CPSC 540: Machine Learning

Conditional Random Fields, Latent Dynamics

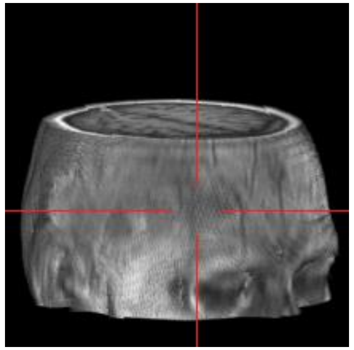
Winter 2016

Admin

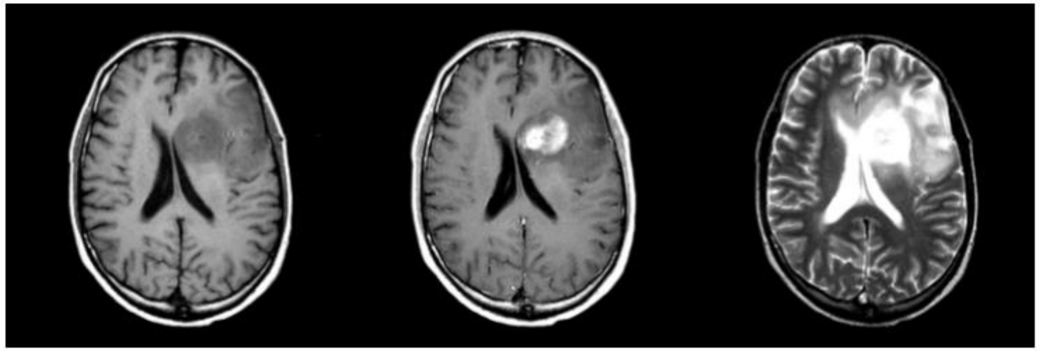
- Assignment 5:
  - Due in 1 week.
- Project:
  - Due date moved again to April 26 (so that undergrads can graduate).
  - Graduate students graduating in May must submit by April 21.
- No tutorial Friday (or in subsequent weeks).
- Final help session Monday.
- Thursday class may go long.

Motivation: Automatic Brain Tumor Segmentation

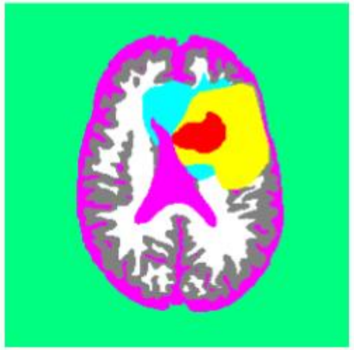
- Task: segmentation tumors and normal tissue in multi-modal MRI data.



Input:



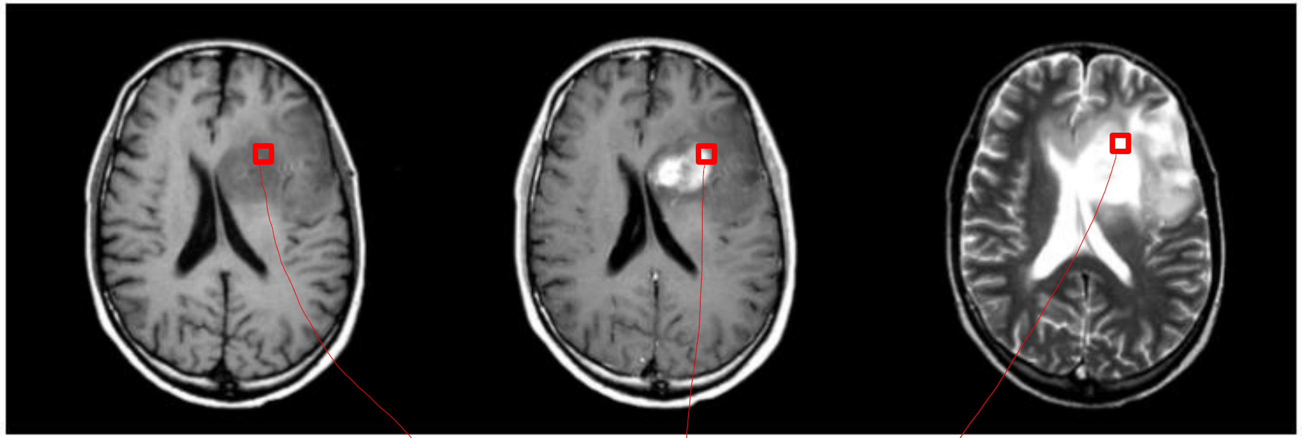
Output:



- Applications:
  - Radiation therapy target planning, quantifying treatment responses.
  - Mining growth patterns, image-guided surgery.
- Challenges:
  - Variety of tumor appearances, similarity to normal tissue.
  - "You are never going to solve this problem."

Naïve Approach: Voxel-Level Classifier

- We could treat classifying a voxel as supervised learning:

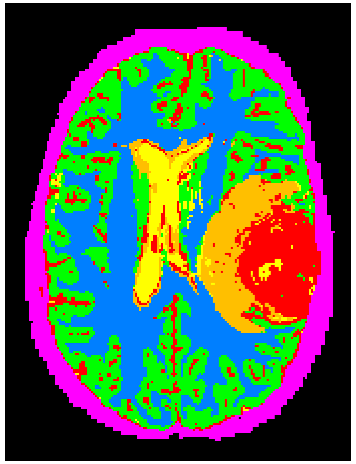


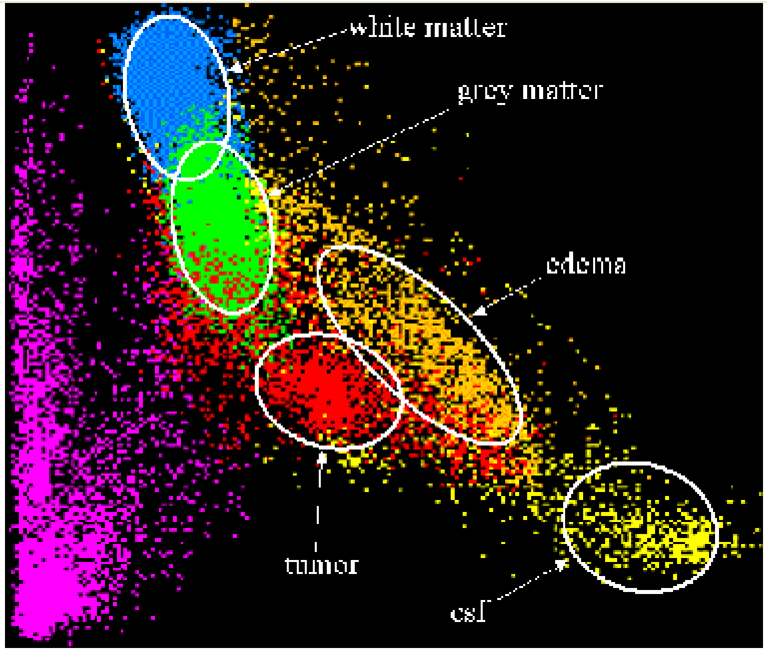
$x^i = <98, 187, 246>$ $\quad$ $y^i =$ "tumor"

- "Learn" model that predicts y^i given x^i: model can classify new voxels.
- Advantage: we can apply machine learning, and ML is cool.
- Disadvantage: it doesn't work at all.

Naïve Approach: Voxel-Level Classifier

- Even in "nice" cases, significant overlap between tissues:
  – Mixture of Gaussians and "outlier" class:



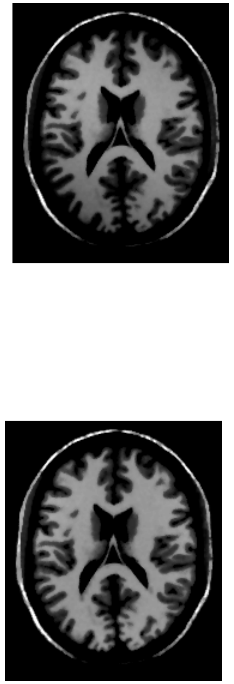


- Problems with naïve approach:
  – Intensities not standardized.
  – Location and texture matter.

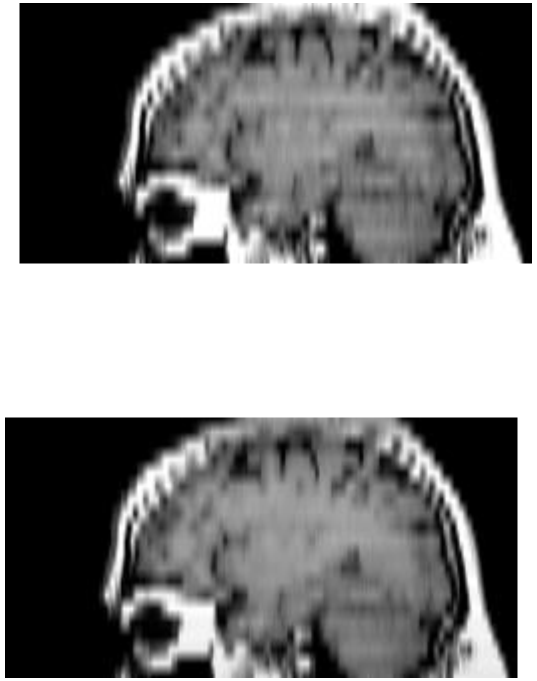
Improvement 1: Intensity Standardization

- Want x^i = <98,187,246> to mean same thing in different places.
- Pre-processing to normalize intensities:

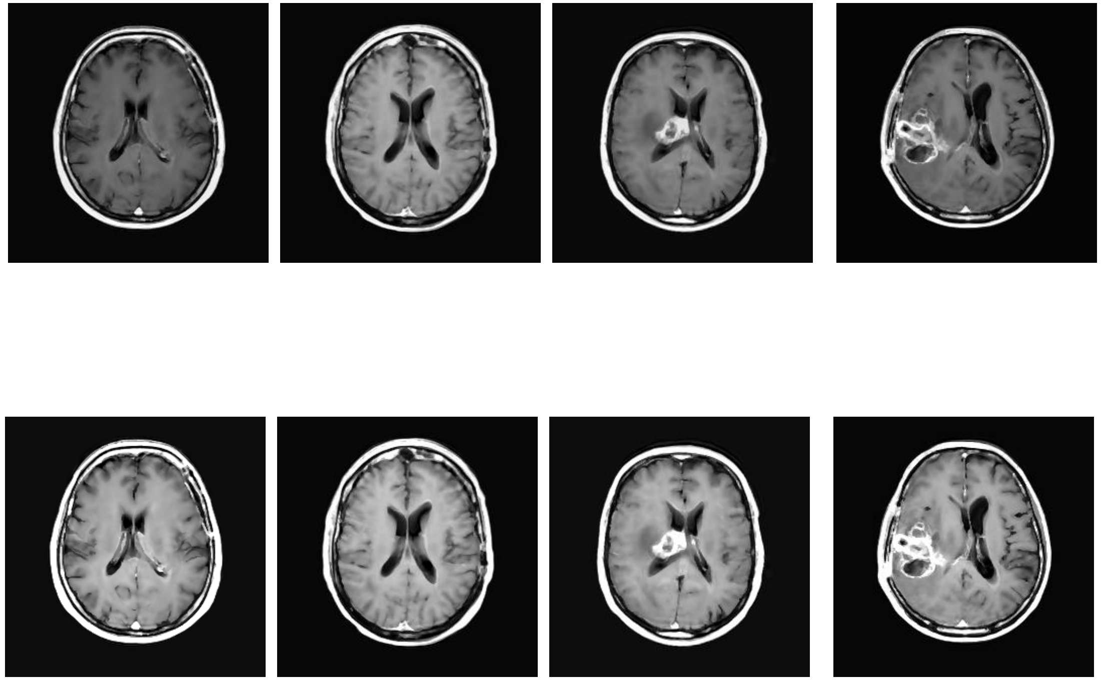
Within Images:



Between slices:

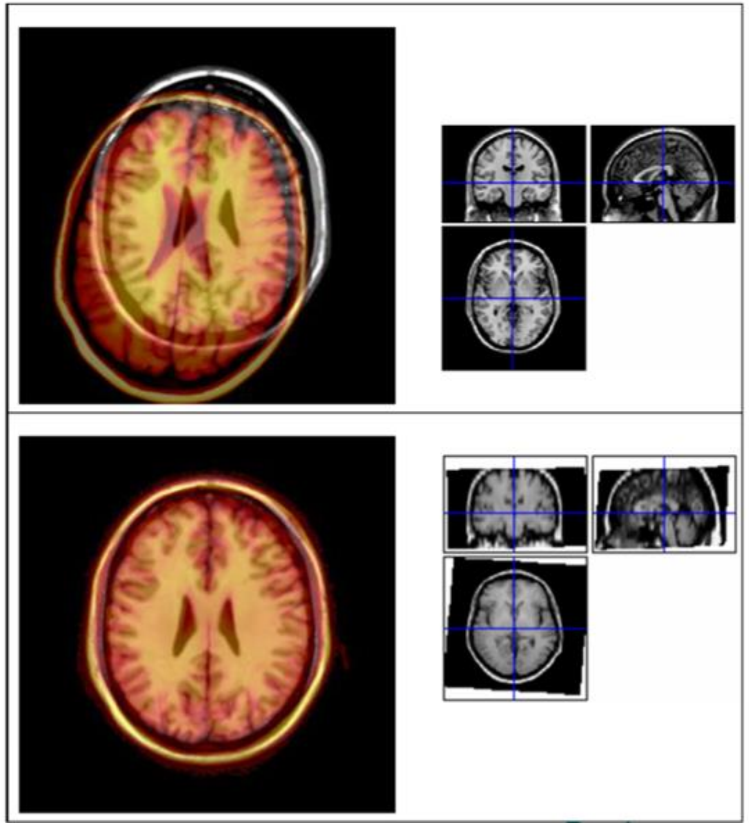


Between people:



Improvement 2: Template Alignment

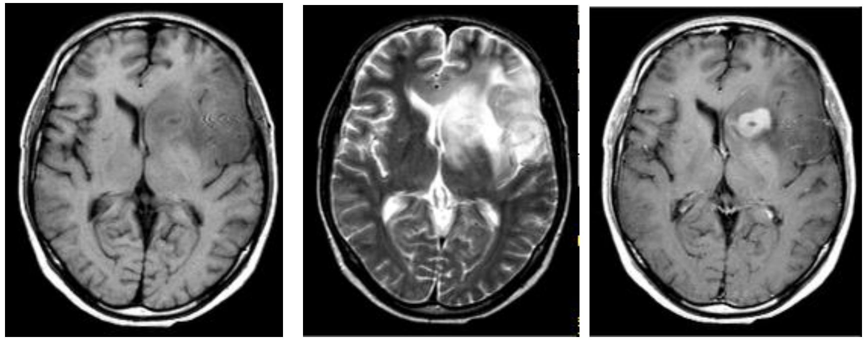
- Location matters:
  - Seeing xi =<98,187,246> in one area of head is different than in other areas.
- Alignment to standard coordinates system:



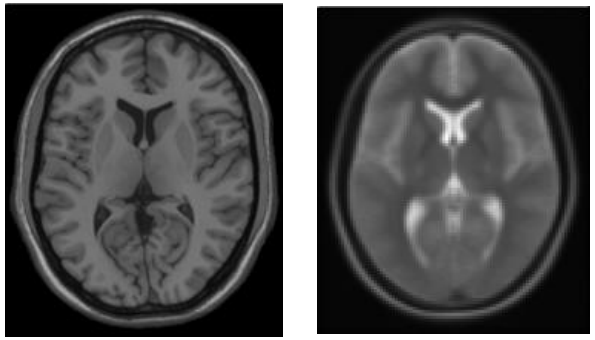
Improvement 2: Template Alignment

- Add spatial features that take into account location information:

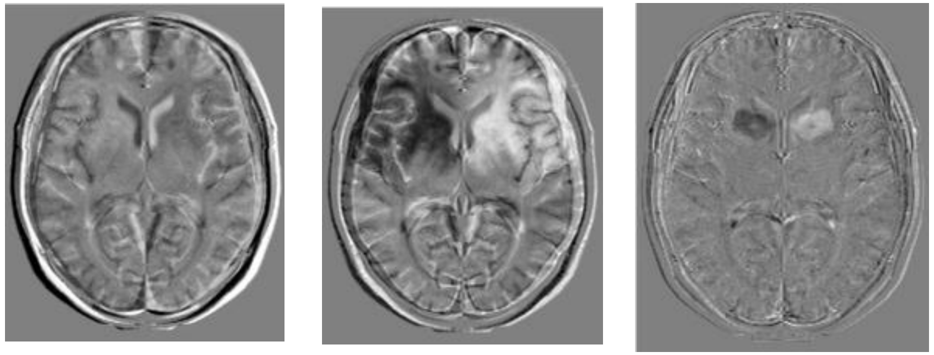
Aligned input images:



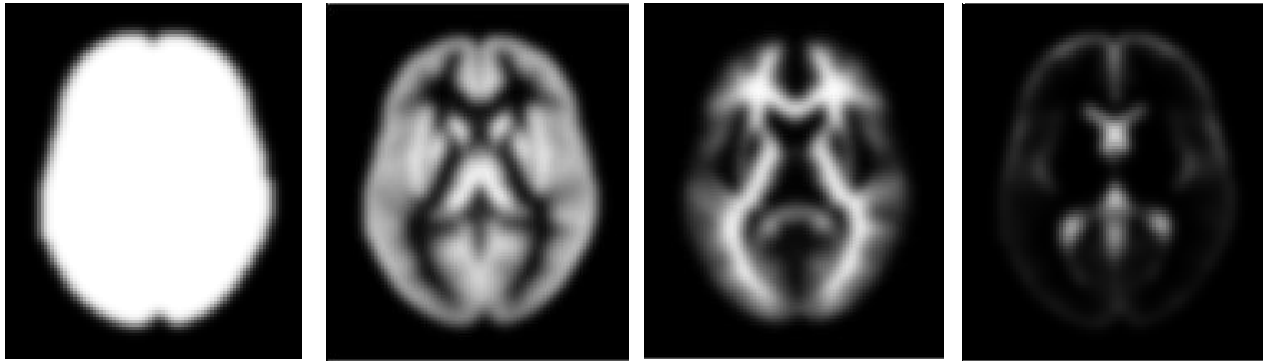
Template images:



Bilateral symmetry based on known axis:

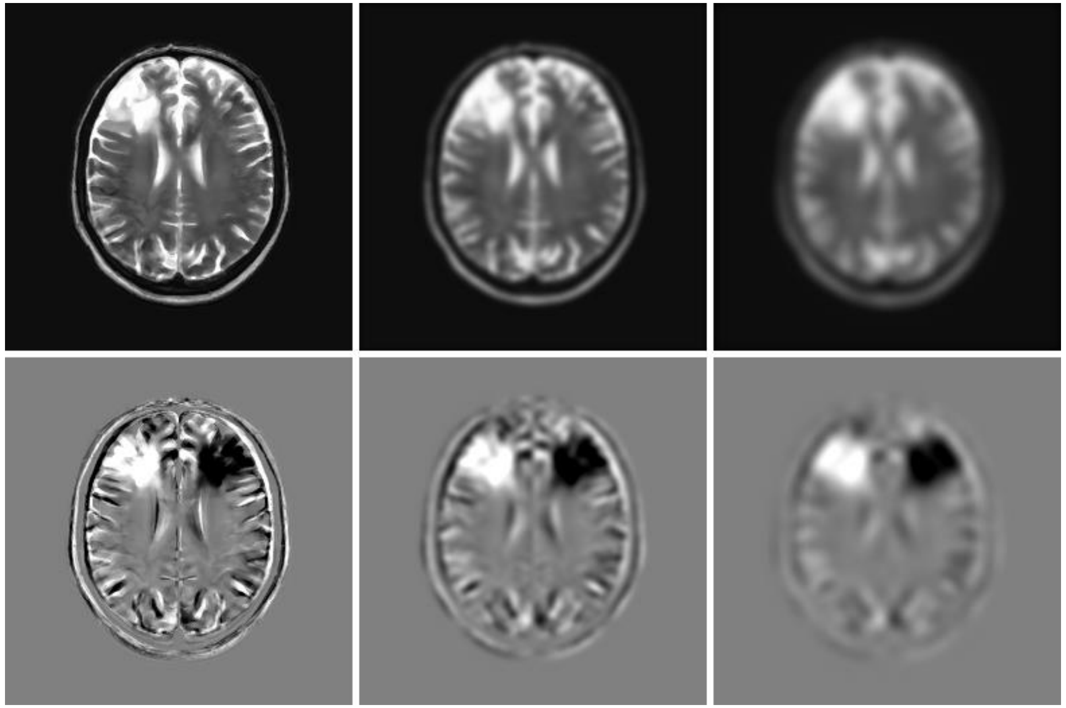


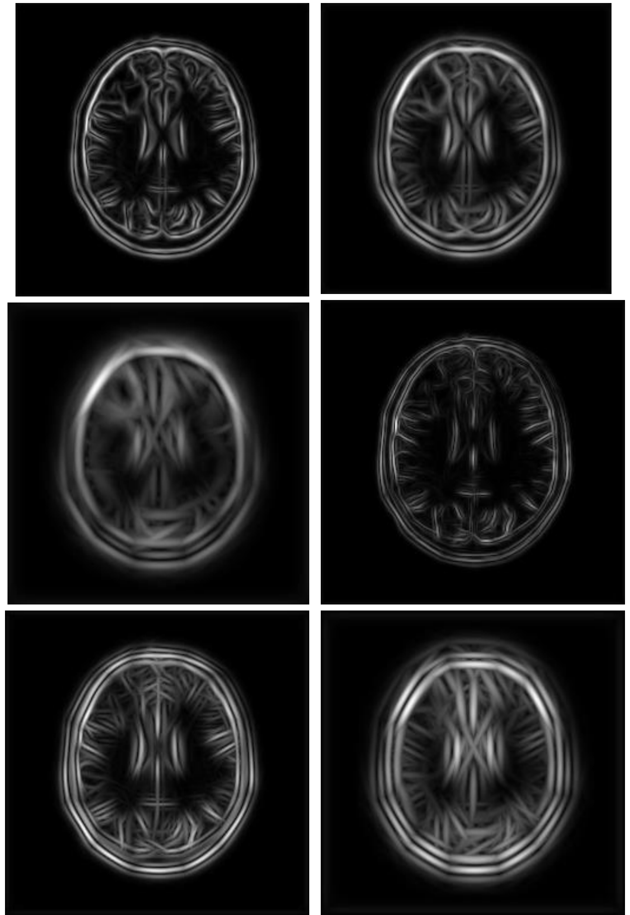
Priors for normal tissue locations:

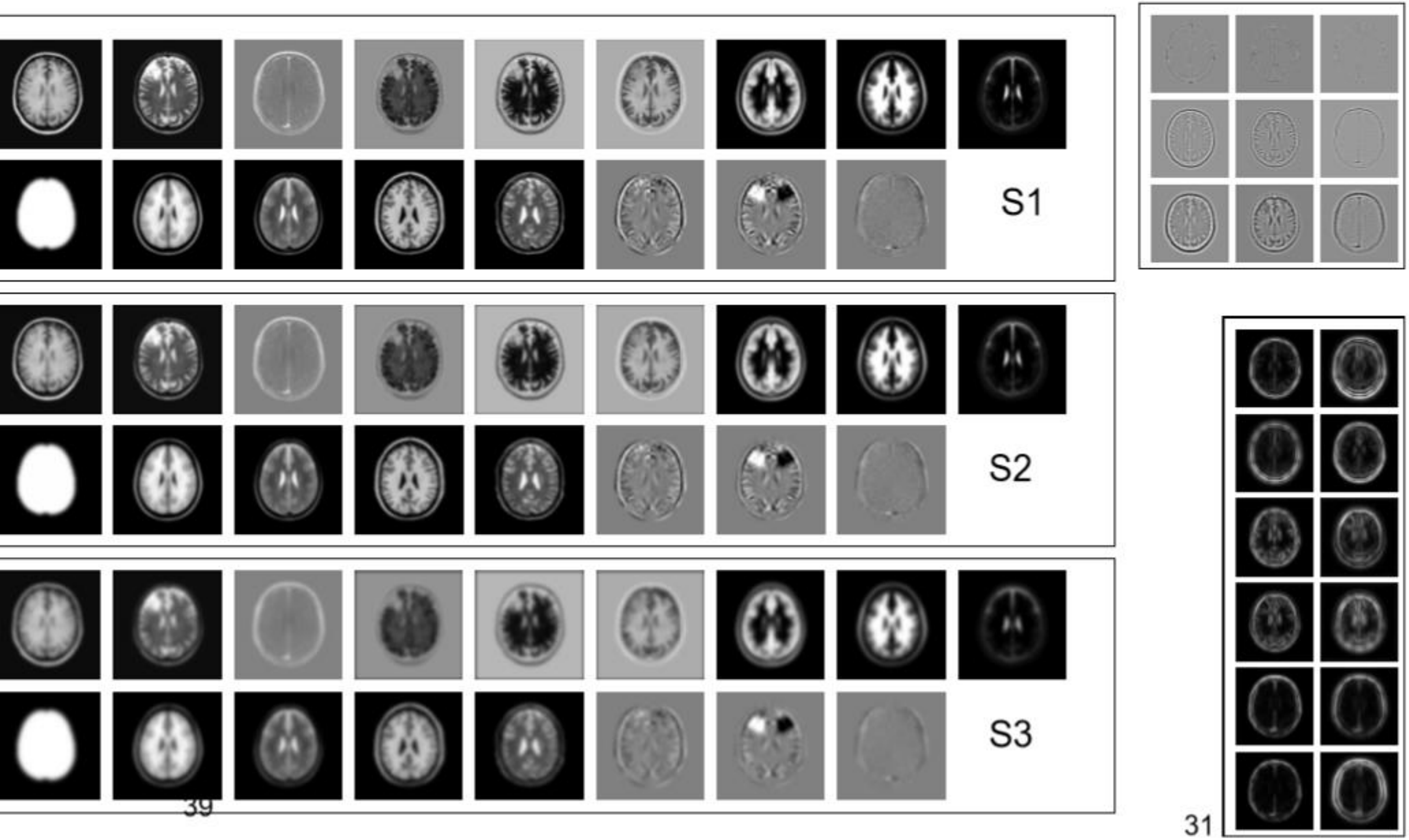


Improvement 3: Convolutions

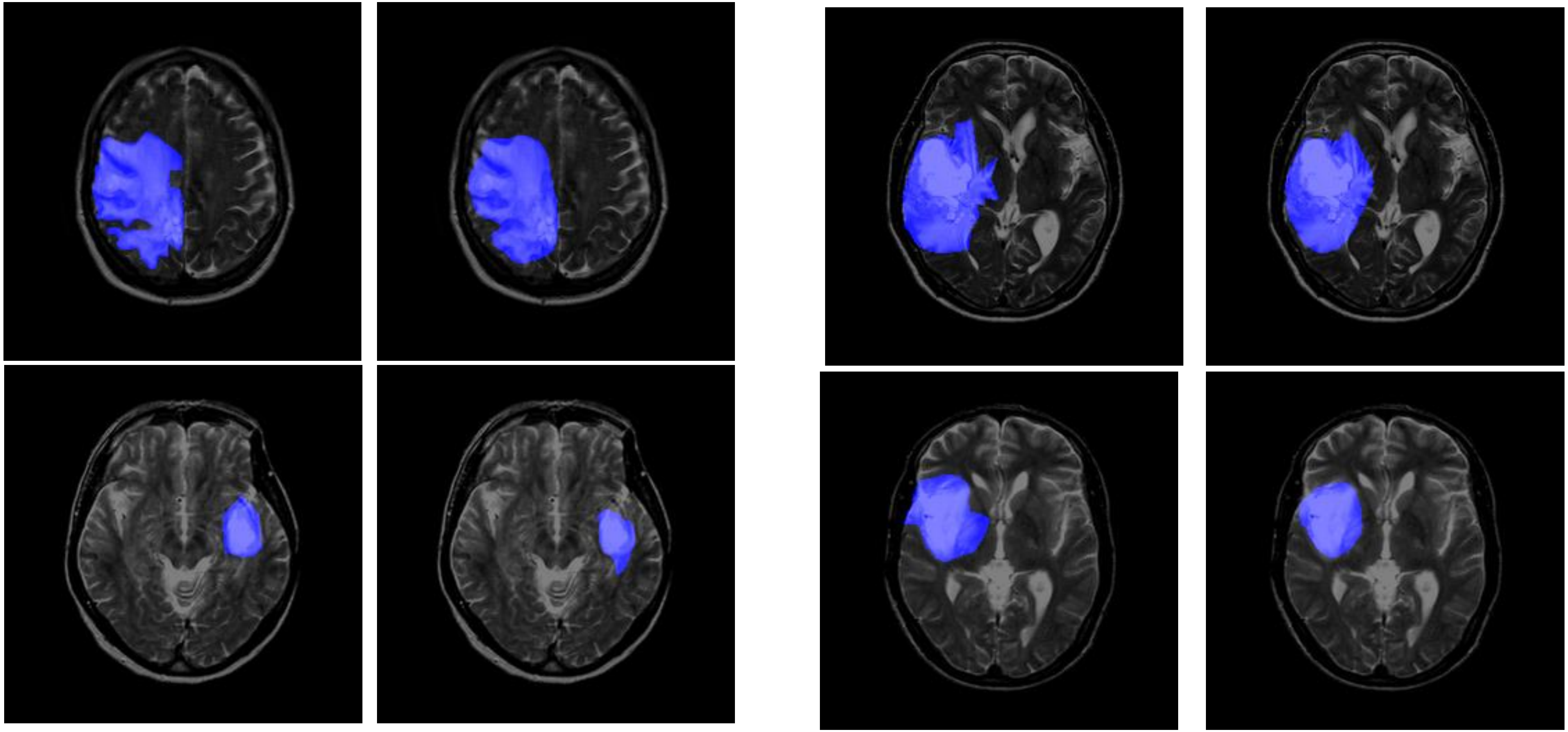
- Use convolutions to incorporate neighborhood information.
  - We used fixed convolutions, now you would try to learn them.





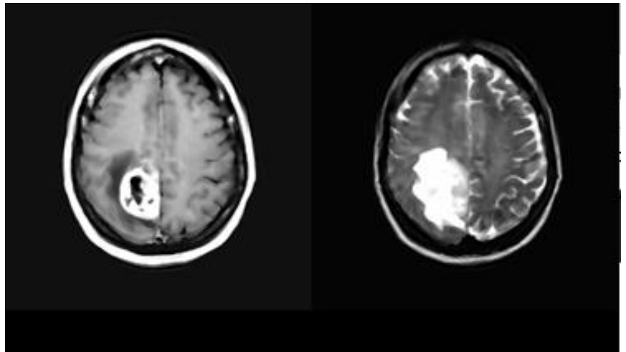

S1
S2
S3
39
31

Performance of Final System



Challenges

- Final system used linear classifer, and typically worked well.
- But several ML challenges arose:
  1. Time: 14 hours to train logistic regression on 10 images.
     - Lead to quasi-Newton, stochastic gradient, and SAG work.
  2. Overfitting: using all features hurt, so we used manual feature selection.
     - Lead to regularization, L1-regularization, and structured sparsity work.
  3. Relaxation: post-processing by filtering and `hole-filling of labels.
     - Lead to conditional random fields, shape priors, and structure learning work.



Outline

- Motivation
- Conditional Random Fields Clean Up
- Latent/Deep Graphical Models

Multi-Class Logistic Regression: View 1

- Recall that multi-class logistic regression makes decisions using:

$$\hat{y} = \underset{y \in \{1,2,\dots,k\}}{\operatorname{argmax}} \; w_y^T f(x)$$

- Here, f(x) are features and we have a vector w_y for each class 'y'.
- Normally fit w_y using regularized maximum likelihood assuming:

$$p(y \mid x, w_1, w_2, \dots, w_k) \propto \exp(w_y^T f(x))$$

- This softmax function yields a differentiable and convex NLL.

Multi-Class Logistic Regression: View 2

- Recall that multi-class logistic regression makes decisions using:

$$\hat{y} = \underset{y \in \{1,2,\dots,k\}}{\operatorname{argmax}} \; w_y^T f(x)$$

- Claim: can be written using a single 'w' and features of 'x' and 'y'.

$$\hat{y} = \underset{y \in \{1,2,\dots,k\}}{\operatorname{argmax}} \; w^T f(x,y)$$

- To do this, we can use the construction:

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_K \end{bmatrix} \quad f(x,1) = \begin{bmatrix} f(x) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad f(x,2) = \begin{bmatrix} 0 \\ f(x) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \text{Implies that } w^T f(x,y) = w_y^T f(x)$$

Multi-Class Logistic Regression: View 2

- So multi-class logistic regression with new notation uses:

$$\hat{y} = \underset{y \in \{1,2,\dots,k\}}{\operatorname{argmax}} \; w^T f(x,y)$$

- And softmax probabilities gives:

$$p(y \mid x, w) = \frac{\exp(w^T f(x,y))}{\sum_{y'} \exp(w^T f(x,y'))} \propto \exp(w^T f(x,y))$$

- View 2 gives extra flexibility in defining features:
  - For example, we might have different features for class 1 than 2:

$$f(x,1) = \begin{bmatrix} f(x) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad f(x,2) = \begin{bmatrix} 0 \\ g(x) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

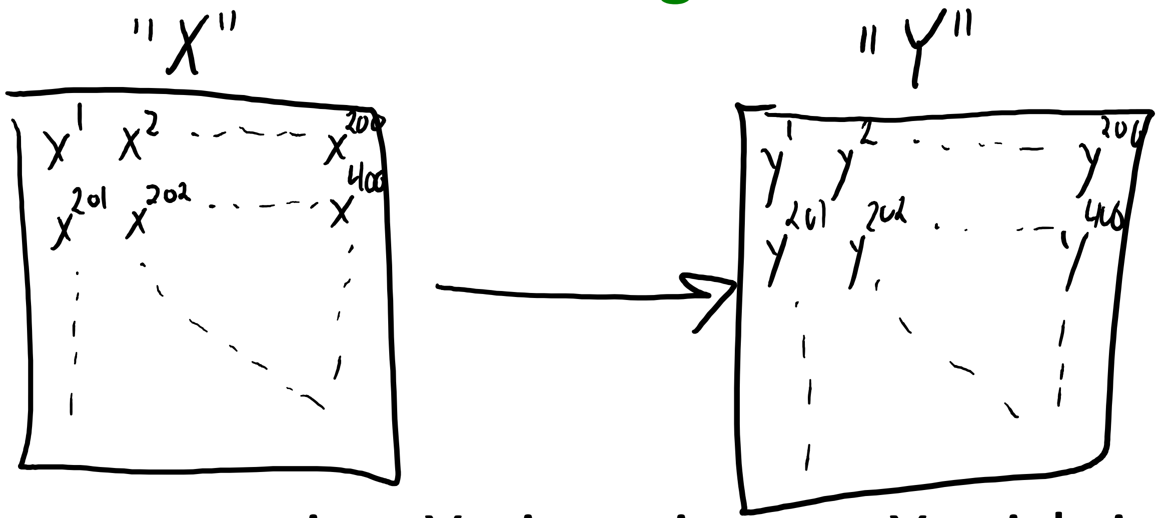
We can even do crazy stuff like: $f(x,3) = \begin{bmatrix} f(x) \\ g(x) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

Multi-Class Logistic Regression for Segmentation

- In brain tumor example, each x^i is the features for one voxel:

Softmax model gives $p(y^i \mid x^i, w)$ for any label y^i of voxel 'i'.

- But we want to label the whole image:

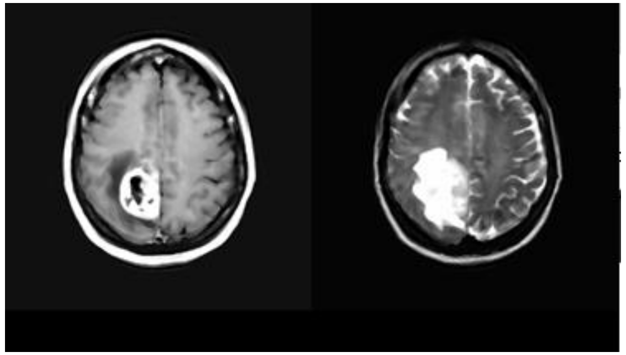


- Probability of segmentation Y given image X with independent model:

$$p(Y \mid X, w) = \prod_{i=1}^{n} p(y^i \mid x^i, w).$$

Conditional Random Fields

- Unfortunately, independent model gives silly results:



- This model of $p(Y|X,w)$ misses the “guilt by association”:
  - Neighbouring voxels are likely to receive the same values.
- The key ingredients of conditional random fields (CRFs):
  - Define features of entire image and labelling $F(X,Y)$:
  - We can model dependencies using features that depend on multiple y^i.

Conditional Random Fields

- Interpretation of independent model as CRF:

$$p(Y \mid X, w) = \prod_{i=1}^{n} p(y^i \mid x^i, w) \propto \prod_{i=1}^{n} \exp(w^T f(x^i, y^i))$$

$$= \exp\left(\sum_{i=1}^{n} w^T f(x^i, y^i)\right)$$

$$= \exp(W^T F(X, Y))$$

$$W = \begin{bmatrix} w \\ w \\ w \\ \vdots \\ w \end{bmatrix} \qquad F(X,Y) = \begin{bmatrix} f(x^1, y^1) \\ f(x^2, y^2) \\ f(x^3, y^3) \\ \vdots \\ f(x^n, y^n) \end{bmatrix}$$

(Using same 'w' for all 'i' is called <u>parameter tieing</u>)

Conditional Random Fields

- Example of modeling dependencies between neighbours as a CRF:

Take $p(Y \mid X, W) \propto \exp(W^T F(X,Y))$

$$W = \begin{bmatrix} w \\ w \\ w \\ \vdots \\ w \\ v \\ v \\ \vdots \\ v \end{bmatrix} \qquad F(X,Y) = \begin{bmatrix} f(X, y^1) \\ f(X, y^2) \\ f(X, y^3) \\ \vdots \\ f(X, y^n) \\ f(X, y^1, y^2) \\ f(X, y^2, y^3) \\ \vdots \\ f(X, y^{n-1}, y^n) \end{bmatrix}$$

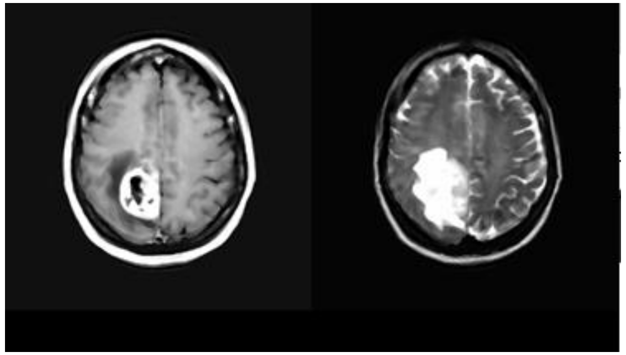
We always condition on X, so y^i features can depend on any part of X.

Features of dependency between y^1 and y^2.

Usually we'll use different parameters for dependency features.

Conditional Random Fields for Segmentation

- Recall the performance with the independent classifier:
  - Features of the form $f(X,y^i)$.



- Consider a CRF that also has pairwise features:
  - Features $f(X,y^i,y^j)$ for all (i,j) corresponding to adjacent voxels.
  - Model "guilt by association":

Conditional Random Fields as Graphical Models

- Seems great: we can now model dependencies in the labels.
  - Why not model threeway interactions with $F(X, y^i, y^j, y^k)$?
  - How about adding things like shape priors $F(X, Y_r)$ for some region 'r'?
- Challenge is that inference and decoding can become hard.
- We can view CRFs as undirected graphical models:

$$p(Y \mid X, w) \propto \prod_{c \in C} \phi_c(Y_c)$$

where we have a potential $\phi_c(Y_c)$ if Y_c appear together in one or more features in $F(X, Y_c)$

- If the graph is too complicated (and we don't have special 'F'):
  - Intractable since we need inference (computing Z/marginals) for training.

Overview of Exact Methods for Graphical Models

- We can do exact decoding/inference/sampling for:
  - Small number of variables (enumeration).
  - Chains (Viterbi, forward-backward, forward-filter backward-sample).
  - Trees (belief propagation).
  - Low treewidth graphs (junction tree).

Variations on variable elimination that let you compute all marginals.

Also known as message-passing algorithms.

- Other cases where exact computation is possible:
  - Semi-Markov chains (allow dependence on time you spend in a state).
  - Context-free grammars (allows potentials on recursively-nested parts of sequence).
  - Binary 'k' and "attractive" potentials (exact decoding via graph cuts).
  - Sum-product networks (restrict potentials to allow exact computation).

Overview of Approximate Methods for Graphical Models

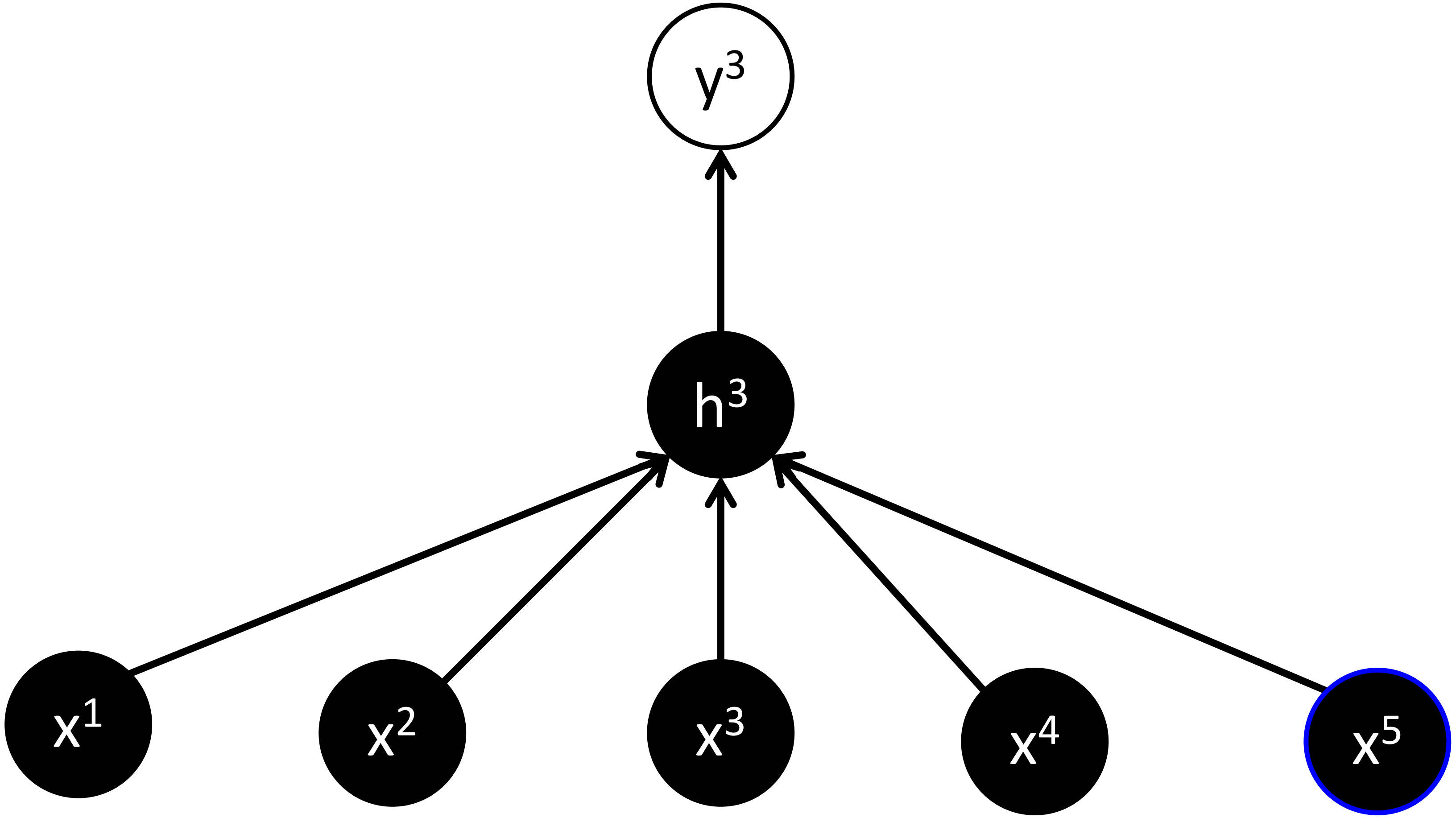
- Approximate decoding with local search:
  - Coordinate descent is called iterated conditional mode (ICM).
- Approximate sampling with MCMC:
  - We saw Gibbs sampling last week.
- Approximate inference with variational methods:
  - Mean field, loopy belief propagation, tree-reweighted belief propagation.
- Approximate decoding with convex relaxations:
  - Linear programming approximation.
- Block versions of all of the above:
  - Variant is alpha-expansions: block moves involving classes.

Overview of Methods for Fitting Graphical Models

- If inference is intractable, there are some alternatives for learning:
  - Variational inference to approximate Z and marginals.
  - Pseudo-likelihood: fast and cheap convex approximation for learning.
  - Structured SVMs: generalization of SVMs that only requires decoding.
  - Younes: alternate between Gibbs sampling and parameter update.
    - Also known as “persistent contrastive divergence”.

- For more details on graphical models, see:
  - UGM software: http://www.cs.ubc.ca/~schmidtm/Software/UGM.html
  - MLRG PGM crash course: http://www.cs.ubc.ca/labs/lci/mlrg

Independent Logistic Regression



Labels $\left(\begin{matrix} w^T h_3(X) \\ \text{or} \\ w^T F(X) \end{matrix}\right)$

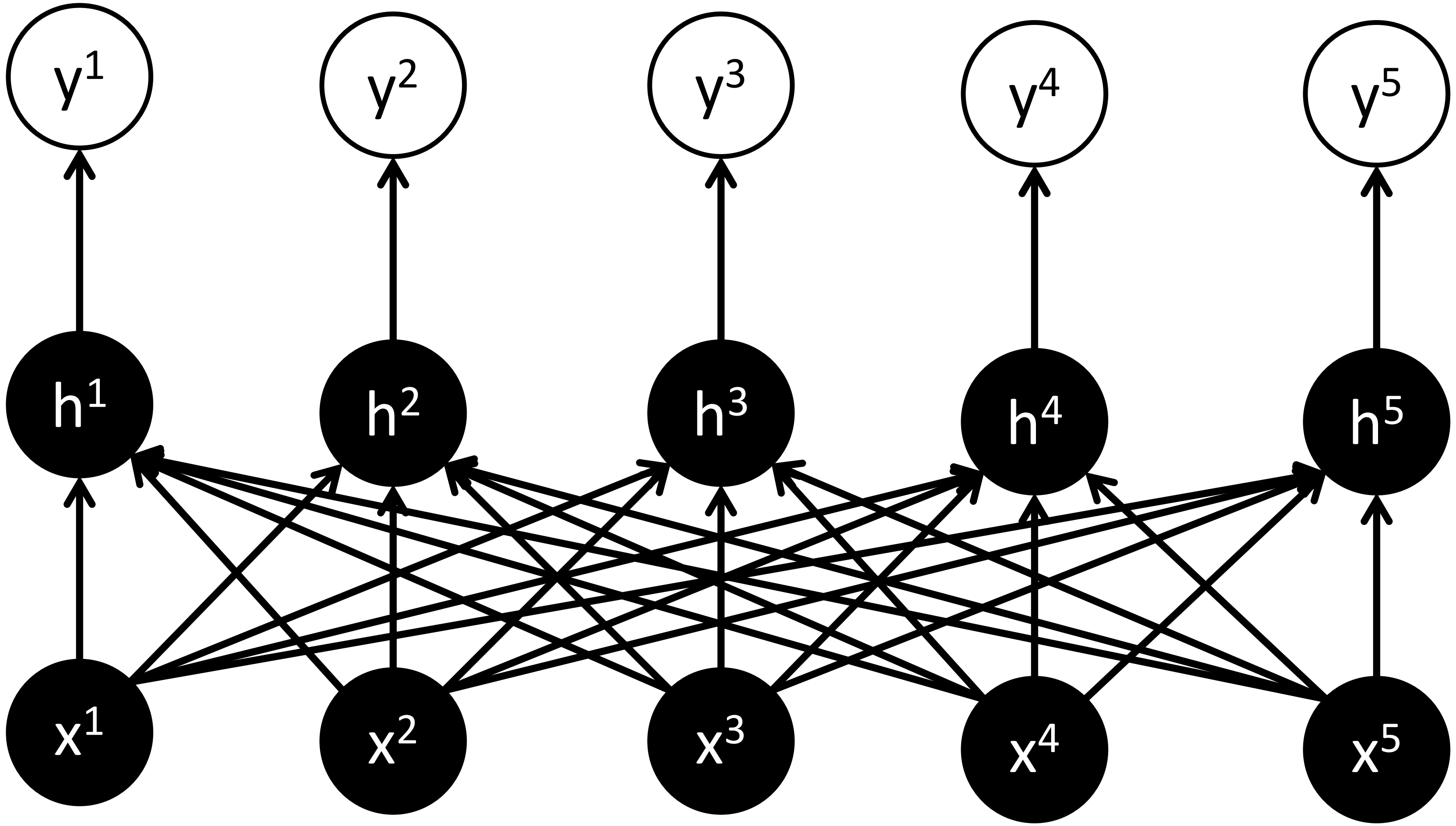
↑

Fixed features $\left(\begin{matrix} \text{Convolutions:} \\ w^T X \end{matrix}\right)$

↑

Input

Independent Logistic Regression

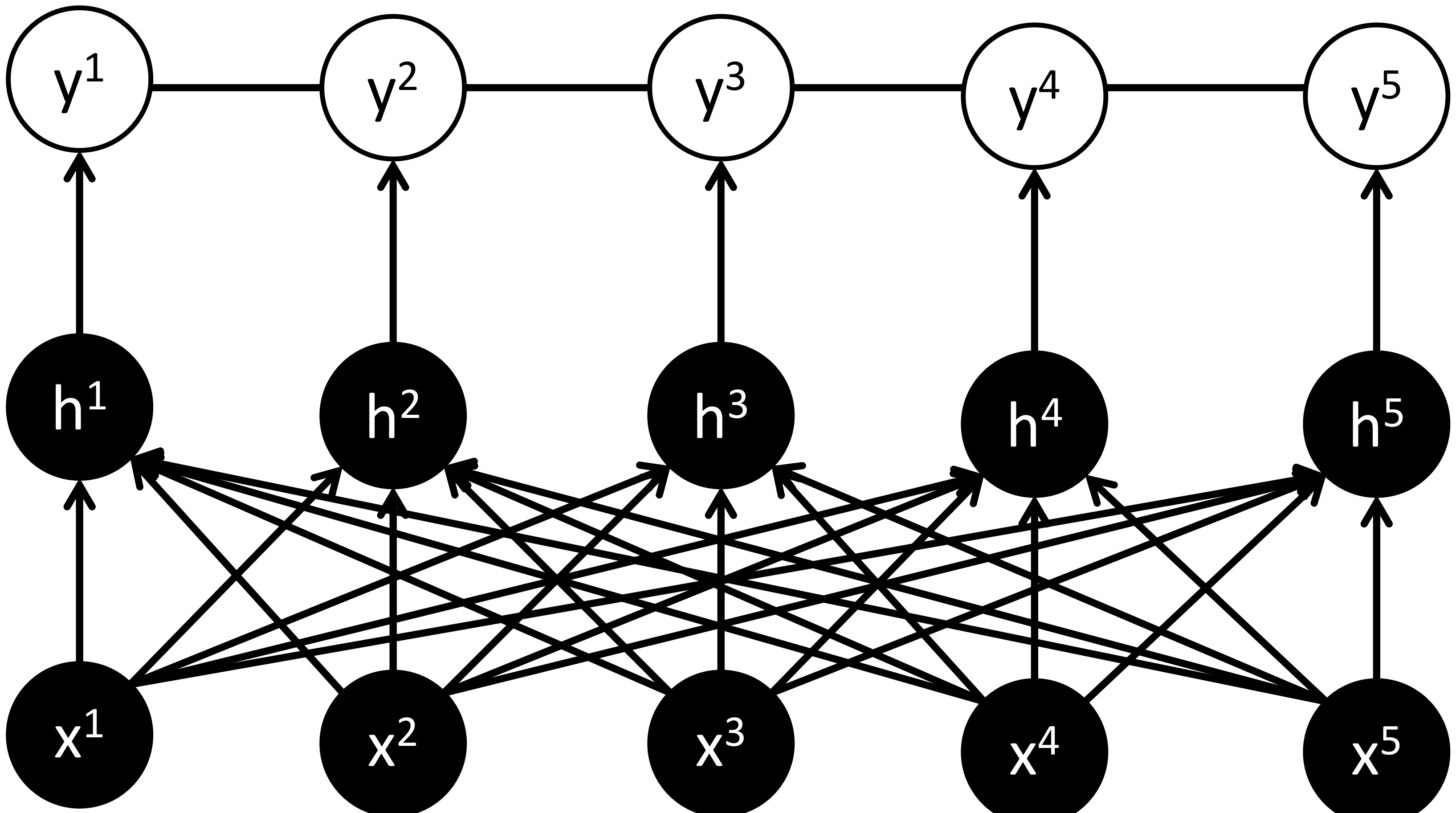


The y^i are treated as random variables, the x^i and h^i are deterministi

We are linear combinations of linear combinations so the h don't increase expressive power of model.

But since h are convolutions they might be doing some compression of input.

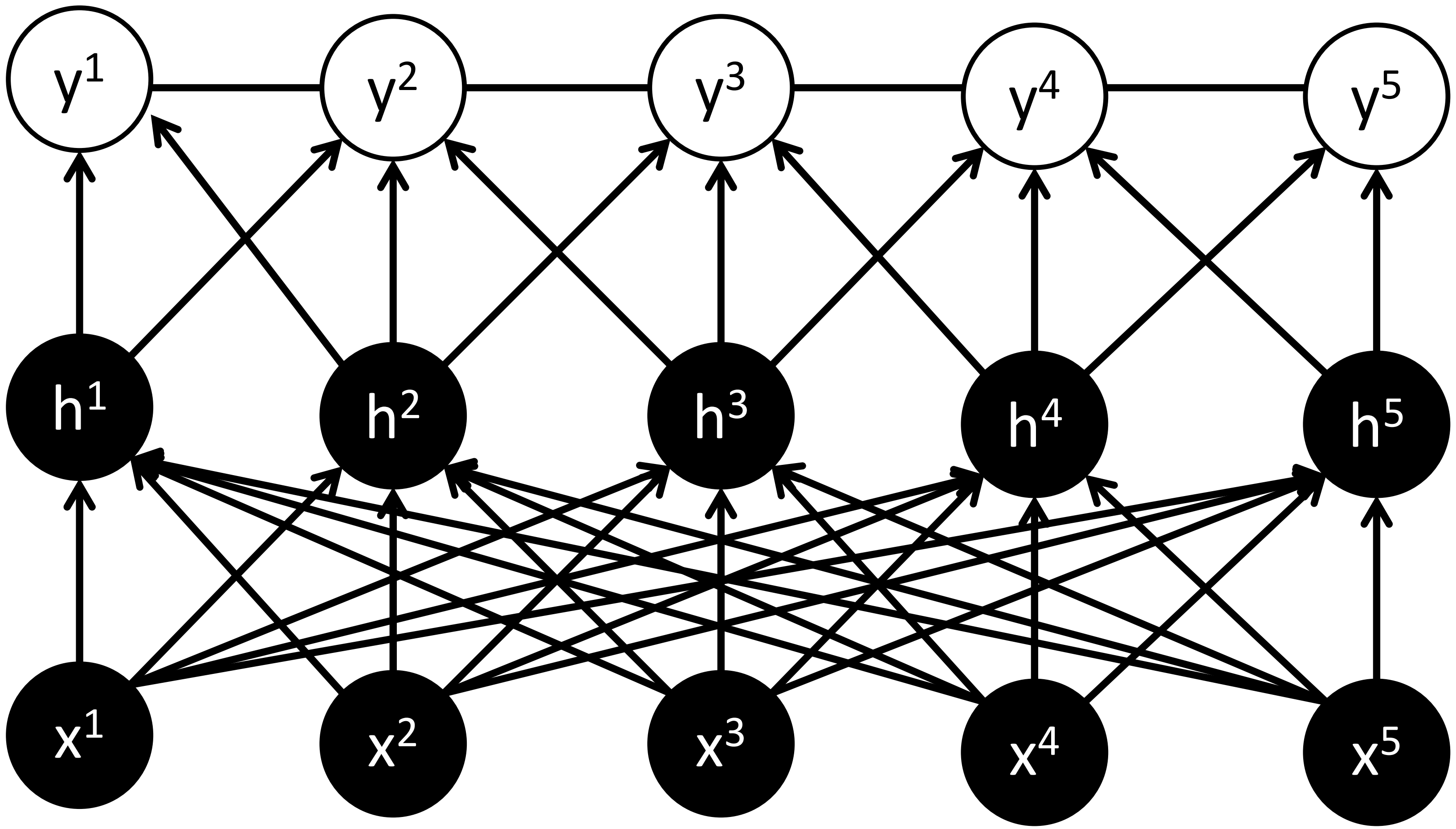
Conditional Random Field (CRF)



UGM on random variables y^i allows it to model <u>dependencies</u>.

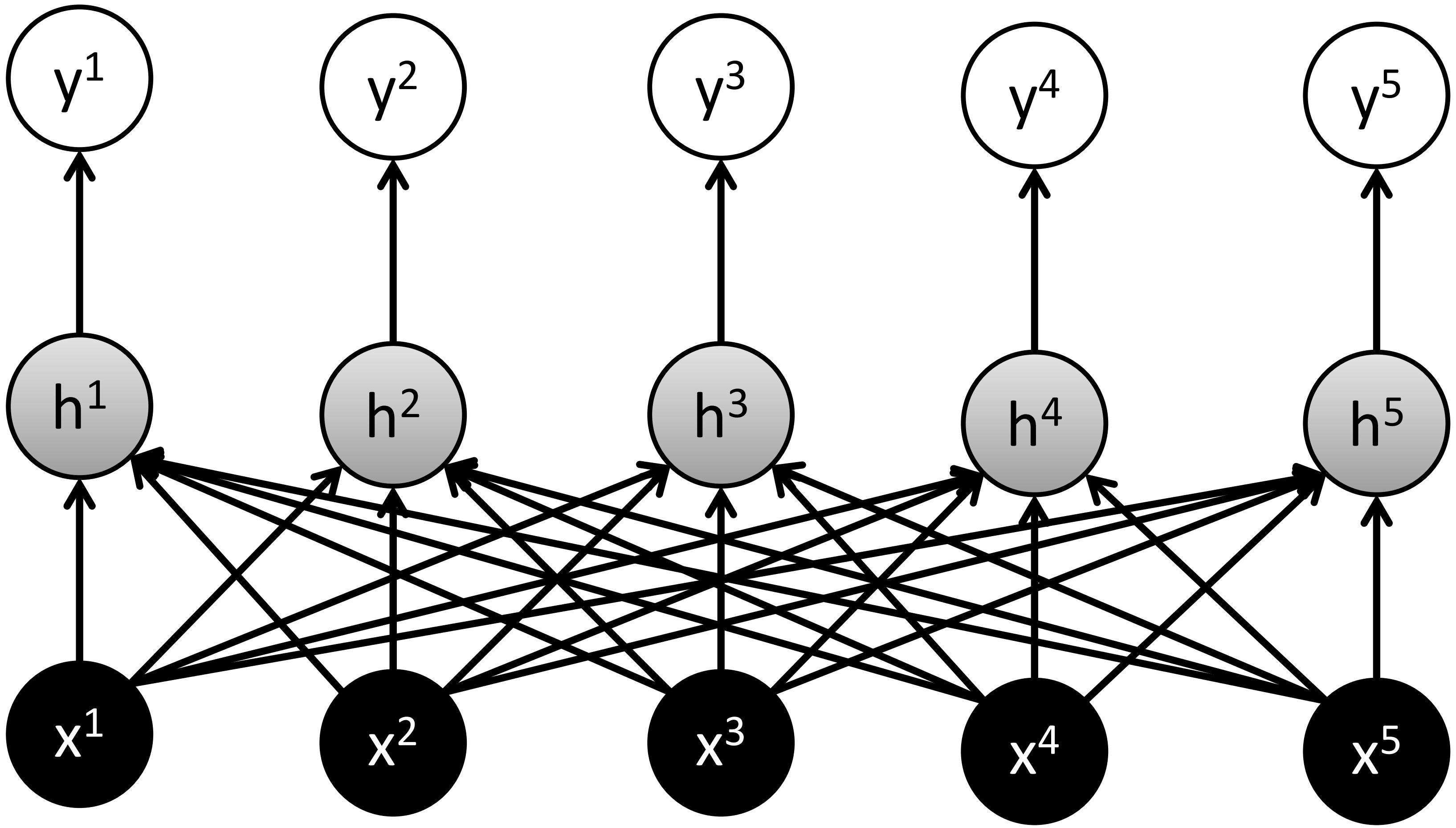
Looks weird to have "directed" parents, but <u>observed</u> parents don't introduce dependencies

Conditional Random Field (CRF)



The edges can depend on any/all h^i, but we'll ignore these edges in our diagrams.

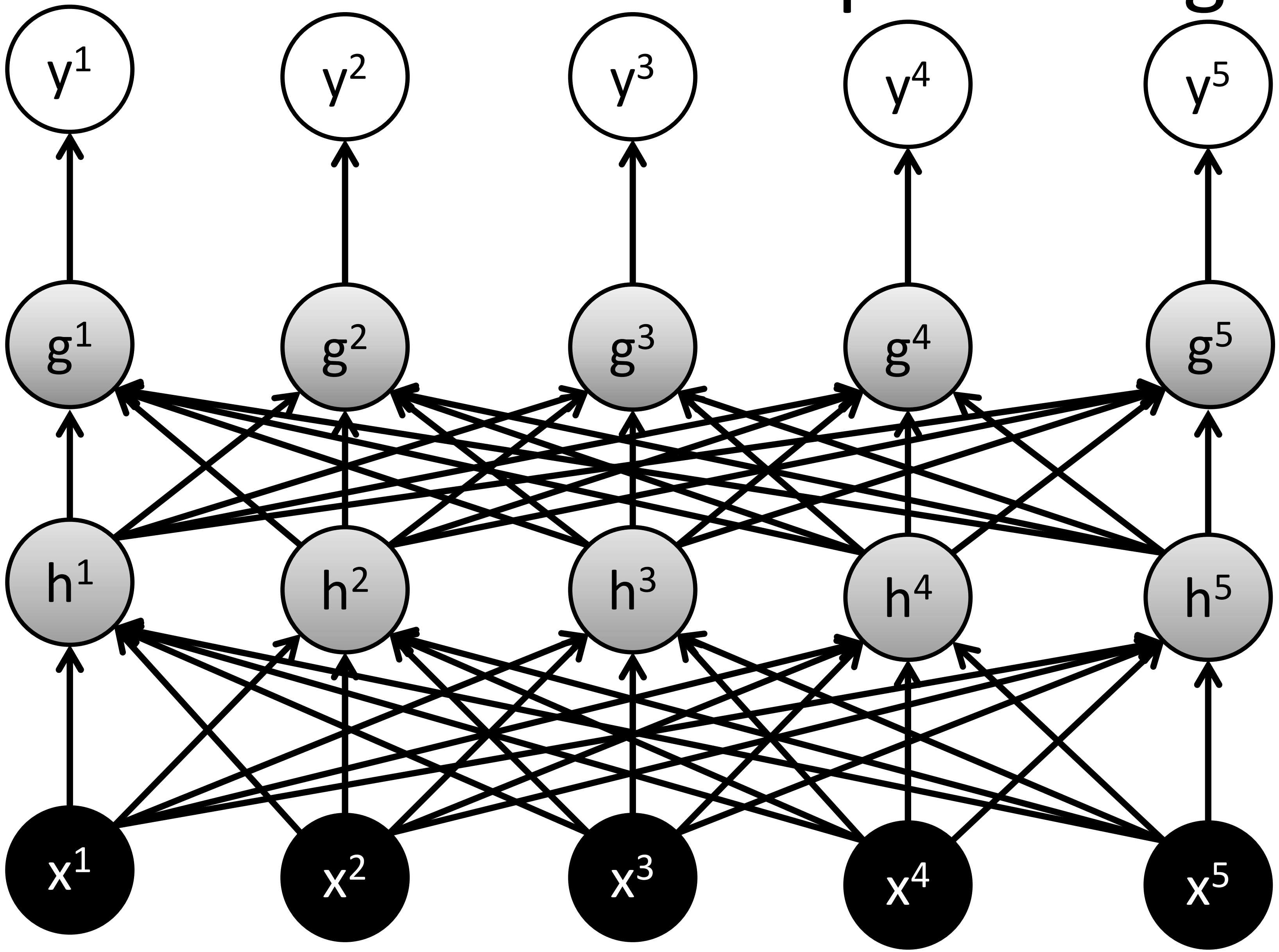
Neural Network



Alternative to <u>fixed</u> features is to <u>learn</u> the features.

- More general than any fixed set, but non-convex.
- The h^i are not observed, but we treat them as deterministic transformations and <u>not</u> random variables.
- To avoid "linear combination of linear combination", apply non-linear transform of h^i (sigmoid, tanh, ReLU).

Deep Learning

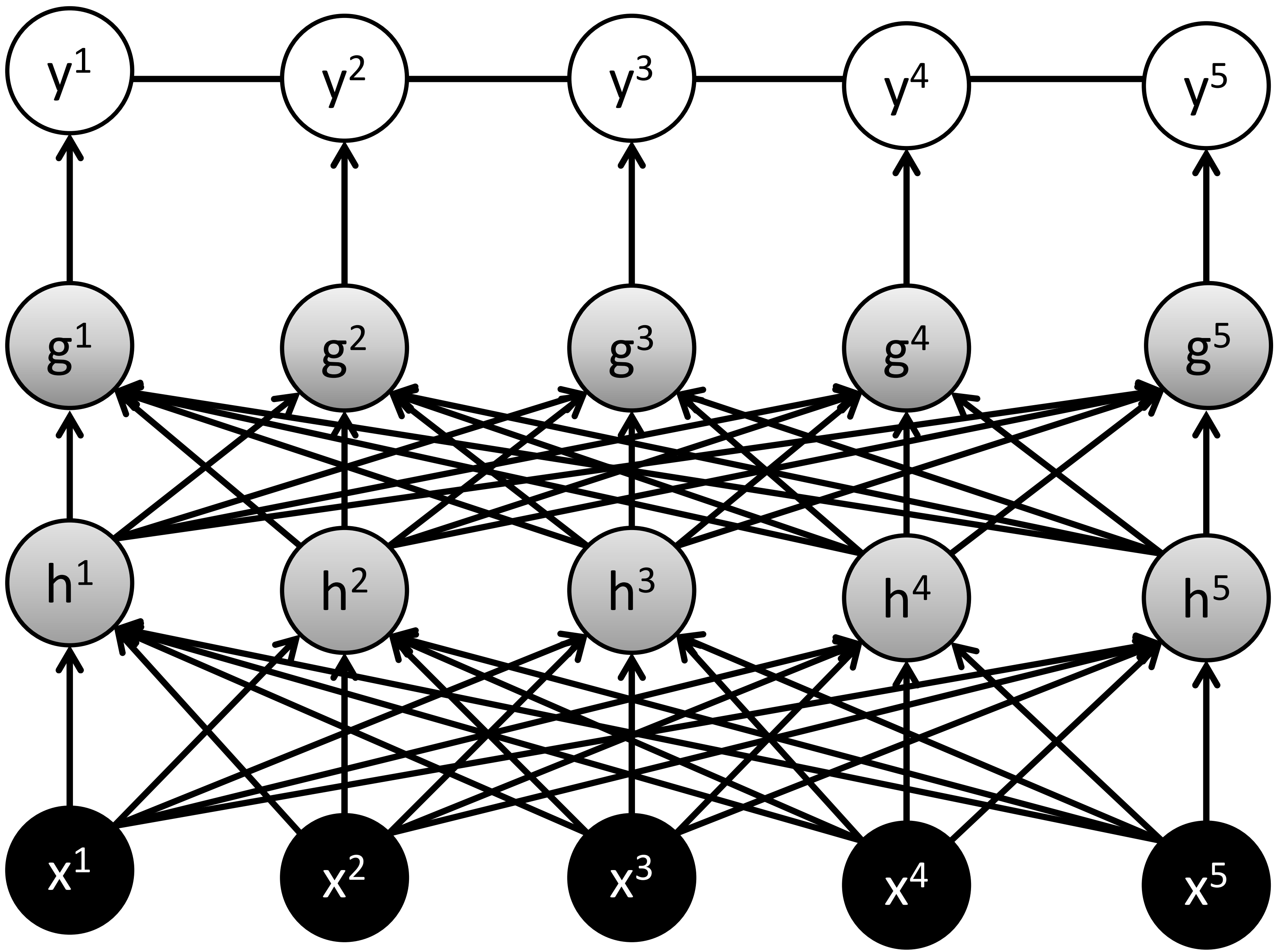


We're not explicitly modeling dependence between y^i in output.

Adding more layers increases expressive power.

Do we have to choose between deep learning and CRFs?

Conditional Neural Field (CNF)



CNFs use deep learning features fed into CRF.

- Not convex but can be trained jointly.
- Because 'g' and 'h' are deterministic transforms, does not increase complexity of inference.

1. Forward propagation to get y^i.
2. Forward-backward to $\nabla \log(Z)$.
3. Backpropagation to get gradient.

Outline

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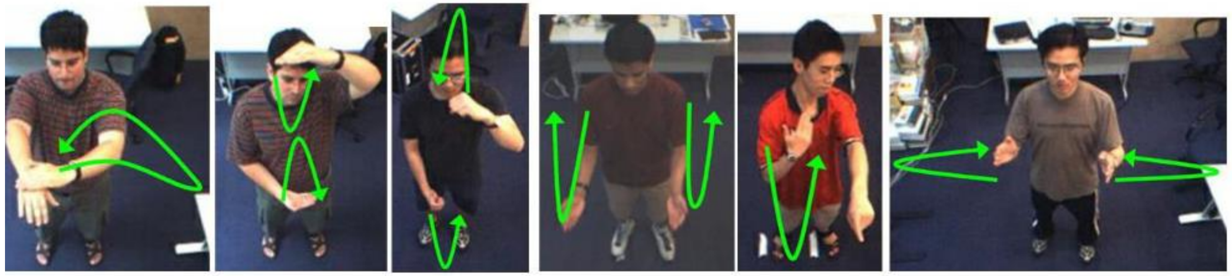
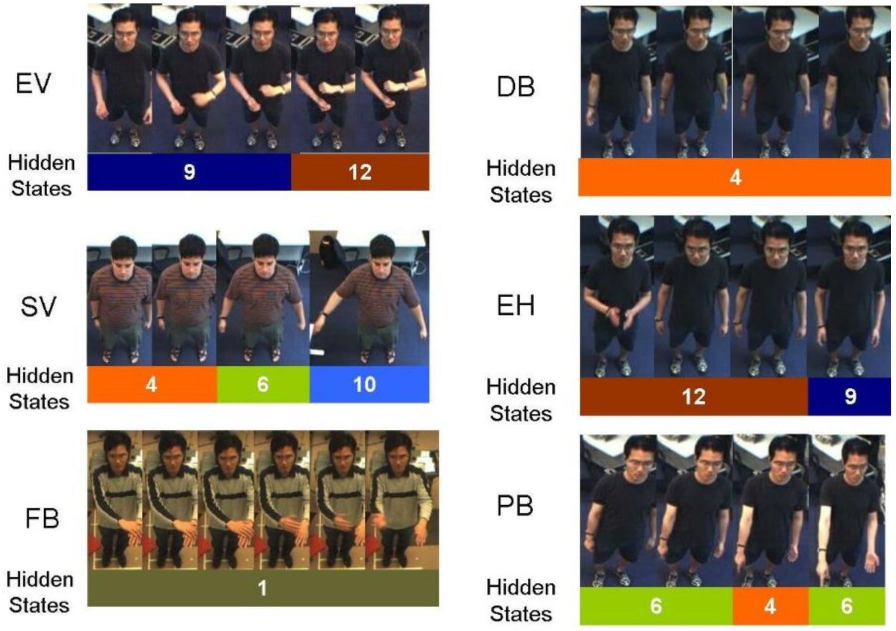
- Latent/Deep Graphical Models

Motivation: Gesture Recognition

- Want to recognize gestures from video:



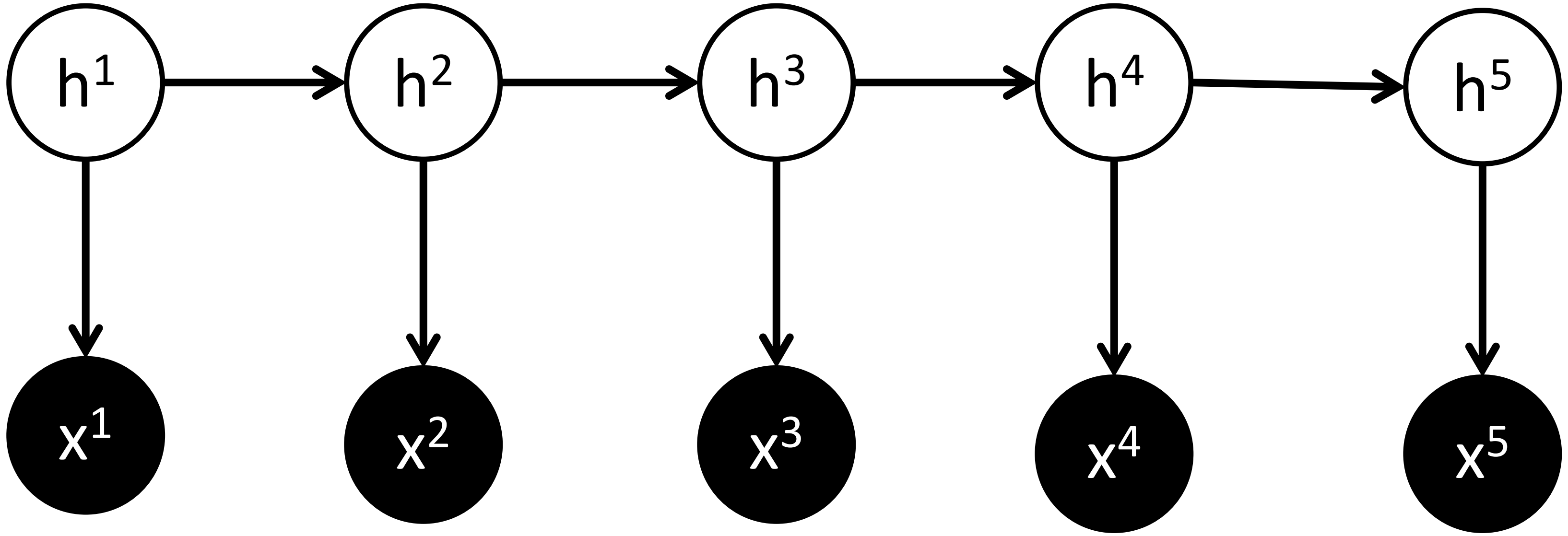
- A gesture is composed of a sequence of parts:
  - Some parts appear in different gestures.



- We have gesture (sequence) labels:
  - But no part labels.
  - We don't know what the parts should be.

Hidden Markov Model (HMM)

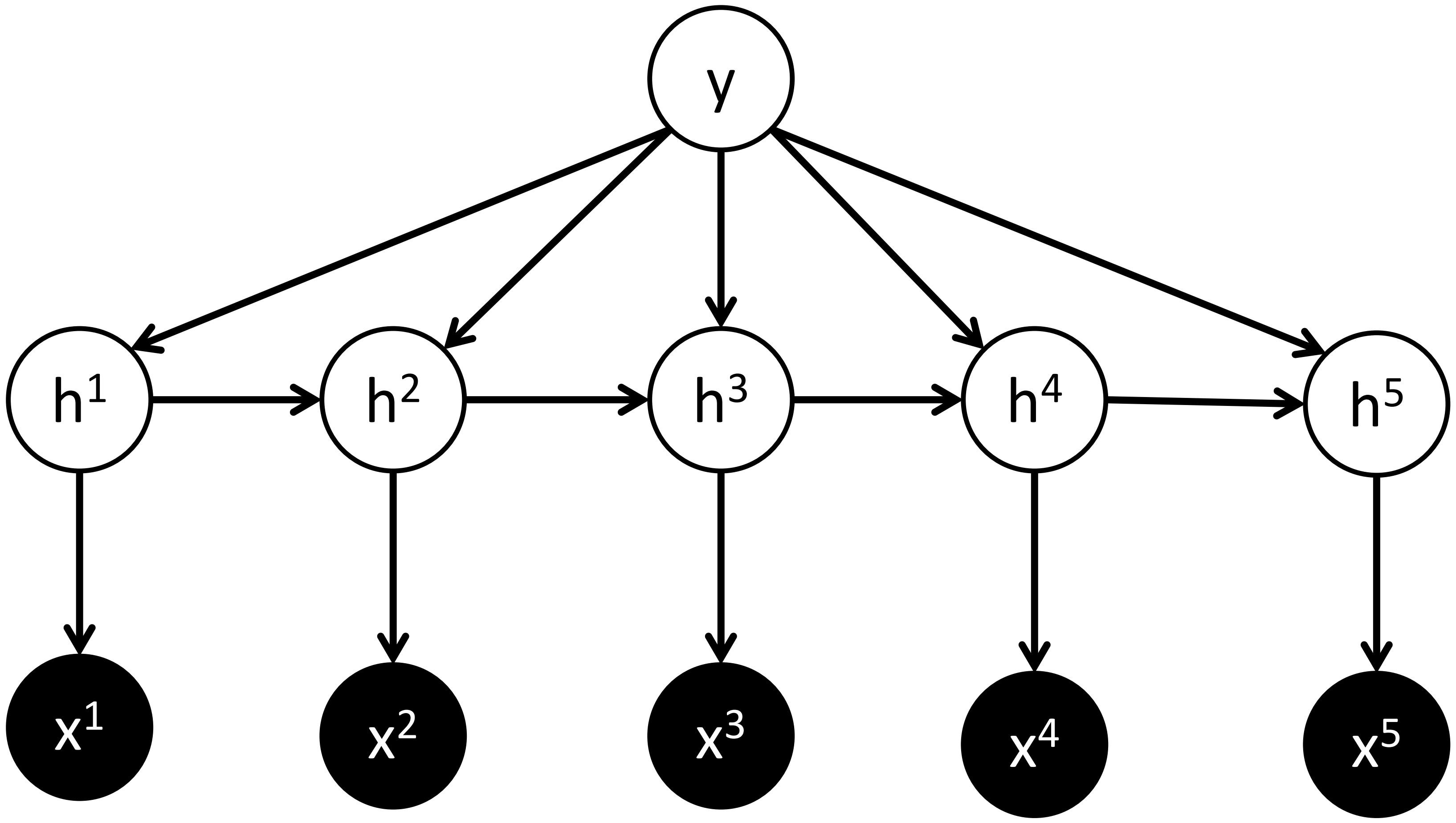
Given a particular gesture, we can model video using an HMM:



Discrete latent 'h' is the "part" at time 'i'.
- we learn $p(x^i | h^i)$ and $p(h^i | h^{i-1})$ for the gesture.

x^i is image at time 'i'.

Generative HMM Classifier

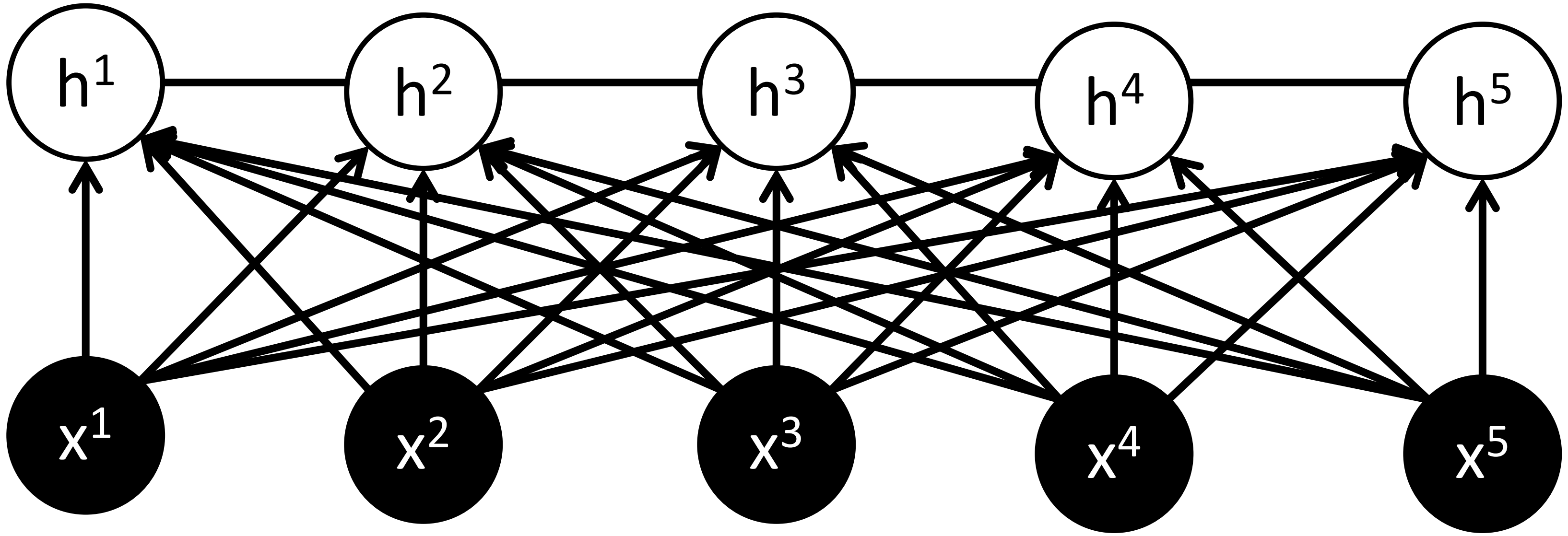


We can use the HMM with a generative classifier:

$$p(y|x) = p(y)p(x|y)$$
$$= p(y)\sum_{h_1}\sum_{h_2}\cdots\sum_{h_n} p(x,h|y).$$

- Correctly models that y is defined by sequence of parts.
- Inference is fast (treewidth=2)
- But we assumed video frames are independent given parts, and even with this modeling $p(x^i|h^i)$ is hard.

Conditional Random Field (CRF)

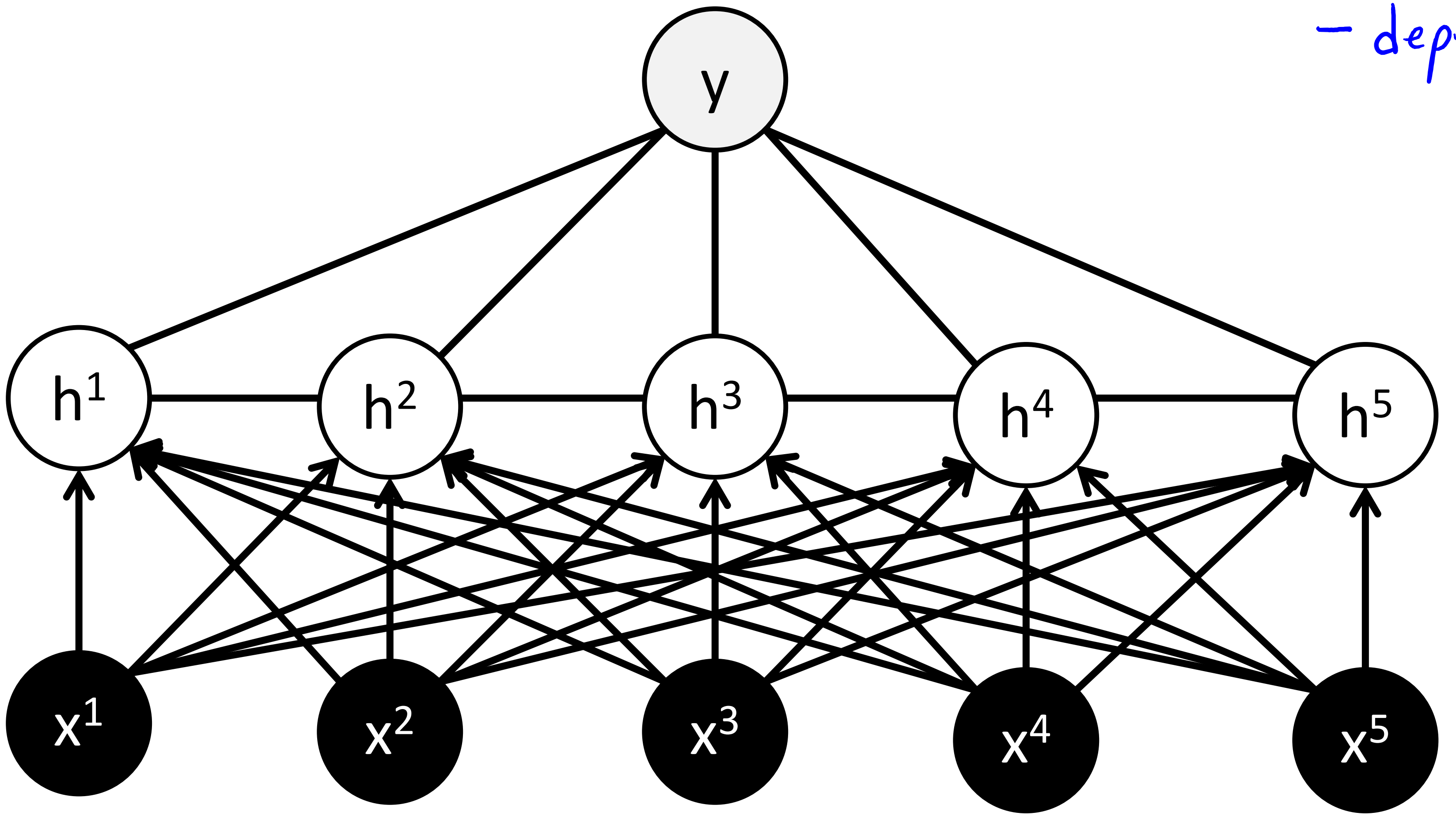


Let's use a CRF instead:

- Treat X as fixed so we don't need to model it.
- But CRFs are supervised and we don't see the h^i.

Hidden Conditional Random Field (HCRF)

UGM that includes
- temporal dependence between parts.
- dependence of gesture y on sequence of parts.



Called "hidden" CRF because we observe 'y' but not the h^i during training.

Treewidth = 2 ⇒ Inference is easy.

Graphical Models with Hidden Variables

- As before we deal with hidden variables by marginalizing:

$$p(Y|X) = \sum_{h^1} \sum_{h^2} \cdots \sum_{h^n} p(Y,H|X)$$

- If we assume a UGM over {Y,H} given X we get:

$$p(Y|X) = \sum_H \frac{\prod_{c\in C} \psi_c(Y,H)}{\sum_{H,Y'} \prod_{c\in C} \psi_c(Y',H)} = \frac{\sum_H \prod_{c\in C} \psi_c(Y,H)}{\sum_{H,Y'} \prod_{c\in C} \psi_c(Y',H)} = \frac{Z(Y)}{Z}$$

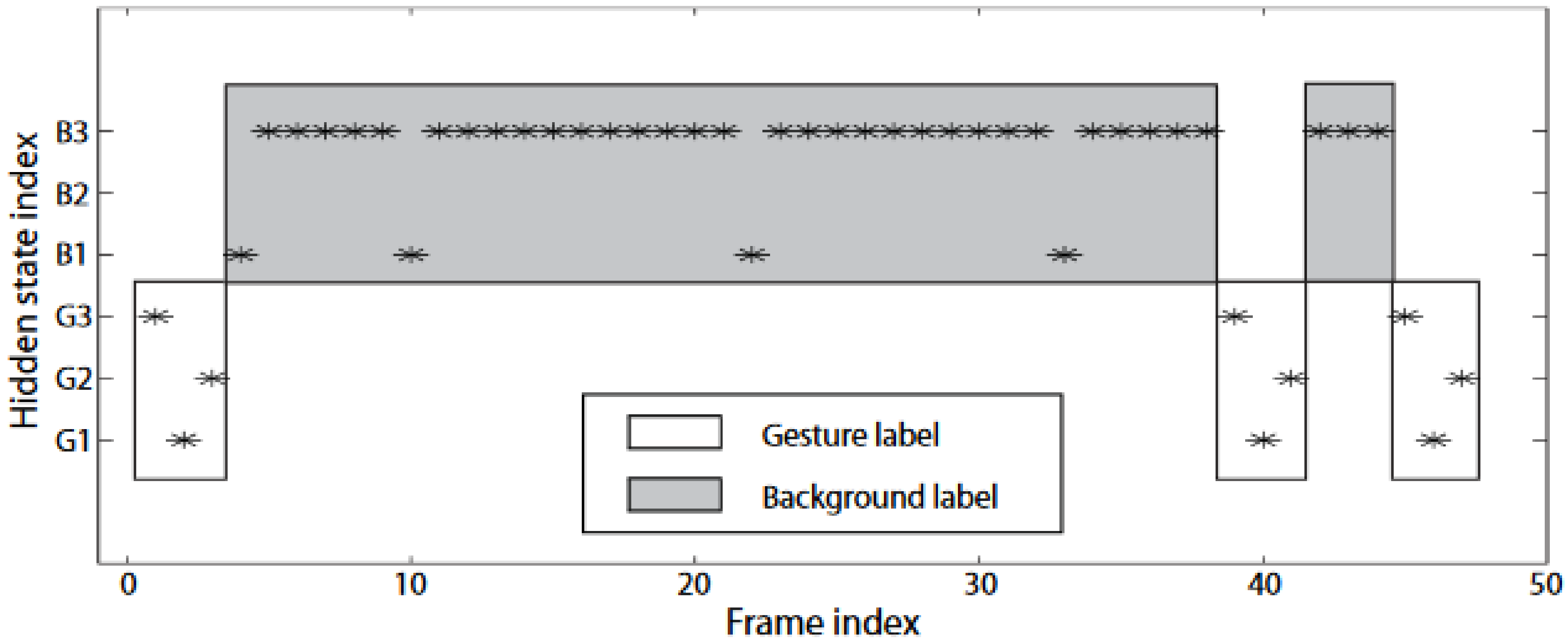
Normalizing constant of UGM over H with Y fixed.

Normalizing constant of UGM over Y and H

- Consider usual choice of log-linear phi:
  - NLL = $\underbrace{-\log(Z(Y))}_{\text{concave}} + \underbrace{\log(Z)}_{\text{convex}}$.

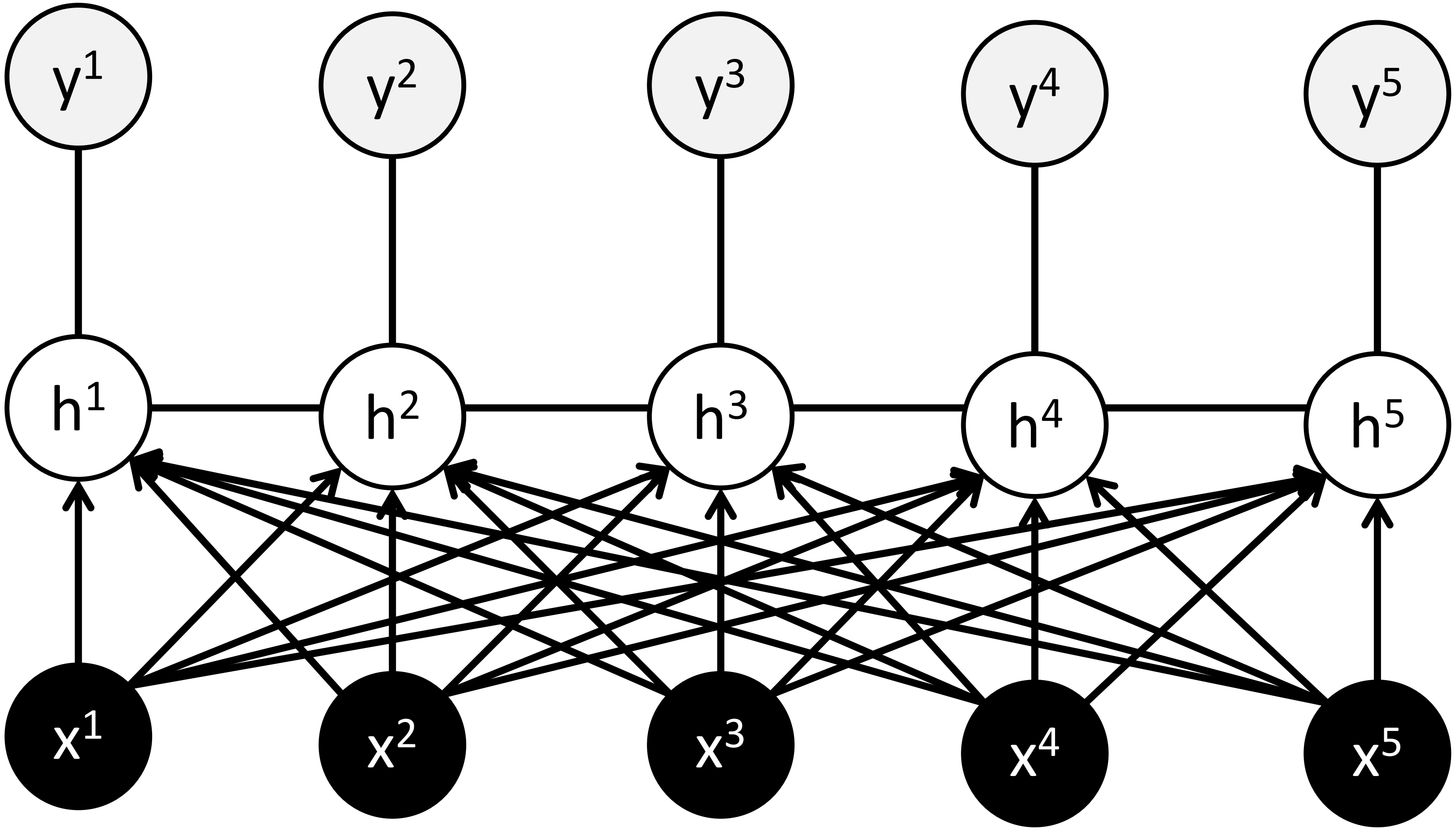
Motivation: Gesture Recognition

- What if we want to label video with multiple potential gestures?
  - Assume we have labeled video sequences.



http://groups.csail.mit.edu/vision/vip/papers/morency_cvpr07.pdf

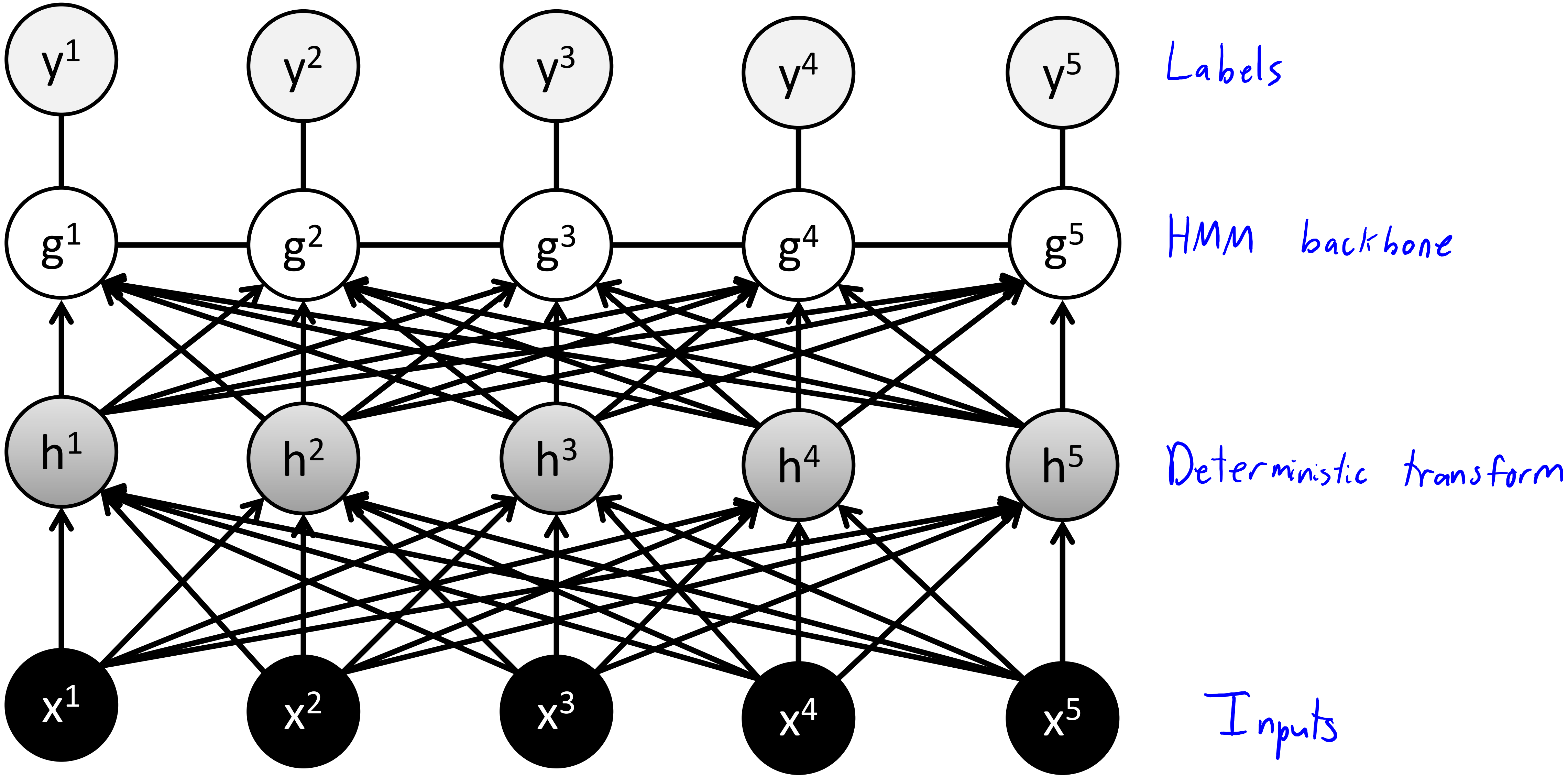
Latent-Dynamic Conditional Random Field (LDCRF)



} Label y^i for each time:

} Capture "latent dynamics"
– usually each y^i associated with possible hidden states.

Latent-Dynamic Conditional Neural Field (LDCNF)



Summary

- Conditional random fields generalize logistic regression:
  - Allows dependencies between labels.
  - Requires inference in graphical model.
- Conditional neural fields combine CRFs with deep learning.
  - Could also replace CRF with conditional density estimators (e.g., DAGs).
- UGMs with hidden variables have nice form: ratio of normalizers.
  - Can do inference with same methods.
- Latent dynamic conditional random/neural fields:
  - Allow dependencies between hidden variables.

- Next time: Boltzmann machines, LSTMs, and beyond CPSC 540.