 350k faces
Remesh: 100k faces



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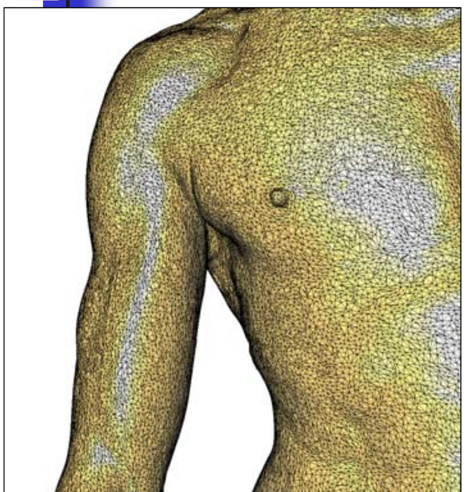
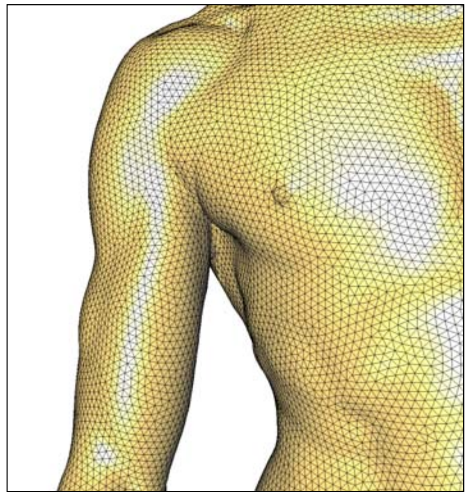
David: Zoom in



Original



Remesh WCVT

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Mesh Adaptation

- Modify existing mesh using sequence of local operations
- Fast
- Simple to implement
  - Depending on choice of local operations
- Hard to find GOOD spacing of vertices
  - WCVD does the trick but at a cost…
- Heuristic
  - How many iterations of each operation to do?

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