Increased Discrimination in Level Set Methods with Embedded Conditional Random Fields

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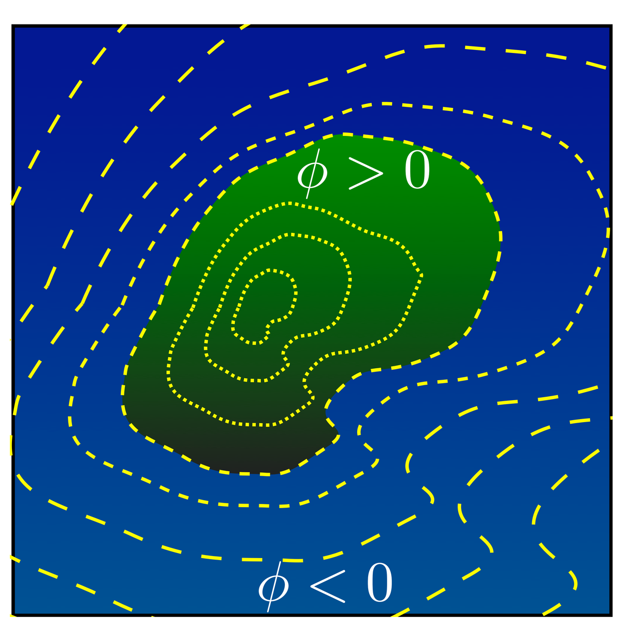
Introduction

- We want to use training data to build an automatic segmentation tool
- Conditional random fields (CRFs):
  - **discriminative model**
  - **models neighbor's correlation**
  - **feature-based edge regularization**
  - Markov assumption on labels
- Level set segmentation:
  - generative model
  - assumes neighbor independence
  - image-based edge regularization
  - **allows non-Markov priors**
- We embed CRFs within a level set framework:
  - a conditional level set method
  - a CRF that allows non-Markov priors

Level Set Segmentation

- Represent contour implicitly as the zero level set of an embedding function
- Minimize the energy by solving the Euler-Lagrange equations

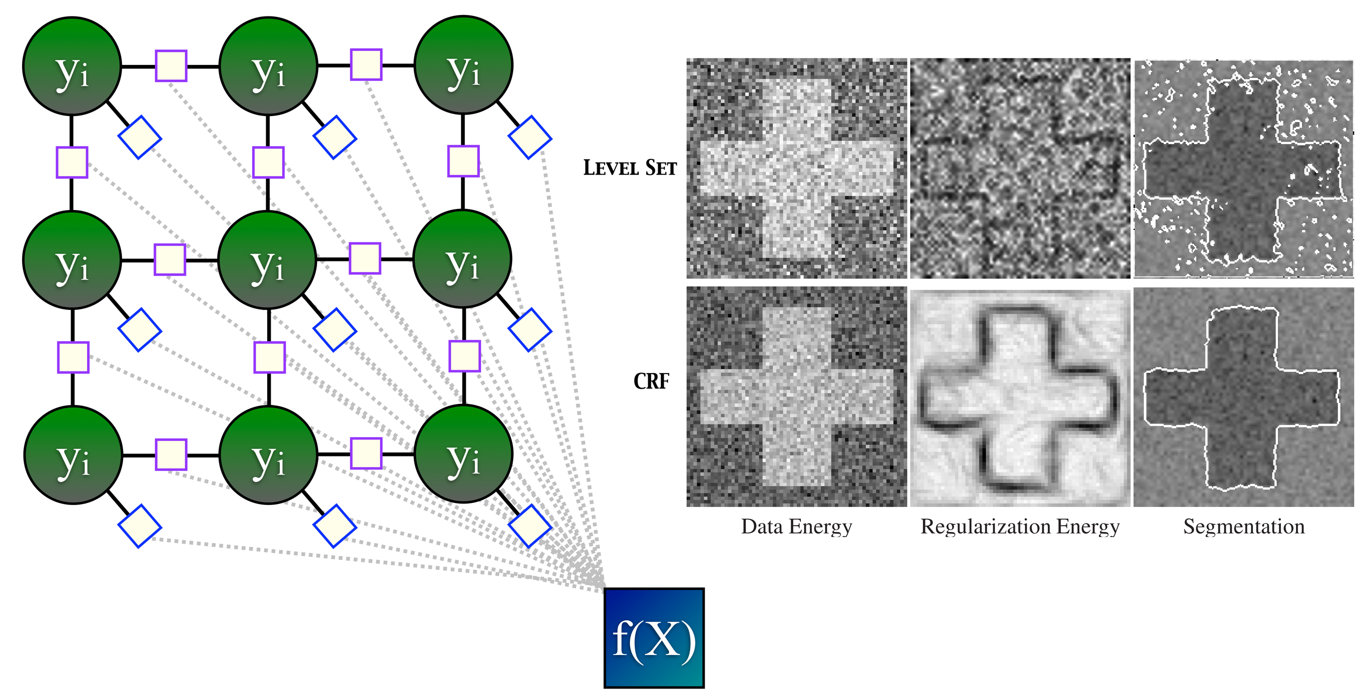


- Chan-Vese energy:

$$E(\Phi) = \int_\Omega -H(\Phi)\log p_1(\mathbf{f}(x),\mathbf{w}) - (1-H(\Phi))\log p_2(\mathbf{f}(x),\mathbf{w}) + v|\nabla H(\Phi)|g(X,\alpha)dx$$

- Parameter estimation: fix regions, fit independent generative pixel model, tune v and α manually.

Conditional Random Fields



- CRFs model the conditional probability of the labels Y given features f(X)

$$P(Y|X) = \frac{1}{Z}\exp\left(\sum_{i\in N} y_i \mathbf{w}^T\mathbf{f}_i(X) + \sum_{i,j\in E} y_{ij}\mathbf{v}^T\mathbf{f}_{ij}(X)\right)$$

- Parameter estimation:
  - is **jointly convex** in **w** and **v**
  - is **efficient** using a conditional pseudo-likelihood
  - is **discriminative**; there is no image model P(X)
  - **models correlations** between neighboring pixels
  - learns edge **regularization related to labels**

Associative CRFs

- To embed the CRF within a level set method:
  - we convert to a {0,1} representation
  - we use associative edge features

$$f_{ijk}(X) \triangleq \frac{1}{1+|f_{ik}(X) - f_{jk}(X)|}$$

  - we require **v** to be non-negative

$$\max_{\mathbf{w},\mathbf{v}} \frac{1}{Z}\exp\left(\sum_i y_i\mathbf{w}^T\mathbf{f}_i(X) + \sum_{ij}(1-|y_i-y_j|)\sum_k v_k f_{ijk}(X)\right) \quad \text{subject to} \quad \mathbf{v}\geq 0$$

- We can efficiently solve this optimization problem with a bound-constrained L-BFGS method

Continuous-Domain CRFs

- The associative CRF can be embedded into a continuous model that has the same energy:

	CRF	cont. CRF
node labels	y_i	$H(\Phi(x))$
edge labels	$1-\|y_i-y_j\|$	$1-\|\nabla H(\Phi(x))\|$
node features	$\mathbf{f}_i(X)$	$\mathbf{f}(x)$
edge features	$f_{ij} = F(\mathbf{f}_i(X),\mathbf{f}_j(X))$	$F(\nabla\mathbf{f}(x))$

- Energy functional:

$$E(\Phi) = \int_\Omega -H(\Phi)(\mathbf{w}^T\mathbf{f}) + (1-H(\Phi))(\mathbf{w}^T\mathbf{f}) + |\nabla H(\Phi)|\sum_k v_k\frac{1}{1+|\nabla f_k|}dx$$

- Euler-Lagrange equations:

$$\frac{\partial\Phi}{\partial t} = -2\delta(\Phi)\mathbf{w}^T\mathbf{f} + \delta(\Phi)\,\mathrm{div}\left(\left(\sum_k v_k\frac{1}{1+|\nabla f_k|}\right)\frac{\nabla\Phi}{|\nabla\Phi|}\right)$$

Training:
Given: a set of images $X_1, X_2, \ldots, X_n$
Extract features $\mathbf{f}(X_1), \mathbf{f}(X_2), \ldots, \mathbf{f}(X_n)$
Compute optimal node and edge parameters $\{\mathbf{w},\mathbf{v}\}$ by maximizing the constrained pseudo-likelihood of the CRF
Segmentation:
Given: one image X
Extract features $\mathbf{f}(X)$
Compute segmentation by evolving a curve driven by the Euler-Lagrange equations

Shape Priors

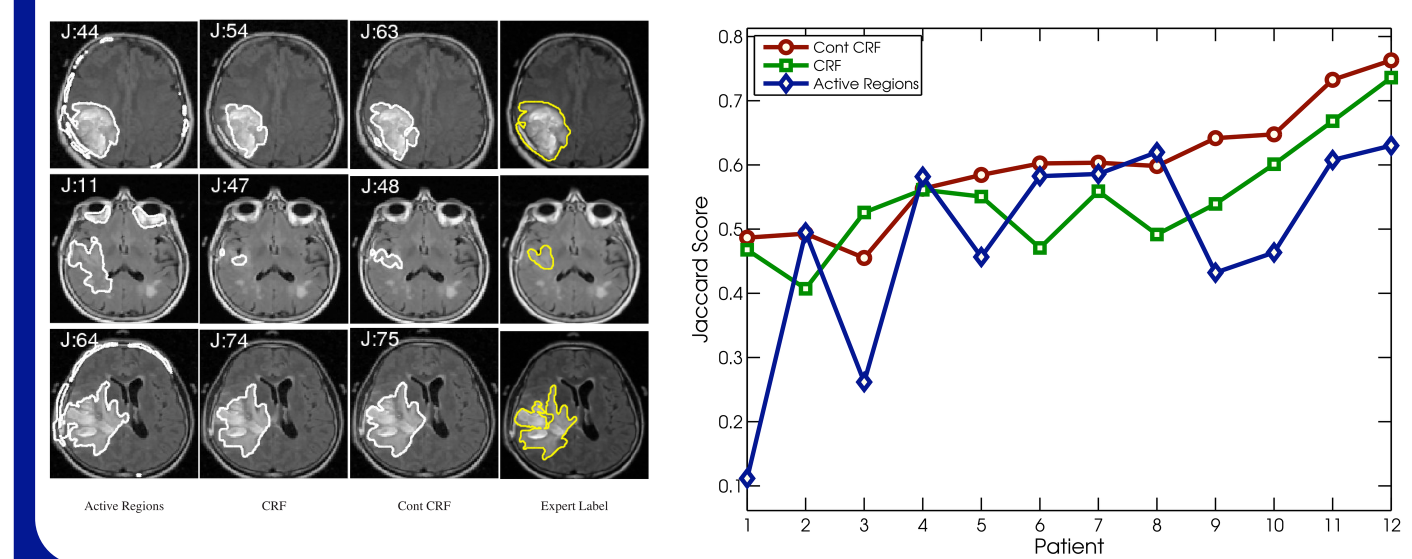
- We can add a non-Markov shape prior to the continuous CRF as an extra term in the energy:

$$E_s(\Phi) = \int_\Omega \beta H(\Phi)\left(s\Phi - \Phi_s(\mathcal{A}(x))\right)^2 dx$$

- $A(x)$ is an affine transformation with scale s of the shape prior level set Φ_s, and β is the shape regularization strength.

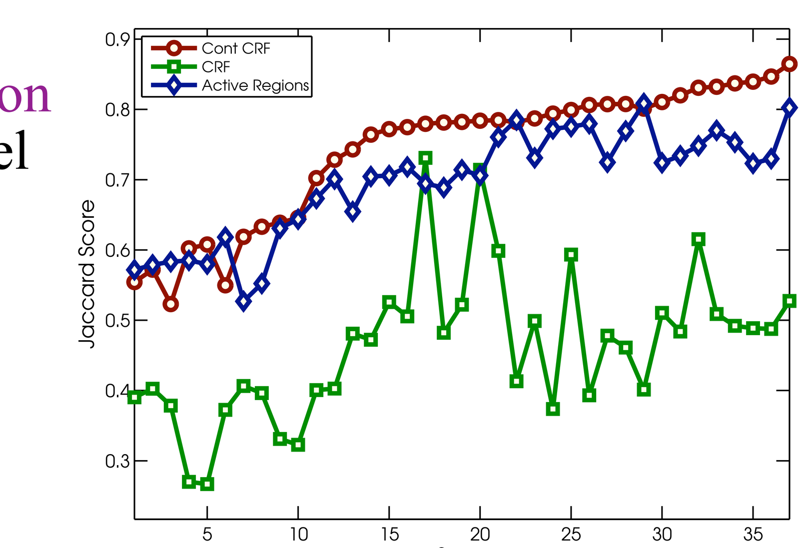
Brain Tumor Segmentation

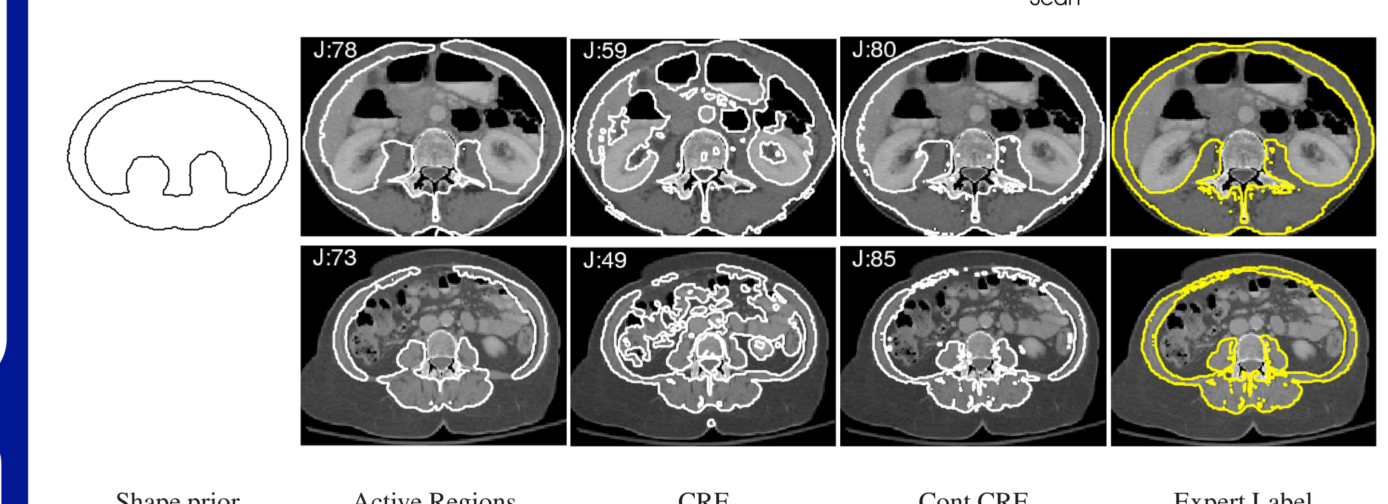
- Results on 3D MRI brain tumor segmentation data



Skeletal Muscle Segmentation

- Results on 2D CT muscle segmentation data where the level set methods use a shape prior





Discussion

- Unlike most work on level set methods, we require no manual initialization or parameter tuning, and do not need a generative model of the image.
- Other non-Markov terms can easily be added, such as the intensity inhomogeneity field.