Support Vector Random Fields

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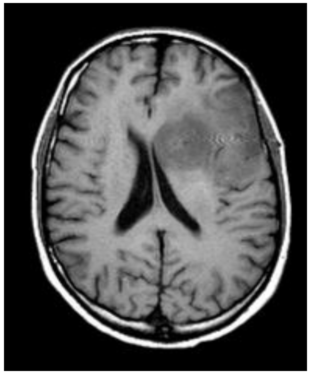
Overview

- Introduction
- Background
  - Markov Random Fields (MRFs)
  - Conditional Random Fields (CRFs) and Discriminative Random Fields (DRFs)
  - Support Vector Machines (SVMs)
- Support Vector Random Fields (SVRFs)
- Experiments
- Conclusion

Introduction

- Classification Tasks
  - Scalar Classification: class label depends only on features:
    - IID data
  - Sequential Classification: class label depends on features and 1D structure of data:
    - strings, sequences, language
  - **Spatial Classification**: class label depends on features and 2D+ structure of data:
    - images, volumes, video



Notation

- Through this presentation, we use
  - X: an Input (e.g. an Image with m by n elements)
  - Y: a joint labeling for the elements of X
  - S: a set of nodes (pixels)
  - x_i: an observation in node I
  - y_i: an class label in node I

Problem Formulation

- For an instance:
  - X = {x1,….,xn}
- Want the most likely labels:
  - Y = {y1,…,yn}
- Optimal Labeling if data is independent:
  - Y = {y1|x1,…,yn|xn} (Support Vector Machine)

- Labels in Spatial Data are NOT independent!
  - spatially adjacent labels are often the same (Markov Random Fields and Conditional Random Fields)
  - spatially adjacent elements that have similar features often receive the same label (Conditional Random Fields)
  - spatially adjacent elements that have different features may not have correlated labels (Conditional Random Fields)

Background:
Markov Random Fields (*MRFs*)

- Traditional technique to model spatial dependencies in the labels of neighboring element
- Typically uses a generative approach: model the joint probability of the features at elements $X = \{x_1, . . . , x_n\}$ and their corresponding labels $Y=\{y_1, . . . , y_n\}$: $P(X,Y)=P(X|Y)P(Y)$

- Main Issue:
  - Tractably calculating the joint requires major simplifying assumptions: (ie. $P(X|Y)$ is Gaussian and factorized as $\prod_i p(x_i|y_i)$, and $P(Y)$ is factored using H-C theorum).
  - Factorization makes **restrictive independence assumptions**, AND **does not allow modeling of complex dependencies between the features and the labels**

MRF vs. SVM

- MRFs model dependencies between:
  - the features of an element and its label
  - **the labels of adjacent elements**
- SVMs model dependencies between:
  - the features of an element and its label

Background:
Conditional Random Fields (*CRFs*)

- A CRF
  - A discriminative alternative to the traditionally generative MRFs
  - Discriminative models directly model the posterior probability of hidden variables given observations: P(Y|X)
    - No effort is required to model the prior. ☺
  - Improve the factorized form of a MRF by relaxing many of its major simplifying assumptions
  - Allows the tractable modeling of complex dependencies

MRF vs. CRF

- MRFs model dependencies between:
  - the features of an element and its label
  - the labels of adjacent elements
- CRFs model decencies between:
  - the features of an element and its label
  - the labels of adjacent elements
  - **the labels of adjacent elements and their features**

Background:
Discriminative Random Fields (DRFs)

- DRFs are a 2D extension of 1D CRFs:

$$P(Y \mid X) \propto \prod_{i \in S} A_i(y_i, X) \prod_{j \in N_i} I_{ij}(y_i, y_j, X)$$

- A_i models dependencies between X and the label at i (GLM vs. GMM in MRFs)
- I_{ij} models dependencies between X and the labels of i and j (GLM vs. counting in MRFs)
- Simultaneous parameter estimation as convex optimization
- Non-linear interactions using basis functions

Backgrounds: Graphical Models

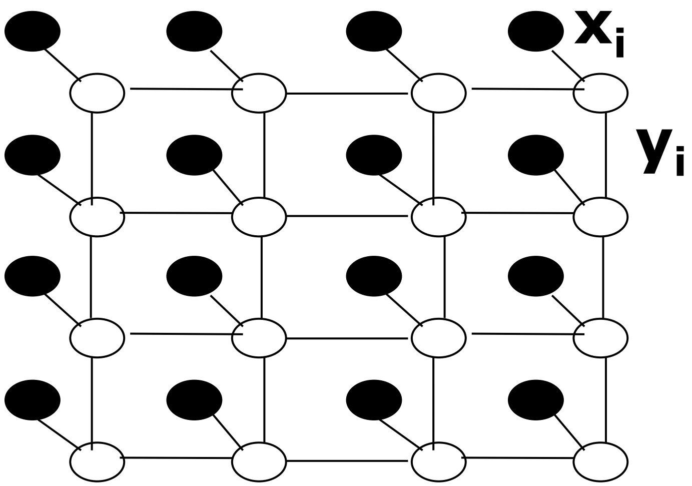


Fig. 1. A MRF. Shaded nodes (x_i) are the observation nodes (pixels) and unshaded nodes (y_i) are hidden variables (labels).

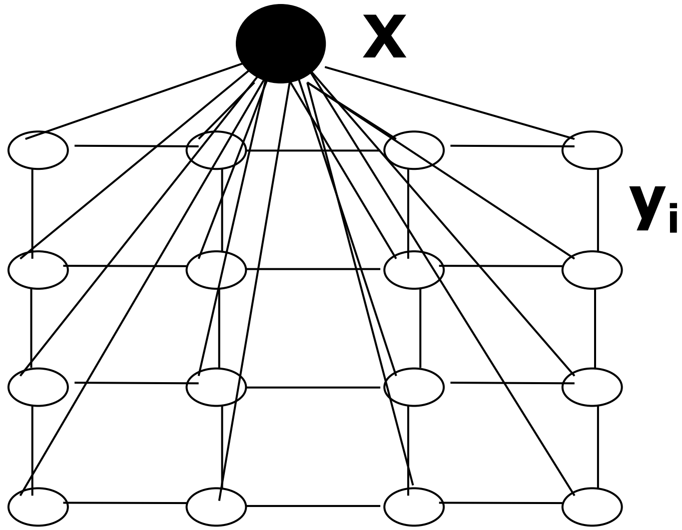


Fig. 2. Graphical structure of a DRF, the extension of a CRF in the 2-dim lattice structure

Background:
Discriminative Random Fields (DRFs)

- Issues
  - initialization
  - overestimation of neighborhood influence (edge degradation)
  - termination of inference algorithm (due to above problem)
  - GLM may not estimate appropriate parameters for:
    - high-dimensional feature spaces
    - highly correlated features
    - unbalanced class labels
  - Due to properties of error bounds, SVMs often estimate better parameters than GLMs
- Due to the above issues, ‘stupid’ SVMs can outperform ‘smart’ DRFs at some spatial classification tasks

Support Vector Random Fields

- We want:
  - the appealing generalization properties of SVMs
  - the ability to model different types of spatial dependencies of CRFs
- Solution:

  Support Vector Random Fields

Support Vector Random Fields: Formulation

$$P(Y \mid X) = \frac{1}{Z} \exp \left\{ \sum_{i \in S} \log(O(y_i, \Gamma_i(X))) + \sum_{i \in S} \sum_{j \in N_i} V(y_i, y_j, X) \right\}$$

- $\Gamma_i(X)$ is a function that computes features from the observations X for location i,
- *O(yi, i(X))* is an SVM-based Observation-Matching potential
- *V (yi, yj ,X)* is a (modified) DRF pairwise potential.

Support Vector Random Fields: Observation-Matching Potential

- SVMs decision functions produce a (signed) ‘distance to margin’ value, while CRFs require a strictly positive potential function
- Used a modified* version of [Platt, 2000] to convert the SVM decision function output to a positive probability value that satisfies positivity

- *Addresses minor numerical issues

Support Vector Random Fields: Local-Consistency Potential

- We adopted a DRF potential for modeling label-label-feature interactions:

  $$V(y_i, y_j, x) = y_i y_j (\eta \cdot \Phi_{ij}(x))$$

- Φ in DRFs is unbounded. In order to encourage continuity, we used

  $$\Phi_{ij} = (\max(T(x)) - |T_i(x) - T_j(x)|) / \max(T(X))$$

- Pseudolikelihood used to estimate η

Support Vector Random Fields: Sequential Training Strategy

1. Solve for Optimal SVM Parameters
   (Quadratic Programming)
2. Convert SVM Decision Function to Posterior Probability
   (Newton w/ Backtracking)
3. Compute Pseudolikelihood with SVM Posterior fixed
   (Gradient Descent)

- Bottleneck for low dimensions: Quadratic Programming
- Note: Sequential Strategy removes the need for expensive CV to find appropriate L2 penalty in pseudolikelihood

Support Vector Random Fields: Inference

1. Classify all pixels using posterior estimated from SVM decision function
2. Iteratively update classification using pseudolikelihood parameters and SVM posterior (Iterated Condition Modes)

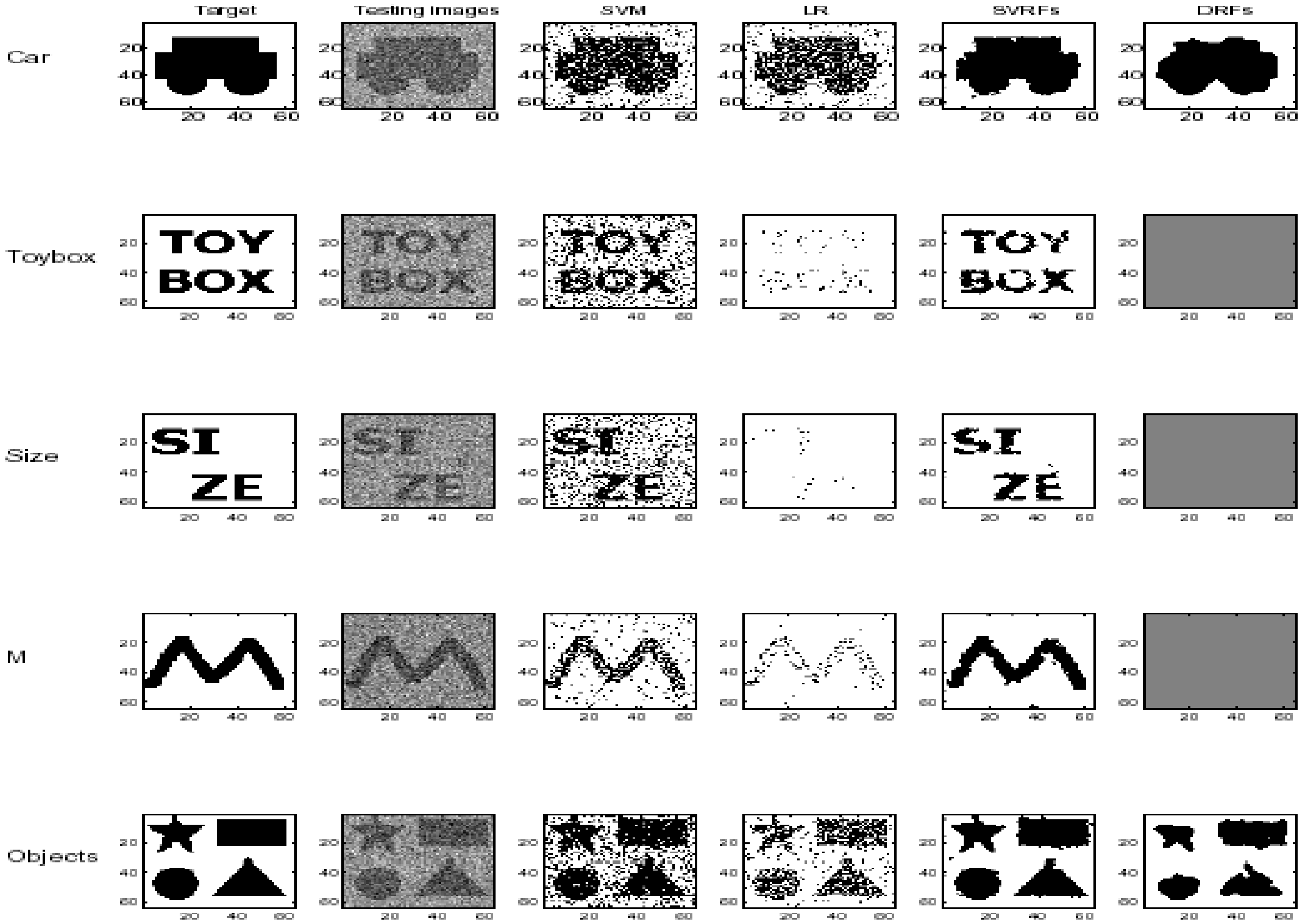
SVRF vs. AMN

- Associative Markov Network:
  - another strategy to model spatial dependencies using Max Margin approach
- Main Difference?
  - SVRF: use ‘traditional’ maximum margin hyperplane between classes in feature space
  - AMN: multi-class maximum margin strategy that seeks to maximize margin between best model and runner-up
- Quantitative Comparison:
  - Stay tuned...

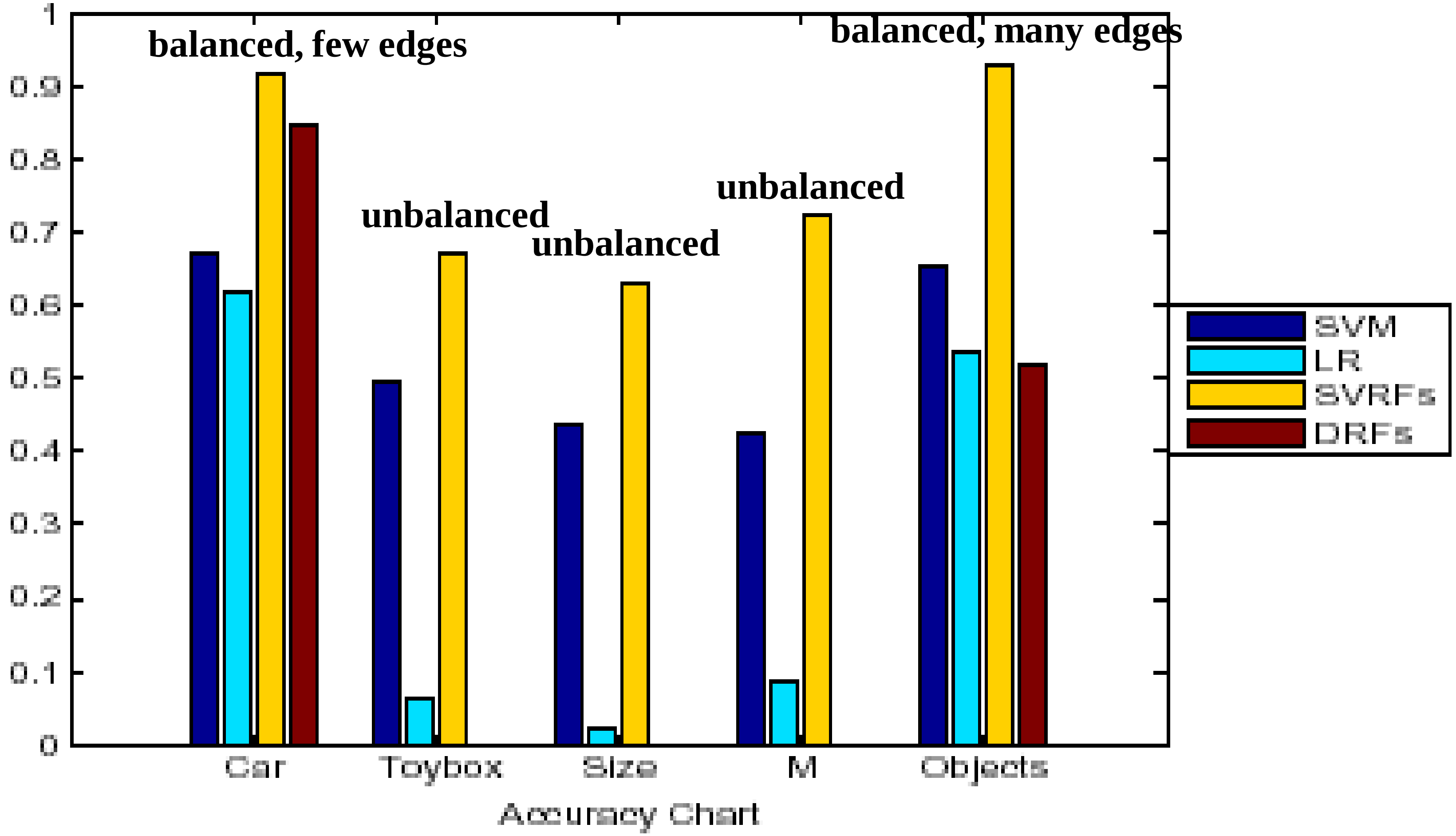
Experiments: Synthetic

- Toy problems:
  - 5 toy problems
  - 100 training images
  - 50 test images
- 3 unbalanced data sets: Toybox, Size, M
- 2 balanced data sets: Car Objects

Experiments: Synthetic



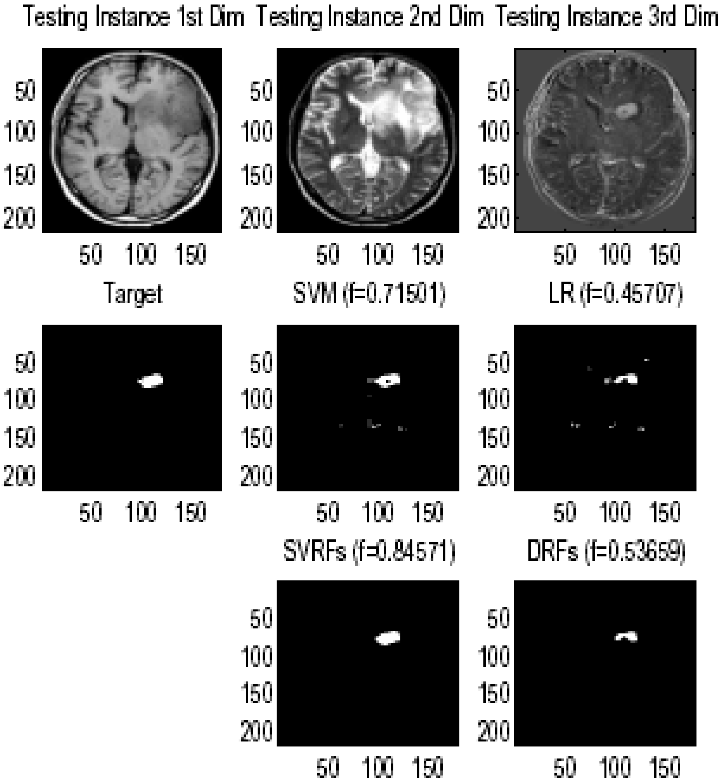
Experiments: Synthetic

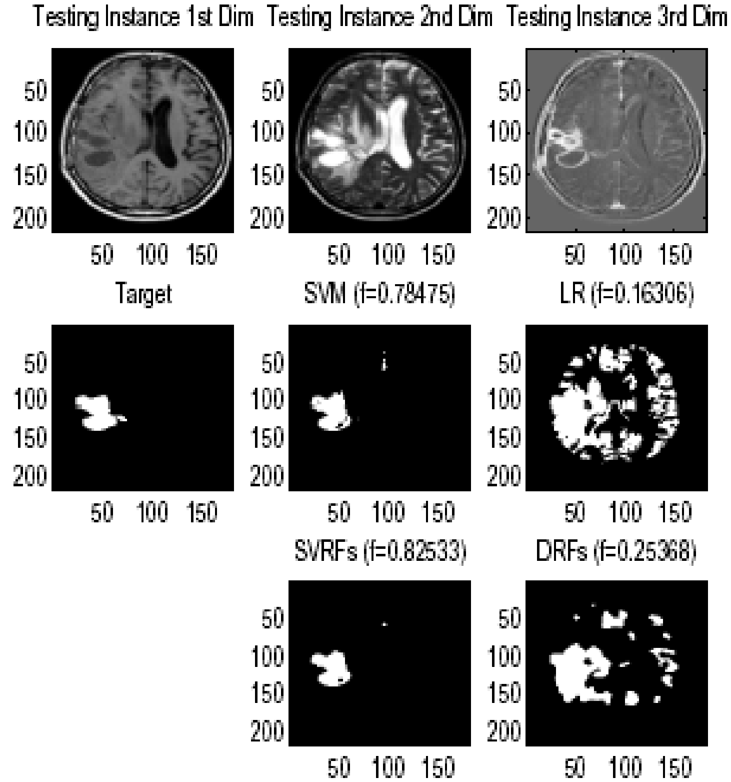


Experiments: Real Data

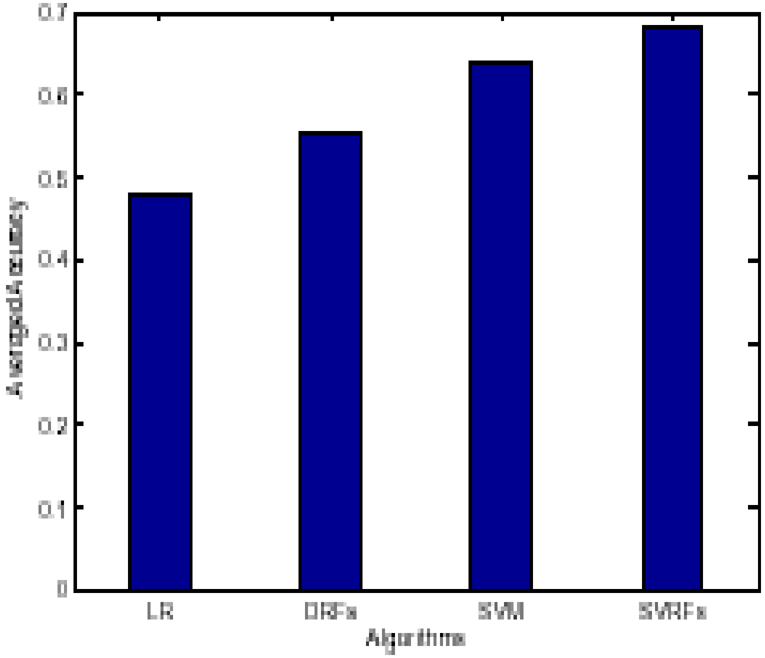
- Real problem:
  - Enhancing brain tumor segmentation in MRI
  - 7 Patients
  - Intensity inhomogeneity reduction done as preprocessing
  - Patient-Specific training: Training and testing are from different slices of the same patient (different areas)
  - ~40000 training pixels/patient
  - ~20000 test pixels/patient
  - 48 features/pixel

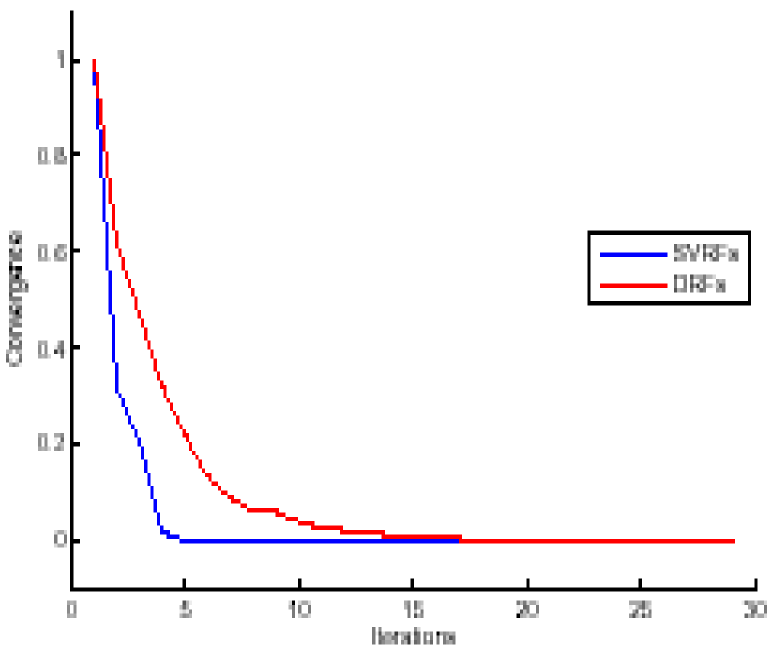
Experiment: Real problem





Experiment: Real problem





(a) Accuracy: Jaccard score TP/(TP+FP+FN)

(b) Convergence for SVRFs and DRFs

Conclusions

- Proposed SVRFs, a method to extend SVMs to model spatial dependencies within a CRF framework
- Practical technique for structured domains for d >= 2
- Did I mention kernels and sparsity?
- The end of (SVM-based) ‘pixel classifiers’?

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