CPSC 340: Machine Learning and Data Mining

More Deep Learning

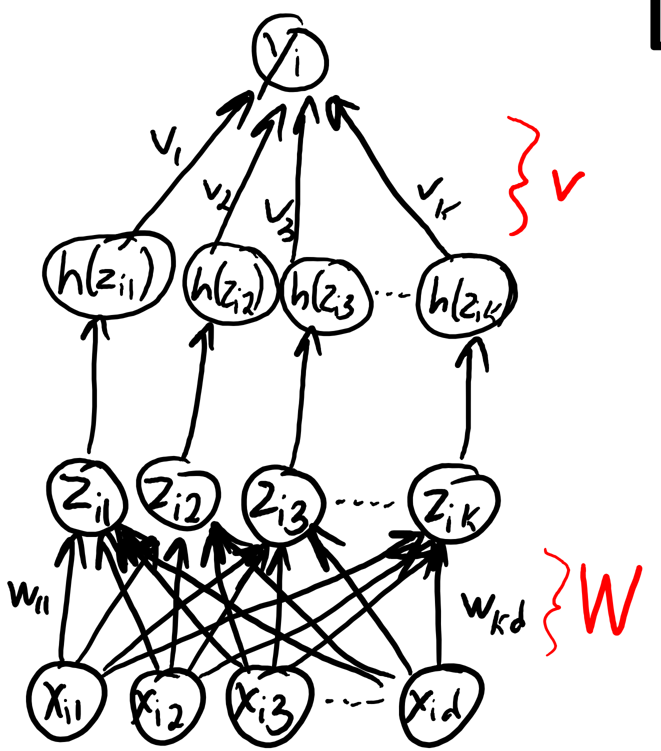
Fall 2017

Admin

- Assignment 5:
  - Due Monday, 1 late day for Wednesday, 2 for next Friday.
- Final:
  - Details and previous exams posted on Piazza.
- Extra office hours:
  - 3:00 next Thursday (with me).
  - Monday/Tuesday (with TAs).

Last Time: Deep Learning

Neural network:

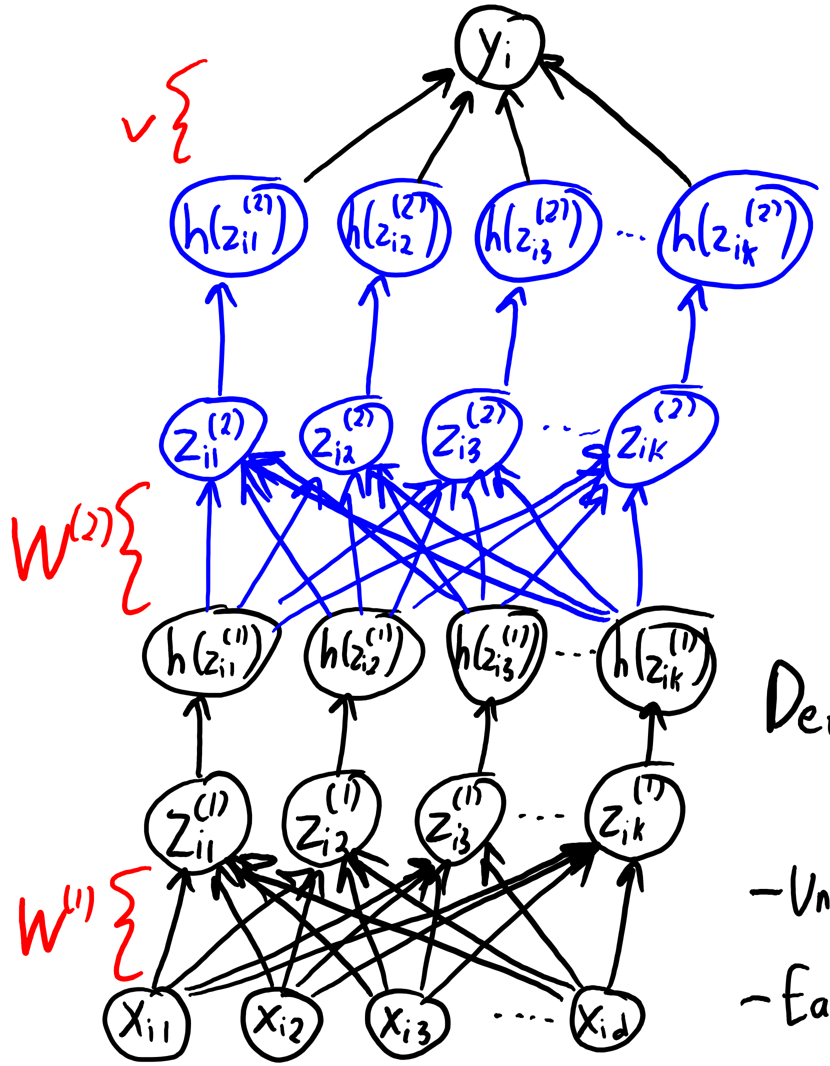


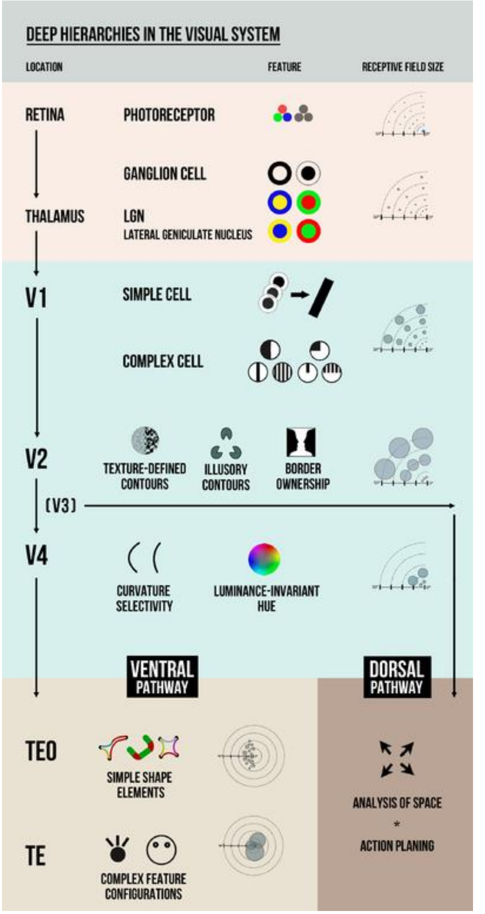
$y_i = v^T h(Wx_i)$

Learn 'W' and 'v' together.

– learn features for supervised learning.

– Non-linear 'h' makes it a universal approximator for large 'K'





Deep neural networks:

$y_i = v^T h(W^{(2)} h(W^{(1)} x_i))$

– Unprecedented performance on difficult problems.

– Each layer combines "parts" from previous layer.

https://en.wikibooks.org/wiki/Sensory_Systems/Visual_Signal_Processing

Deep Learning Practicalities

- This lecture focus on deep learning practical issues:
  - Backpropagation to compute gradients.
  - Stochastic gradient training.
  - Regularization to avoid overfitting.

- Next lecture:
  - Special ‘W’ restrictions to further avoid overfitting.

But first: Adding Bias Variables

- Recall fitting line regression with a bias:

$$\hat{y}_i = \sum_{j=1}^{d} w_j x_{ij} + \beta$$

  – We avoided this by adding a column of ones to X.

- In neural networks we often want a bias on the output:

$$\hat{y}_i = \sum_{c=1}^{k} v_c h(w_c^T x_i) + \beta$$

- But we also often also include biases on each z_{ic}:

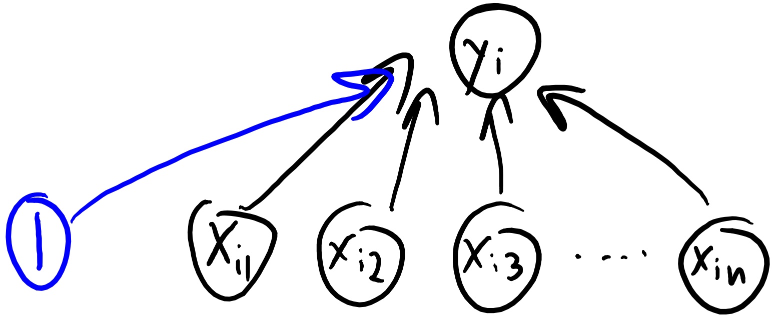
$$\hat{y}_i = \sum_{c=1}^{k} v_c h(w_c^T x_i + \beta_c) + \beta$$

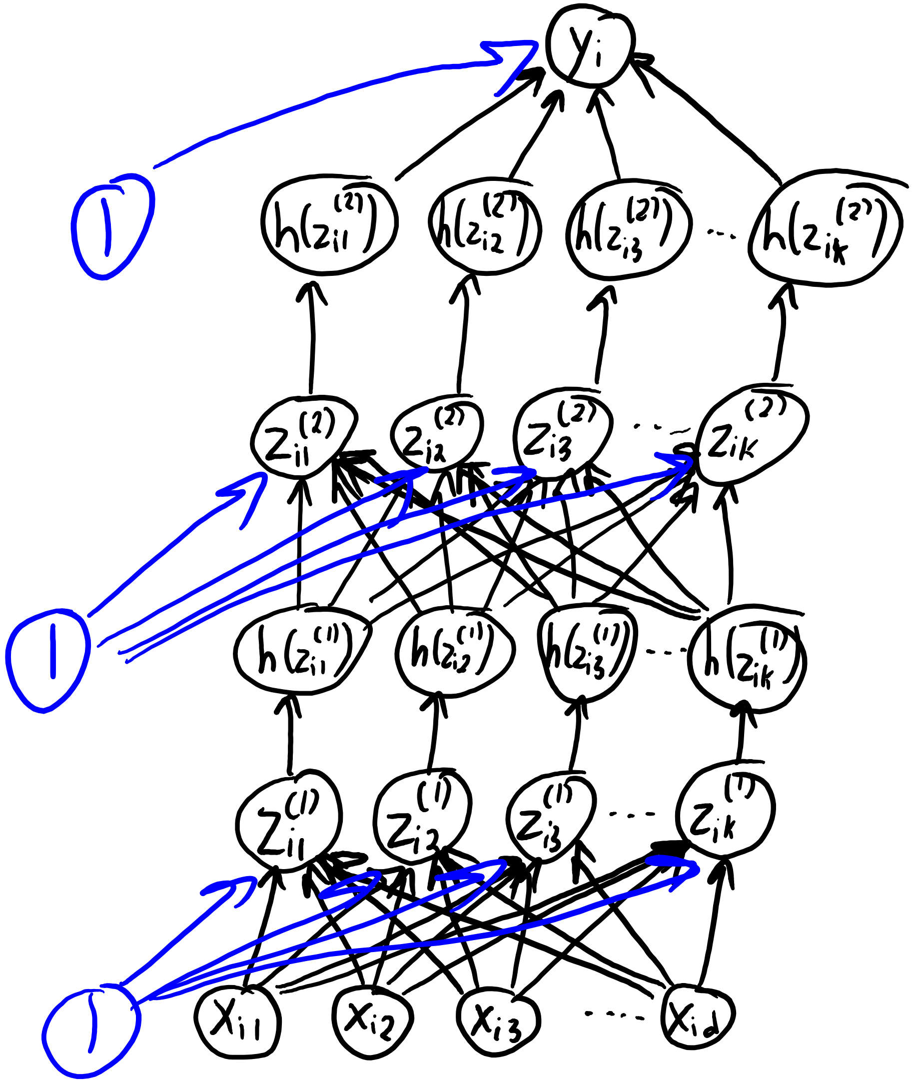
  – A bias towards this $h(z_{ic})$ being either 0 or 1.

- Equivalent to adding to vector $h(z_i)$ an extra value that is always 1.
  – For sigmoids, you could equivalently make one row w_c be equal to 0.

But first: Adding Bias Variables

Linear model with bias:





Artificial Neural Networks

- With squared loss, our objective function is:

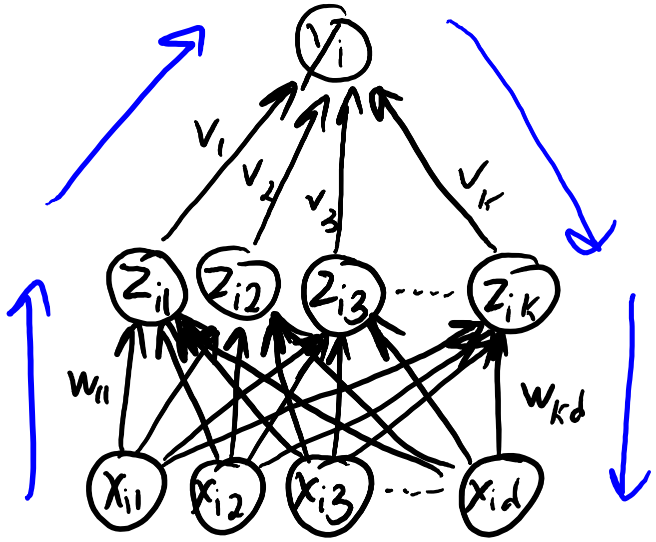
$$f(v, W) = \frac{1}{2}\sum_{i=1}^{n}(v^T h(Wx_i) - y_i)^2$$

- Usual training procedure: stochastic gradient.
  - Compute gradient of random example ‘i’, update both ‘v’ and ‘W’.
  - Highly non-convex and can be difficult to tune.

- Computing the gradient is known as “backpropagation”.
  - Video giving motivation here.

Backpropagation

- Overview of how we compute neural network gradient:
  - Forward propagation:
    - Compute $z_i^{(1)}$ from x_i.
    - Compute $z_i^{(2)}$ from $z_i^{(1)}$.
    - …
    - Compute $\hat{y}_i$ from $z_i^{(m)}$, and use this to compute error.
  - Backpropagation:
    - Compute gradient with respect to regression weights 'v'.
    - Compute gradient with respect to $z_i^{(m)}$ weights $W^{(m)}$.
    - Compute gradient with respect to $z_i^{(m-1)}$ weights $W^{(m-1)}$.
    - …
    - Compute gradient with respect to $z_i^{(1)}$ weights $W^{(1)}$.
- "Backpropagation" is the chain rule plus some bookkeeping for speed.



Backpropagation

- Let's illustrate backpropagation in a simple setting:
  - 1 training example, 3 hidden layers, 1 hidden "unit" in layer.

$$f(W^{(1)}, W^{(2)}, W^{(3)}, v) = \frac{1}{2}\underbrace{(\hat{y}_i - y_i)}_{r}^2 \quad \text{where} \quad \hat{y}_i = vh(W^{(3)}h(W^{(2)}h(W^{(1)}x_i)))$$

$$\frac{\partial f}{\partial v} = r\,h(W^{(3)}h(W^{(2)}h(W^{(1)}x_i))) = r\,h(z_i^{(3)})$$

$$\frac{\partial f}{\partial W^{(3)}} = r\,v\,h'(W^{(3)}h(W^{(2)}h(W^{(1)}x_i)))\,h(W^{(2)}h(W^{(1)}x_i)) = r\,v\,h'(z_i^{(3)})\,h(z_i^{(2)})$$

$$x_i \xrightarrow{W^{(1)}} z_i^{(1)} \to h(z_i^{(1)}) \xrightarrow{W^{(2)}} z_i^{(2)} \to h(z_i^{(2)}) \xrightarrow{W^{(3)}} z_i^{(3)} \to h(z_i^{(3)}) \xrightarrow{v} \hat{y}_i$$

Backpropagation

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$$\frac{\partial f}{\partial W^{(2)}} = r\,v\,h'(W^{(3)}h(W^{(2)}h(W^{(1)}x_i)))\,W^{(3)}h'(W^{(2)}h(W^{(1)}x_i))\,h(W^{(1)}x_i) = \underbrace{r^{(3)}W^{(3)}h'(z_i^{(2)})}\,h(z_i^{(1)})$$

$$\frac{\partial f}{\partial W^{(1)}} = r\,v\,h'(W^{(3)}h(W^{(2)}h(W^{(1)}x_i)))\,W^{(3)}h'(W^{(2)}(W^{(1)}x_i))\,W^{(2)}h'(W^{(1)}x_i)\,x_i = r^{(2)}W^{(2)}h'(z_i^{(1)})\,x_i$$

Backpropagation

- Let's illustrate backpropagation in a simple setting:
  - 1 training example, 3 hidden layers, 1 hidden "unit" in layer.

$$\frac{\partial f}{\partial v} = r\, h(z_i^{(3)})$$

$$\frac{\partial f}{\partial W^{(3)}} = r\, v\, h'(z_i^{(3)})\, h(z_i^{(2)})$$

$$\frac{\partial f}{\partial W^{(2)}} = r^{(3)} W^{(3)} h'(z_i^{(2)})\, h(z_i^{(1)})$$

$$\frac{\partial f}{\partial W^{(1)}} = r^{(2)} W^{(2)} h'(z_i^{(1)})\, x_i$$

$$\frac{\partial f}{\partial v_c} = r\, h(z_{ic}^{(3)})$$

$$\frac{\partial f}{\partial W^{(3)}_{c'c}} = \underbrace{r\, v_c\, h'(z_{ic'}^{(3)})}\, h(z_{ic}^{(2)})$$

$$\frac{\partial f}{\partial W^{(2)}_{c'c}} = \underbrace{\left[\sum_{c''=1}^{k} r_{c''}^{(3)} W_{c''c'}^{(3)}\right] h'(z_{ic'}^{(2)})}\, h(z_{ic}^{(1)})$$

$$\frac{\partial f}{\partial W^{(1)}_{cj}} = \left[\sum_{c''=1}^{k} r_{c''}^{(2)} W_{c''c}^{(2)}\right] h'(z_{ic}^{(1)})\, x_j$$

  - Only the first 'r' changes if you use a different loss.
  - With multiple hidden units, you get extra sums.
    - Efficient if you store the sums rather than computing from scratch.

Backpropagation

- I've marked those backprop math slides as bonus.
- Do you need to know how to do this?
  - Exact details are probably not vital (there are many implementations), but understanding basic idea helps you know what can go wrong.
  - See discussion here by a neural network expert.

- You should know cost of backpropagation:
  - Forward pass dominated by matrix multiplications by $W^{(1)}$, $W^{(2)}$, $W^{(3)}$, and 'v'.
    - If have 'm' layers and all z_i have 'k' elements, cost would be $O(dk + mk^2)$.
  - Backward pass has same cost as forward pass.
- For multi-class or multi-label classification, you replace 'v' by a matrix:
  - Softmax loss is often called "cross entropy" in neural network papers.

(pause)

Last Time: ImageNet Challenge

- ImageNet challenge:
  - Use millions of images to recognize 1000 objects.
- ImageNet organizer visited UBC summer 2015.
- "Besides huge dataset/model/cluster, what is the most important?"
  1. Image transformations (translation, rotation, scaling, lighting, etc.).
  2. Optimization.
- Why would optimization be so important?
  - Neural network objectives are highly non-convex (and worse with depth).
  - Optimization has huge influence on quality of model.

Stochastic Gradient Training

- Standard training method is stochastic gradient (SG):
  - Choose a random example 'i'.
  - Use backpropagation to get gradient with respect to all parameters.
  - Take a small step in the negative gradient direction.
- Challenging to make SG work:
  - Often doesn't work as a "black box" learning algorithm.
  - But people have developed a lot of tricks/modifications to make it work.
- Highly non-convex, so are the problem local mimina?
  - Some empirical/theoretical evidence that local minima are not the problem.
  - If the network is "deep" and "wide" enough, we think all local minima are good.
  - But it can be hard to get SG to even find a local minimum.

Parameter Initialization

- Parameter initialization is crucial:
  - Can't initialize weights in same layer to same value, or they will stay same.
  - Can't initialize weights too large, it will take too long to learn.
- A traditional random initialization:
  - Initialize bias variables to 0.
  - Sample from standard normal, divided by 10^5 (0.00001*randn).
    - w = .00001*randn(k,1)
  - Performing multiple initializations does not seem to be important.

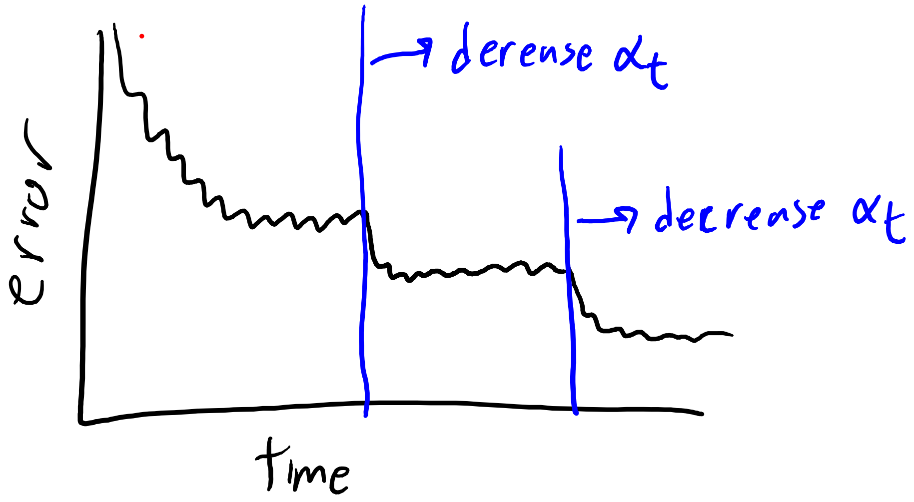
- Popular approach from 10 years ago:
  - Initialize with deep unsupervised model (like "autoencoders" – see bonus).

Parameter Initialization

- Parameter initialization is crucial:
  - Can't initialize weights in same layer to same value, or they will stay same.
  - Can't initialize weights too large, it will take too long to learn.
- Also common to standardize data:
  - Subtract mean, divide by standard deviation, "whiten", standardize y_i.
- More recent initializations try to standardize initial z_i:
  - Use different initialization in each layer.
  - Try to make variance of z_i the same across layers.
  - Use samples from standard normal distribution, divide by sqrt(2*nInputs).
  - Use samples from uniform distribution on [-b,b], where $b = \frac{\sqrt{6}}{\sqrt{k^{(m)} + k^{(m-1)}}}$

Setting the Step-Size

- Stochastic gradient is very sensitive to the step size in deep models.
- Common approach: manual "babysitting" of the step-size.
  - Run SG for a while with a fixed step-size.
  - Occasionally measure error and plot progress:



  - If error is not decreasing, decrease step-size.

Setting the Step-Size

- Stochastic gradient is very sensitive to the step size in deep models.
- Bias step-size multiplier: use bigger step-size for the bias variables.
- Momentum:
  - Add term that moves in previous direction:

$$w^{t+1} = w^t - \alpha^t \nabla f_i(w^t) + \underbrace{\beta^t(w^t - w^{t-1})}_{\text{keep going in the old direction}}$$

  - Usually $\beta^t = 0.9$.

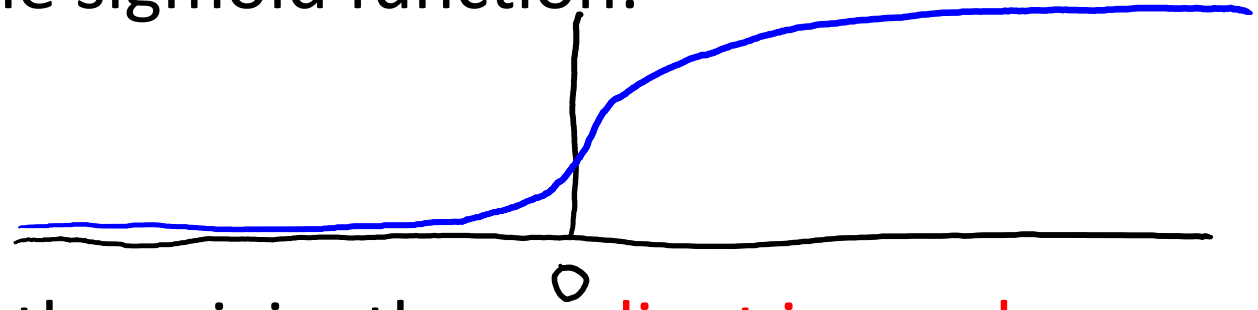
Setting the Step-Size

- Automatic method to set step size is Bottou trick:
  1. Grab a small set of training examples (maybe 5% of total).
  2. Do a binary search for a step size that works well on them.
  3. Use this step size for a long time (or slowly decrease it from there).

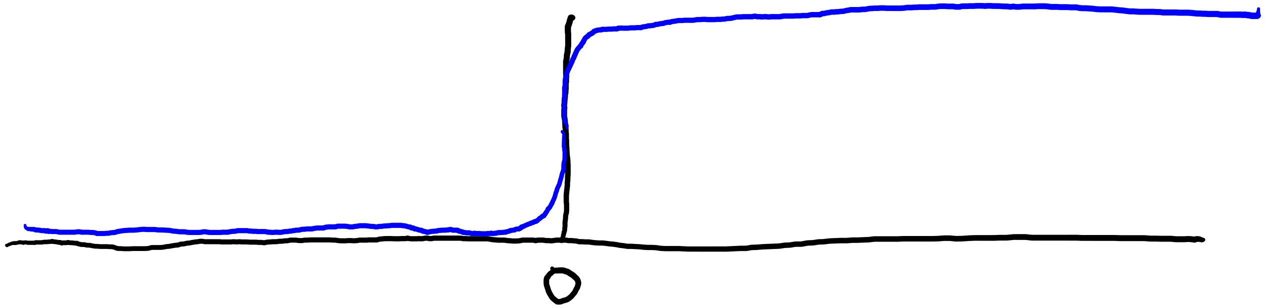
- Several recent methods using a step size for each variable:
  – AdaGrad, RMSprop, Adam (often work better “out of the box”).
  – Seem to be losing popularity to stochastic gradient (often with momentum).
    - Often yields lower test error but this requires more tuning of step-size.
- Batch size (number of random examples) also influences results.
  – Bigger batch sizes often give faster convergence but to worse solutions.
- Another recent trick is batch normalization:
  – Try to “standardize” the hidden units within the random samples as we go.

Vanishing Gradient Problem

- Consider the sigmoid function:



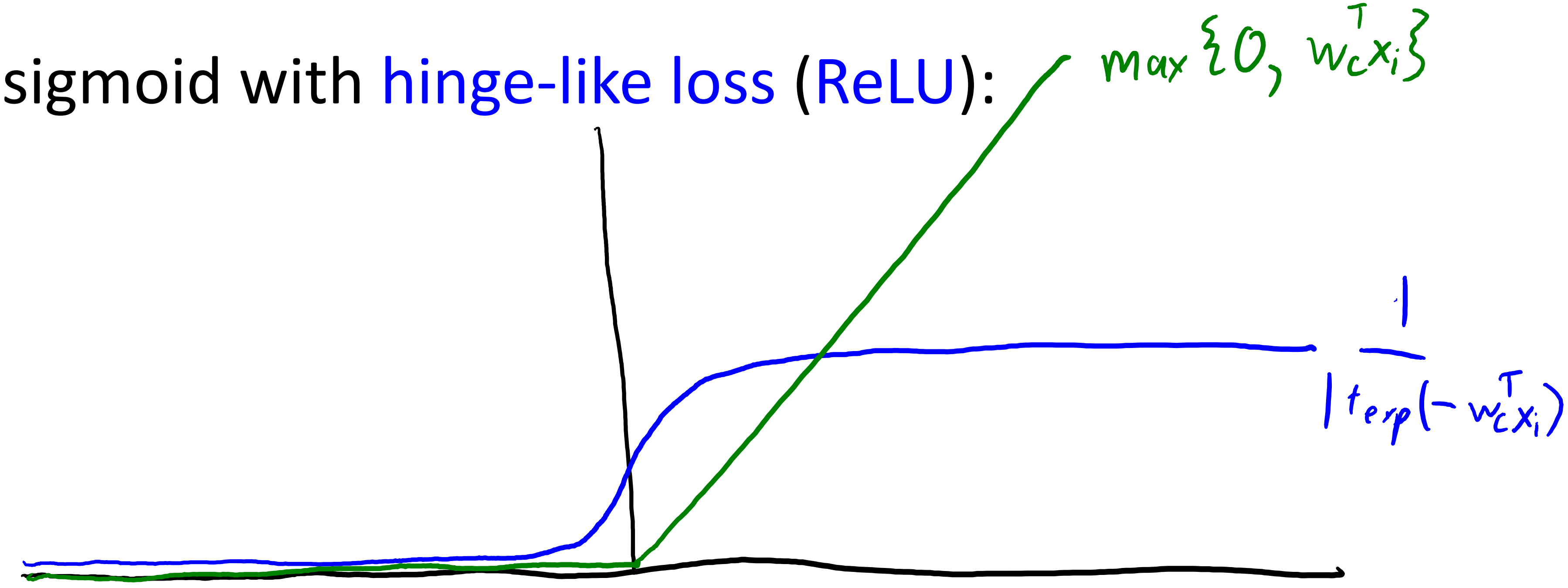
- Away from the origin, the gradient is nearly zero.
- The problem gets worse when you take the sigmoid of a sigmoid:



- In deep networks, many gradients can be nearly zero everywhere.

Rectified Linear Units (ReLU)

- Replace sigmoid with hinge-like loss (ReLU):



- Just sets negative values z_{ic} to zero.
  - Fixes vanishing gradient problem.
  - Gives sparser of activations.
  - Not really simulating binary signal, but could be simulating rate coding.

Deep Learning and the Fundamental Trade-Off

- Neural networks are subject to the fundamental trade-off:
  - As we increase the depth, training error decreases.
  - As we increase the depth, training error no longer approximates test error.

- We want deep networks to model highly non-linear data.
  - But increasing the depth leads to overfitting.

- How could GoogLeNet use 22 layers?
  - Many forms of regularization and keeping model complexity under control.

Standard Regularization

- We typically add our usual L2-regularizers:

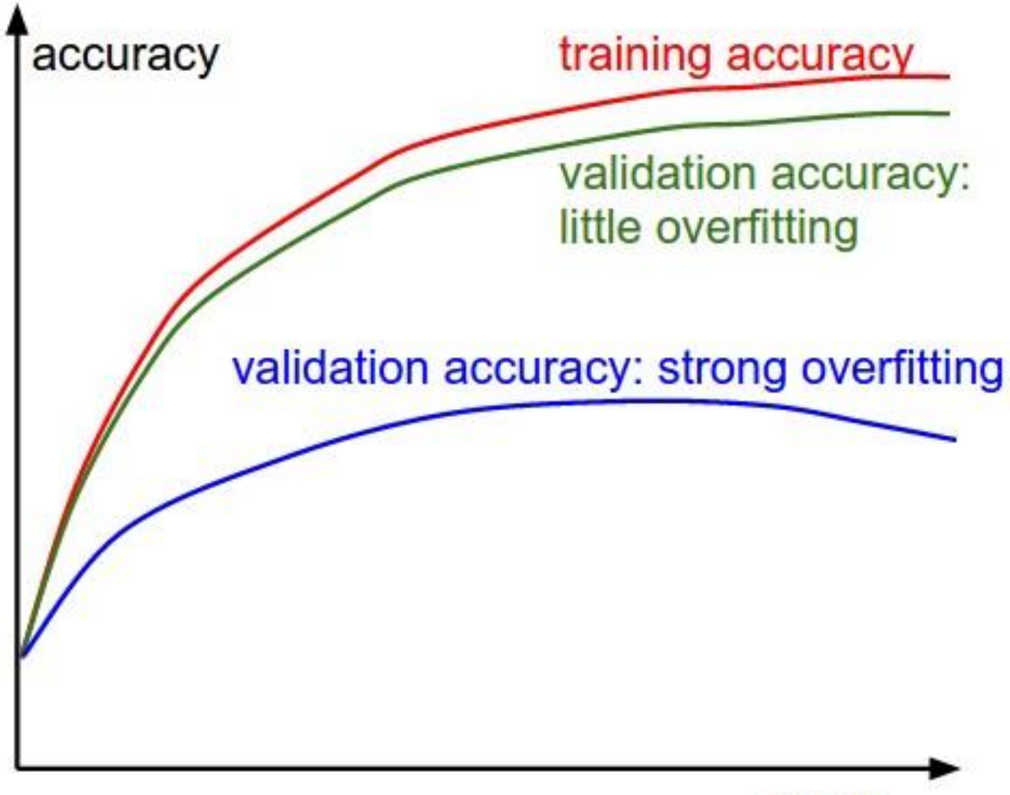
$$f(v, W^{(3)}, W^{(2)}, W^{(1)}) = \frac{1}{2}\sum_{i=1}^{n}\left(v^T h(W^{(3)} h(W^{(2)} h(W^{(1)} x_i))) - y_i\right)^2 + \frac{\lambda_4}{2}\|v\|^2 + \frac{\lambda_3}{2}\|W^{(3)}\|_F^2 + \frac{\lambda_2}{2}\|W^{(2)}\|_F^2 + \frac{\lambda_1}{2}\|W^{(1)}\|_F^2$$

- L2-regularization is called "weight decay" in neural network papers.
  - Could also use L1-regularization.
- "Hyper-parameter" optimization:
  - Try to optimize validation error in terms of λ_1, λ_2, λ_3, λ_4.

- Unlike linear models, typically use multiple types of regularization.

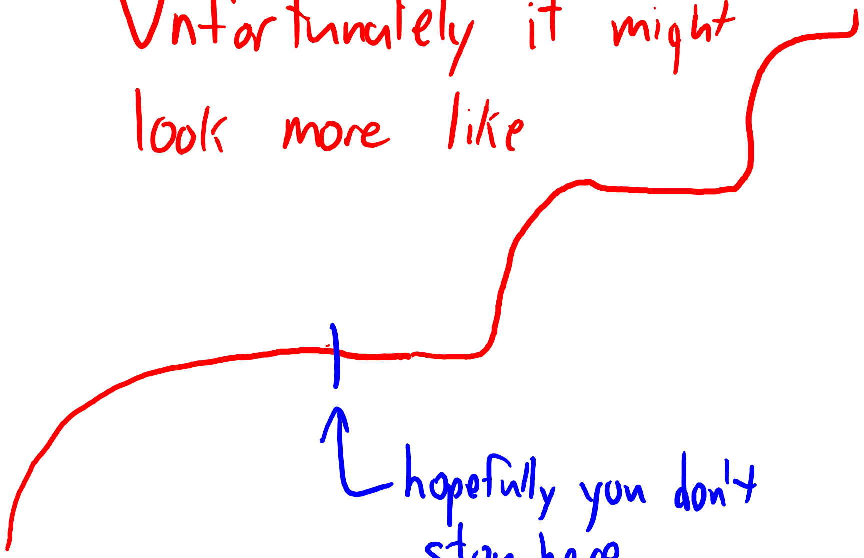
Early Stopping

- Second common type of regularization is "early stopping":
  - Monitor the validation error as we run stochastic gradient.
  - Stop the algorithm if validation error starts increasing.



http://cs231n.github.io/neural-networks-3/

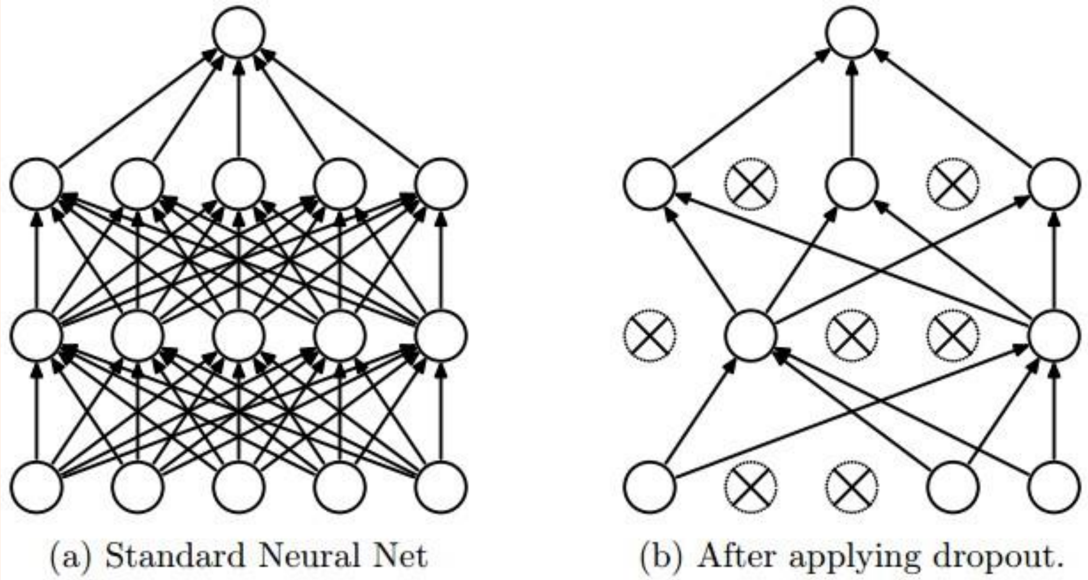
Unfortunately it might look more like



hopefully you don't stop here.

Dropout

- Dropout is a more recent form of regularization:
  - On each iteration, randomly set some x_i and z_i to zero (often use 50%).



  - Encourages distributed representation rather than using specific z_i.
  - Like ensembling a lot of models but without the high computational cost.
  - After a lot of success, dropout may already be going out of fashion.

http://www.cs.toronto.edu/~rsalakhu/papers/srivastava14a.pdf

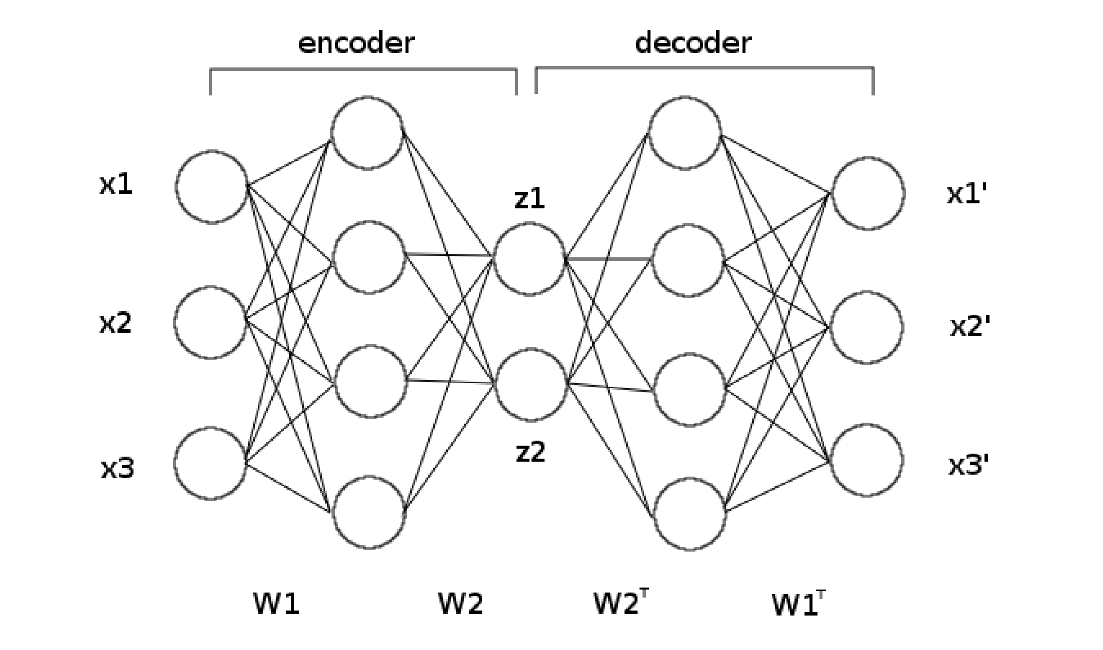
Summary

- Backpropagation computes neural network gradient via chain rule.
- Parameter initialization is crucial to neural net performance.
- Optimization and step size are crucial to neural net performance.
- Regularization is crucial to neural net performance:
  - L2-regularization, early stopping, dropout.

- Next time:
  - The other crucial piece to get these working for vision problems.

Autoencoders

- Autoencoders are an unsupervised deep learning model:
  – Use the inputs as the output of the neural network.

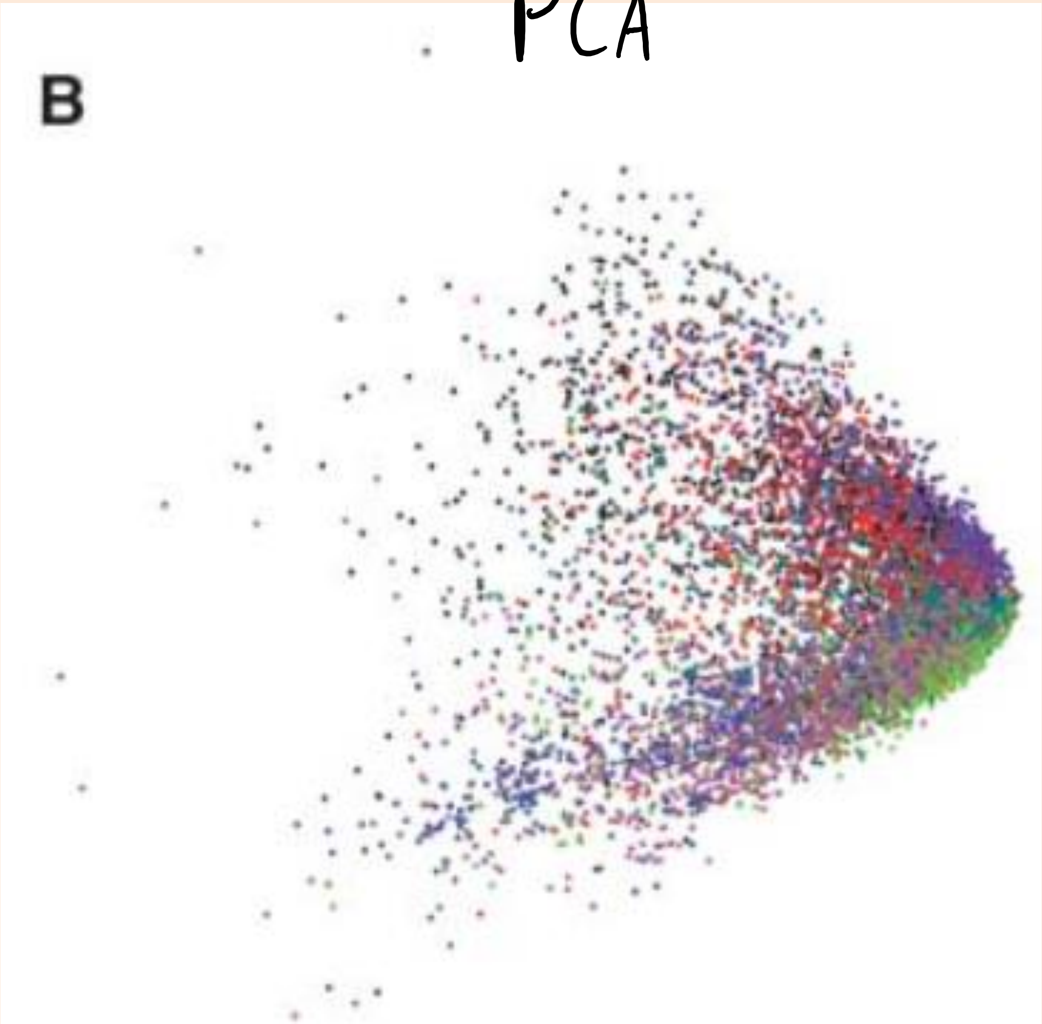


  – Middle layer could be latent features in non-linear latent-factor model.
    - Can do outlier detection, data compression, visualization, etc.
  – A non-linear generalization of PCA.

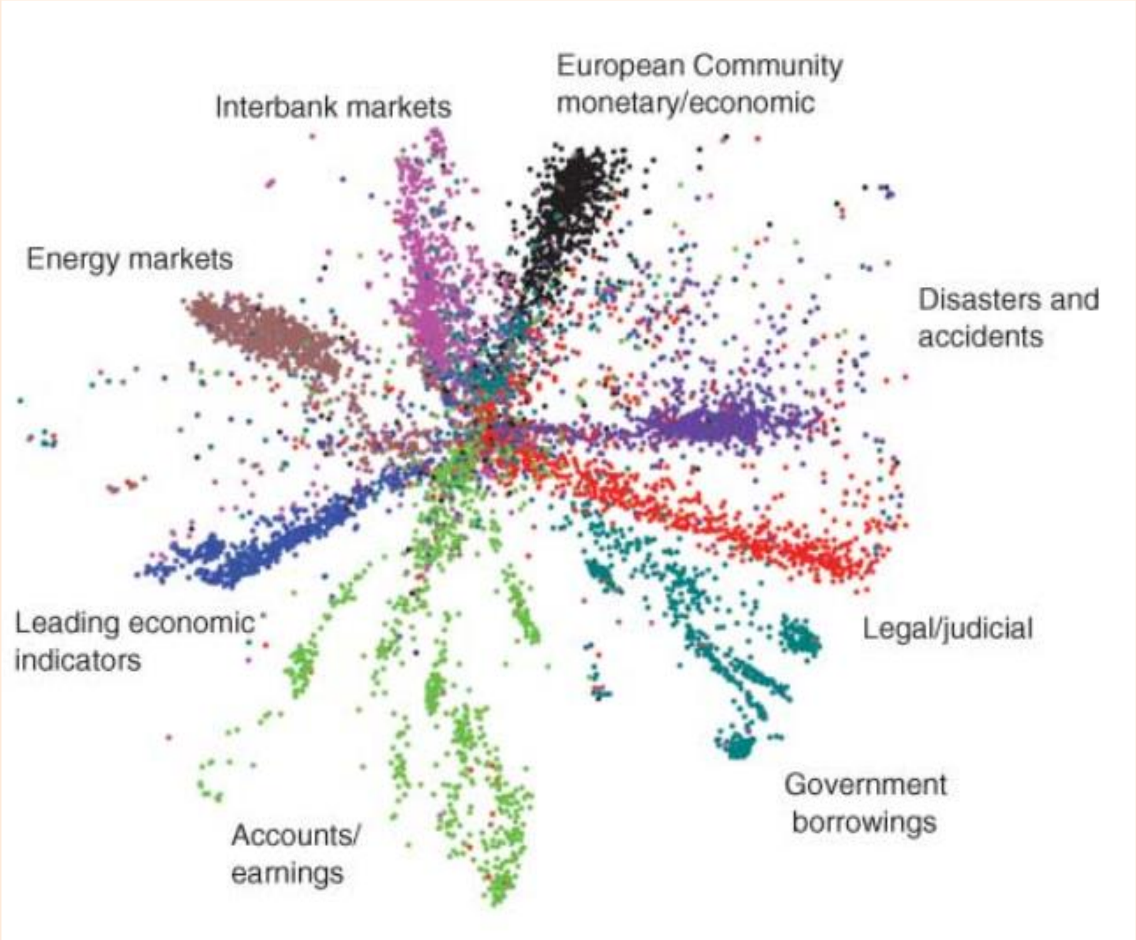
http://inspirehep.net/record/1252540/plots

Autoencoders

PCA



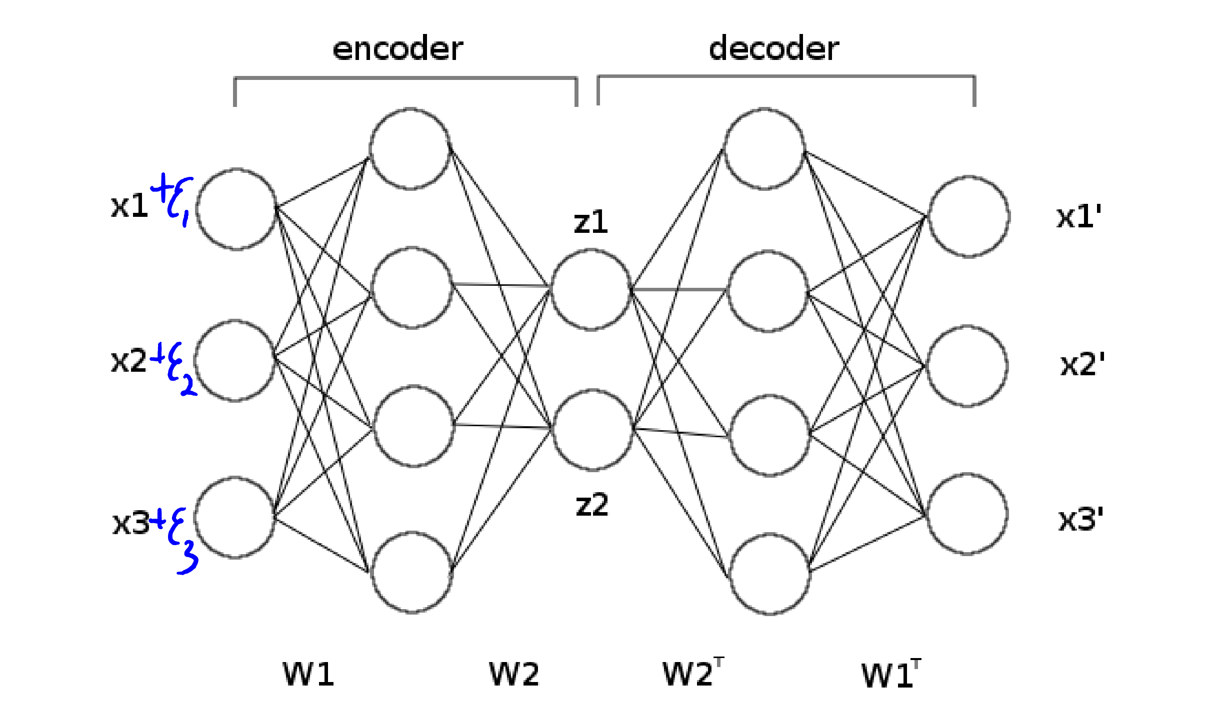
Autoencoder



https://www.cs.toronto.edu/~hinton/science.pdf

Denoising Autoencoder

- Denoising autoencoders add noise to the input:



  – Learns a model that can remove the noise.

http://inspirehep.net/record/1252540/plots