CPSC 340
Assignment 5 Tutorial
Questions 1 to 2.2
Question 1 - Sparse Latent Features

- `example_faces.m` shows an example of using PCA on images
- generates 5 plots:

Figure 1 - Original Images
Question 1 - Sparse Latent Features

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- generates 5 plots:

Figure 1 - Original Images

\[ X \]

32x32 each vector or 1024
Question 1 - Sparse Latent Features

- `example_faces.m` - Figure 2: \( \mu = \text{mean}(X) \);
Question 1 - Sparse Latent Features

- example_faces.m - Figure 3: \[ \text{SVD}(X) = U, \Sigma, V^T \]

\[ W = V[1 : ; 1:k] \]

\[ \frac{1}{2} \| X - ZW \|_2^2 \]
Question 1 - Sparse Latent Features

- `example_faces.m` - Figure 3:

\[
SVD(X) = U, \Sigma, V^T
\]

\[
W = V[1 :, 1 : k]
\]

\[
\frac{1}{2} \| X - ZW \|^2_2
\]

Each eigenvector is 32x32

Eigenvectors (Eigenfaces)
Question 1 - Sparse Latent Features

- example_faces.m - Figure 3: $\text{SVD}(X) = U, \Sigma, V^T$
  $$W = V[1:, 1: k]$$
  $$\frac{1}{2} ||X - ZW||_2^2$$

Change the value of “k” to get more eigenvectors

Each eigenvector is 32x32

Eigenvectors (Eigenfaces)
Question 1 - Sparse Latent Features

- example_faces.m - Figure 4: \[ \mathbf{Z} = \mathbf{XW}^T \]

- Compressed data
- Original data
- Eigenvectors of \( \mathbf{X}^T \mathbf{X} \)

\[ \begin{align*} \mathbf{X} & \quad \text{n x d} \\ \mathbf{W} & \quad k \times d \\ \mathbf{Z} & \quad \text{n x k} \end{align*} \]
Reconstructed $X$

$\frac{1}{2} \| X - ZW \|_2^2$

$\hat{X} = ZW$
Question 1.1 - Sparse Latent Features

If you re-run the script, you may get different principal components, even though all that changes between runs is the order of the training examples. What is the specific difference between the principal components that are obtained between different runs of the algorithm?

Observe what changes between these two

\[ X \sim Z \sim W \]

\[ n \times d \quad n \times k \quad k \times d \]
Question 1.2 - Non-Negative Matrix Factorization

non-negative constraints on $W$. Using `dimRedPCA_alternate` as a template, write a function `dimRedNMF` that implements the non-negative matrix factorization (NMF) model. Hand in your code and hand in a plot of the latent factors (Figure 3) obtained when $k = 100$.

NMF is for optimizing $ZW$ in $\frac{1}{2} \|X - ZW\|_2^2$ such that $Z$ and $W$ have non-zero terms
Question 1.2 - use NMF

a. Optimization stage

Initialize without negative values

$$\frac{1}{2} \| X - ZW \|^2_2$$

findMin uses gradient descent (no constraints)
Question 1.2 - use NMF

a. Optimization stage

Initialize without negative values

\[ \frac{1}{2} \| X - ZW \|_2^2 \]

findMin uses gradient descent (no constraints)

Use gradient descent that enforces non-negative parameters (findMinNN) instead!
Uses least squares - we don’t want that!
Use gradient descent that enforces non-negative parameters (findMinNN) instead!

- NMF results in sparse matrices for $Z$ and $W$ since negative values become zero
- However, the compression ratio is poor - the non-negative constraint strongly limits the model power
Question 1.3 -
Use L1 regularization

a. Optimization stage

We can have negative values!

$$\frac{1}{2} \| X - ZW \|^2_2$$

(findMin) uses gradient descent (no constraints)

Use L1-regularized gradient descent (findMinL1) instead!
Uses least squares - no constraints!

Use L1-regularized gradient descent ([findMinL1](#)) instead!

- L1 Regularization results in sparse matrices for \(Z\) and \(W\)
- The compression ratio is better for using L1 than for using NMF
Question 2 - Recommender Systems

If you run the function `example_movies`, it will load a dataset consisting of movie ratings for different users. The vector $y$ contains the ratings, the first column of $X$ contains the user numbers, and the second column of $X$ contains the movie numbers. The script runs several simple baseline methods, and reports their performance on the validation set.

- No features being represented
- But we can extract latent features that represent the relationships between users, movies, and ratings
Question 2.1 - Latent-Factor Model

We have no features for the user/movies, we must predict the labels based on other labels (collaborative filtering). One way to improve on these methods is with a latent-factor model. Consider a model of the form

$$y_{um} = b_u + b_m + w_m^T z_u,$$
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\[ y_{um} = b_u + b_m + w^T_m z_u \]

- **Latent features (represents hidden relationships)**
  - We can extract them through optimization

- **Bias term, variable for movie ‘m’**

- **Bias term, variable for user ‘u’**
Question 2.1 - Latent-Factor Model

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\[ y_{um} = b_u + b_m + w_m^T z_u \]

We want this equality to hold! Use optimization!

Latent features (represents hidden relationships)
We can extract them through optimization

Bias term, variable for movie ‘\( m \)’

Bias term, variable for user ‘\( u \)’
Question 2.1 - Latent-Factor Model

Consider training this based on the squared loss function, which means that our error for a particular user $u$ and movie $m$ is given by

$$f(b_u, b_m, w_m, z_u) = \frac{1}{2}(y_{um} - (b_u + b_m + w^T_m z_u))^2.$$  

Minimize this objective function

- To optimize this function we can use gradient descent, which involves computing the partial derivatives w.r.t to the unknown variables.
Question 2.1 - Latent-Factor Model

\[ f(b_u, b_m, w_m, z_u) = \frac{1}{2} (y_{um} - (b_u + b_m + w_m^T z_u))^2. \]

Using the notation \( r_{um} = (y_{um} - (b_u + b_m + w_m^T z_u)) \), derive the partial derivative of this expression with respect to (i) \( b_u \), (ii) \( b_m \), (iii) \( (w_m)_i \) for a particular element \( i \) of \( w_m \), and (iv) \( (z_u)_i \) for a particular element \( i \) of \( z_u \).

\[ \frac{\partial f}{\partial b_u} = ? \]
\[ \frac{\partial f}{\partial b_m} = ? \]
\[ \frac{\partial f}{\partial (w_m)_i} = ? \]
\[ \frac{\partial f}{\partial (z_u)_i} = ? \]
Question 2.1 - Optimization problem example

\[ g(x) = x^2 + 1 \]

\[ f(x) = \sqrt{x} \]
Question 2.1 - Optimization problem example

\[ P(x_1, x_2) = \frac{1}{2} (f(x_1) - g(x_2))^2 + \frac{1}{2} (x_1 - x_2)^2 \]
Question 2.1 - Optimization problem example

Algorithm
1. Initialize random values for $x_1, x_2$
2. Update $x_1, x_2$ as follows,

\[ x_1 := x_1 - \alpha \frac{\partial P}{\partial x_1} \]
\[ x_2 := x_2 - \alpha \frac{\partial P}{\partial x_2} \]

\[ \alpha > 0 \]

learning rate

\[ P(x_1, x_2) = \frac{1}{2} (f(x_1) - g(x_2))^2 + \frac{1}{2} (x_1 - x_2)^2 \]

\[ \frac{\partial P}{\partial x_1} = x_1 - x_2 - \frac{1}{\sqrt{x_1}} \left( -\frac{\sqrt{x_1}}{2} + \frac{x_2^2}{2} + \frac{1}{2} \right) \]

\[ \frac{\partial P}{\partial x_2} = -x_1 + 2x_2 (x_2^2 - \sqrt{x_1} + 1) + x_2 \]
Question 2.2 - Gradient descent

- Gradient Descent
  - Uses the complete dataset per iteration
  - Very costly for datasets with over million samples
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Accumulates gradients for each sample
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Accumulates gradients for each sample

Updates variables
Question 2.2 - Gradient descent

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- Use **Stochastic Gradient Descent** instead
  - Uses one random sample at a time to update the variables.

```matlab
% Optimization
maxIter = 10;
alpha = 1e-4;
for iter = 1:maxIter
  % Compute gradient
  gu = zeros(n,1);
gm = zeros(d,1);
gW = zeros(k,d);
gZ = zeros(n,k);
  for i = 1:nRatings
    % Make prediction for this rating based on current model
    u = X(i,1);
m = X(i,2);
yhat = bu(u) + bm(m) + W(:,m)'*Z(u,:);
    % Add gradient of this prediction to overall gradient
    % (follows from chain rule)
r = y(i) - yhat;
gu(u) = gu(u) - r;
gm(m) = gm(m) - r;
gW(i,:) = gW(i,:) - r*Z(u,:);
gZ(u,:) = gZ(u,:) - r*W(:,m);
  end
% Take a small step in the negative gradient directions
bu = bu - alpha*gu;
bm = bm - alpha*gm;
W = W - alpha*gW;
Z = Z - alpha*gZ;
% Compute and output function value
f = 0;
  for i = 1:nRatings
    u = X(i,1);
m = X(i,2);
yhat = bu(u) + bm(m) + W(:,m)'*Z(u,:);
f = f + (1/2)*(y(i) - yhat)^2;
  end
fprintf('Iter - %d, f - %.2e\n',iter,f);
end
```
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Accumulates gradients for each sample

Use the gradient for the chosen sample only!
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Insert the update rules inside the loop

We update every time we choose a random sample