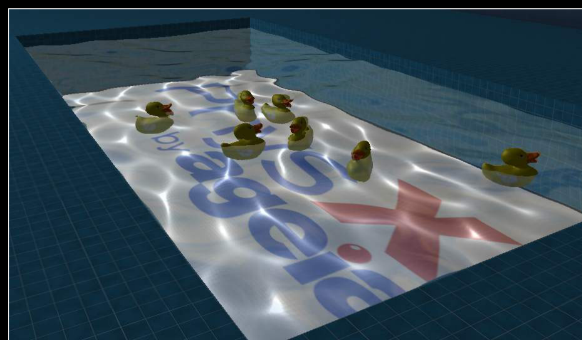
Real Time Fluids in Games



Matthias Müller

ageia

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ageia

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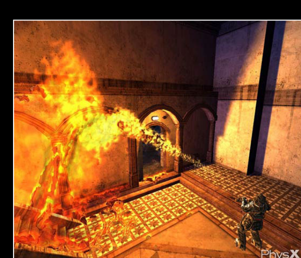
- PhysX accelerator chip (PPU)
  www.ageia.com
- PhysX SDK
  (PPU accelerated, free!)
  - Rigid bodies + joints
  - Cloth simulation
  - Soft bodies
  - Fluid simulation (SPH)
  - Heightfield fluids to come



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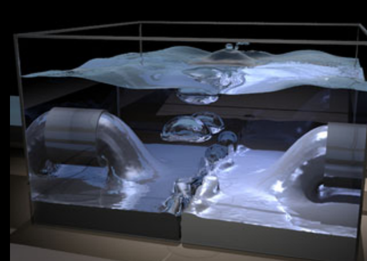
Outline

- Fluids in Games
- Heightfield Fluids
  - A very simple program
  - Physics background
  - Object interaction
- Particle Based Fluids
  - Simple particle systems
  - Smoothed Particle Hydrodynamics (SPH)

Offline Fluid Simulation

- State of the art is impressive!
- Google "Robert Bridson", "Ron Fedkiw", "James O'Brien", …

Gas



Liquids

Offline Simulation Times

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- Typical grid size 256^3 cells
- Linear system with 16 million unknowns!
- Level sets on even finer grids
- Raytracing (reflection / refraction / caustics)
- Photorealistic results
- 10 seconds – 50 minutes per frame!

Game Requirements

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- CHEAP TO COMPUTE!
  - 40-60 fps of which fluid only gets a small fraction
- Stable even in non-realistic settings
  - Game characters sometimes “walk” at 50 mph
- Low memory consumption
  - Must run on consoles
- Challenge:
  - Get as close as possible to offline results while meeting all these constraints!

Reducing Computation Time

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- Reduce resolution (lazy ☹)
  - Simple (use same algorithms)
  - Results look blobby and coarse, details disappear
- Invent new methods (do research ☺)
  - Reduce dimension (e.g. from 3d to 2d)
  - Use different resolutions for physics and appearance
  - Simulate only in interesting, active regions (sleeping)
  - Camera dependent level of detail (LOD)
  - Non-physical animations for specific effects

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Solutions

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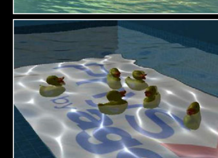
- Procedural Water
  - Unbounded surfaces, oceans
- Heightfield Fluids
  - Ponds, lakes
- Particle Systems
  - Splashing, spray, puddles, smoke



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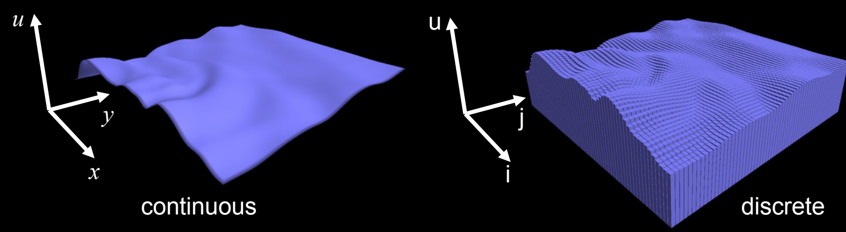
Procedural Animation

- Simulate the effect, not the cause
  [Bridson07], [Yuksel07], [Fournier86], [Hinsinger02]
- No limits to creativity
  - E.g. superimpose sine waves
- Difficult but not impossible
  - Fluid – scene interaction

Heightfield Fluids

Heightfield Fluids



- Represent fluid surface as a 2D function $u(x,y)$
- Pro: Reduction from 3D to 2D
- Cons: One value per (x,y) $\rightarrow$ no breaking waves

Fluid Simulation „Hello World“

- A trivial algorithm with impressive results! Try this at home!
- Initialize `u[i,j]` with some interesting function
- Initialze `v[i,j]=0`

```
loop
   v[i,j] +=(u[i-1,j] + u[i+1,j] + u[i,j-1] + u[i,j+1])/4 – u[i,j]
   v[i,j] *= 0.99
   u[i,j] += v[i,j]
endloop
```

- Clamp on boundary e.g. def. `u[-1,j] = u[0,j]`

Bars Demo

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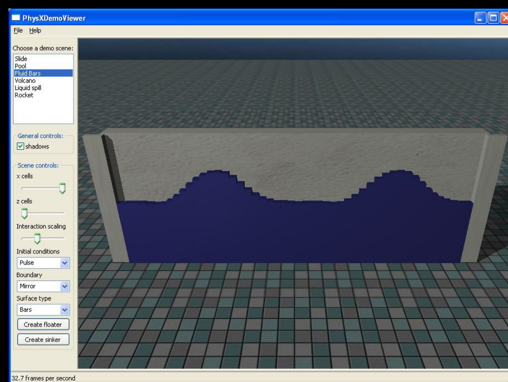

The Physics Behind it

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- We model the water surface as an elastic membrane with low stiffness [Jeffrey02]
- Fairly good approximation
- Better: Derive a more complex surface model from the Navier Stokes Equations [Thuerey07]
- Most games use procedural water today
- Membrane model is an improvement and often sufficient for games.

The Physics Behind It

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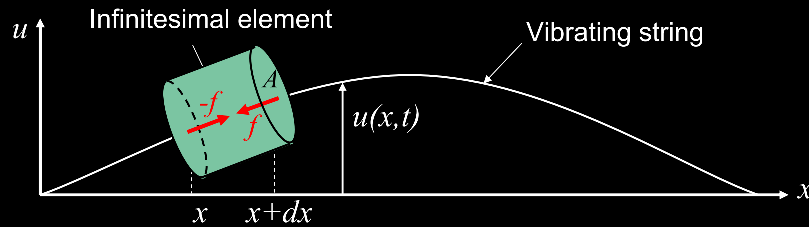
- A one dimensional membrane is a string



- $u(x,t)$ displacement normal to x-axis at time t
- Assuming small displacements and constant stress σ
- Force acting normal to cross section A is $f = \sigma A$

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PDE for the 1D String

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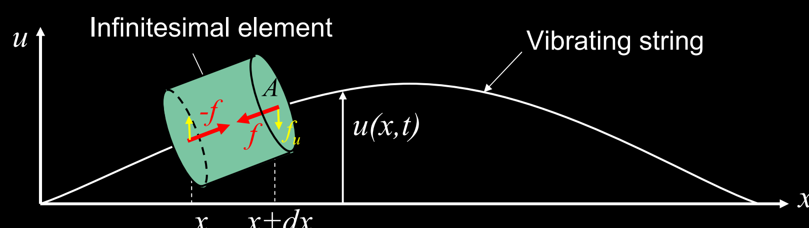

- Component of $f = \sigma A$ in u-direction:
  $f_u \approx u_x \sigma A$ where u_x is derivative of u w.r.t. x
- Newton's 2nd law for an infinitesimal segment

$$(\rho A dx) u_{tt} = \sigma A u_x|_{x+dx} - \sigma A u_x|_x \rightarrow \boxed{\rho u_{tt} = \sigma u_{xx}}$$

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The 1D Wave Equation

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- For the string: $\rho\, u_{tt} = \sigma\, u_{xx}$
- Standard form: $u_{tt} = c^2\, u_{xx}$, where $c^2 = \sigma/\rho$
- Solution: $u(x,t) = a \cdot f(x + ct) + b \cdot f(x - ct)$ for any function f.
- Thus, c is the speed at which waves travel

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The 2D Wave Equation

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- The wave equation generalizes to 2D as

$$u_{tt} = c^2 (u_{xx} + u_{yy})$$

$$u_{tt} = c^2 \nabla^2 u$$

$$u_{tt} = c^2 \Delta u$$

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Discretization

- Replace the 2nd order PDE by two first order PDEs

  $u_t = v$

  $v_t = c^2 (u_{xx} + u_{yy})$

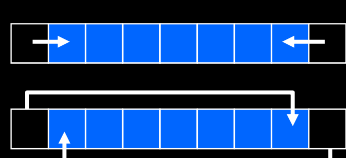
- Discretize in space and time
  (semi-implicit Euler, time step Δt, grid spacing h)

```
v^(t+1)[i,j] = v^t[i,j]
          +Δtc^2(u[i+1,j]+u[i-1,j]+u[i,j+1]+u[i,j-1]-4u[i,j])/h^2
u^(t+1)[i,j] = u^t[i,j] +Δt v^(t+1)[i,j]
```

- We are where we started! (correct scaling, no damping)

Remarks on Heightfields

- The simulation is only conditionally stable
  - Stability condition: $\Delta t < h/c$
- Boundary conditions needed

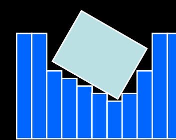


Clamp: Reflection

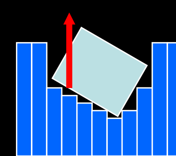
Periodic: Wrap around

Object Interaction

- Object → Water
  - Object pushes bars beneath it down
  - Add the removed water in the vicinity!

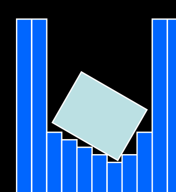


- Water → Object
  - Each bar below the object applies force $\mathbf{f} = -\Delta u\ \rho\ h^2\ \mathbf{g}$ to body at its location
  - Δu is the height replaced by the body, ρ water density, $\mathbf{g}$ gravity



Fully Immersed Bodies

- Body below water surface
- Hole appears above the body
- Non-physical
- See story of divided sea



Solution

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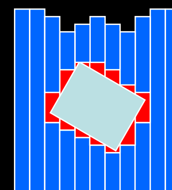
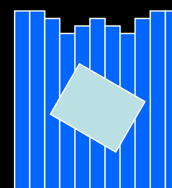
- New state variable `r[i,j]`:
  - Each column stores the part `r[i,j]` of `u[i,j]` currently replaced by solids
- At each time step:
  - `u[i,j]` is not modified directly
  - $\Delta r[i,j] = r^{t}[i,j] - r^{t-1}[i,j]$ is distributed as water `u` to the neighboring columns
  - In case of a negative difference water is removed





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From Bars to Water

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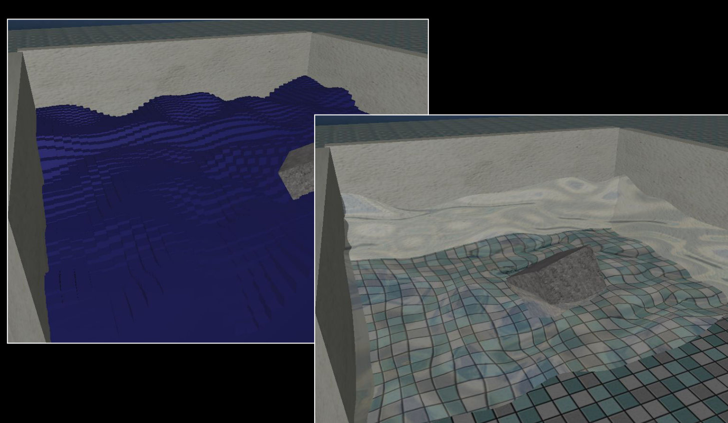

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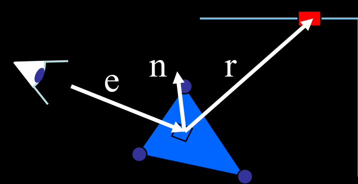
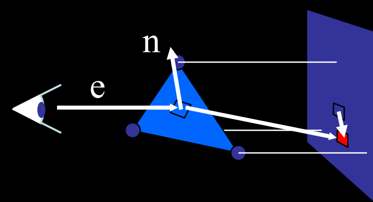
Water Rendering

- Reflection



cube map

- Refraction

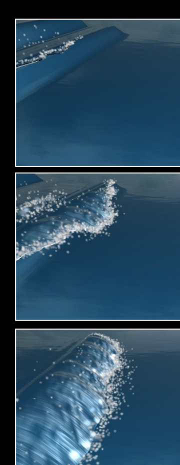


Scene below surface rendered to texture

- Caustics: Cheating - Animated texture

Breaking Waves

- Heightfields cannot capture breaking waves
- Identify steep wave fronts
- Construct and track line along front
- Emit patch of connected particles
- Generate mesh with given thickness for rendering (plus particles for foam)
- See [Thuerey07]



Breaking Waves

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Particle Based Fluids

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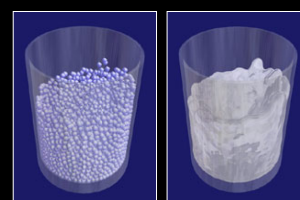
Particle Based Fluids

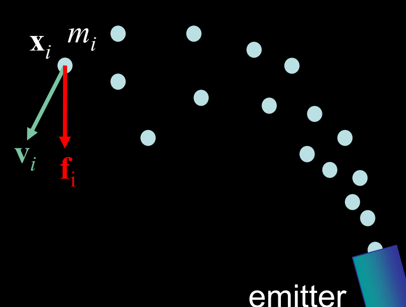
- Particle systems are simple and fast
- Without particle-particle interaction
  - Spray, splashing

- With particle-particle interaction
  - Small puddles, blood, runnels
  - Small water accumulations



Simple Particle Systems

- Particles store
  mass, position, velocity,
  external forces, lifetimes
- Integrate
  $d/dt\ \mathbf{x}_i = \mathbf{v}_i$
  $d/dt\ \mathbf{v}_i = \mathbf{f}_i/m_i$
- Generated by emitters,
  deleted when lifetime is exceeded



Particle-Particle Interaction

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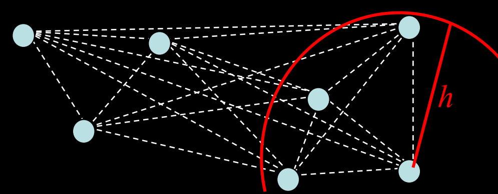

- No interaction → decoupled system → fast
- For n particles $O(n^2)$ potential interactions!
- To reduce to linear complexity $O(n)$ define interaction cutoff distance h

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Spatial Hashing

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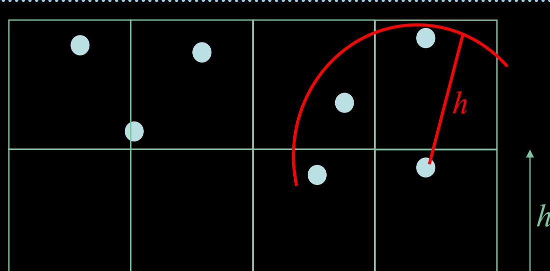

- Fill particles into grid with spacing h
- Only search potential neighbors in adjacent cells
- Map cells $[i,j,k]$ into 1D array via hash function $h(i,j,k)$ [Teschner03]

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Lennard-Jones Interaction

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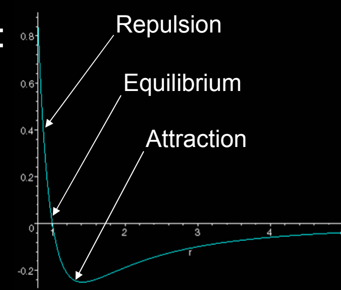
- For simple fluid-like behavior:



$$\mathbf{f}(\mathbf{x}_i, \mathbf{x}_j) = \left(\frac{k_1}{|\mathbf{x}_i - \mathbf{x}_j|^m} - \frac{k_2}{|\mathbf{x}_i - \mathbf{x}_j|^n} \right) \cdot \frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|}$$

- k_1, k_2, m, n control parameters

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Solving Navier-Stokes Eqn.

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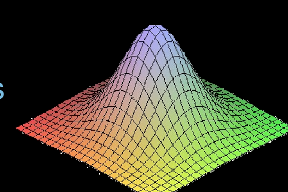
- How formulate Navier-Stokes Eqn. on particles?
- We need continuous fields, e.g. $\mathbf{v}(\mathbf{x})$
- Only have $\mathbf{v}_1, \mathbf{v}_2, .. \mathbf{v}_n$ sampled on particles
- Basic idea:
  - Particles induce smooth local fields
  - Global field is sum of local fields



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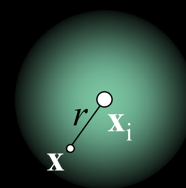
SPH

- Smoothed Particle Hydrodynamics
- Invented for the simulation of stars [Monaghan92]
- Often used for real-time fluids in CG [Müller03]
- Use scalar kernel function $W(r)$
  - $W_i(\mathbf{x}) = W(|\mathbf{x}-\mathbf{x}_i|)$
  - Normalized: $\int\int\int W_i(\mathbf{x})\, d\mathbf{x} = 1$



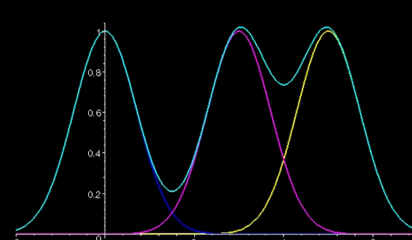
- Example [Müller03]

$$W(r,h) = \frac{315}{64\pi h^9}(h^2 - r^2)^3 \quad 0 \le r \le h$$

Density Computation

- Global density field

$$\rho(\mathbf{x}) = \sum_j m_j W(\mathbf{x} - \mathbf{x}_j)$$



- Density of each particle

$$\rho_i = \rho(\mathbf{x}_i)$$

- Mass conservation guaranteed

$$\int \rho(\mathbf{x})d\mathbf{x} = \sum_j \left(m_j \int W(\mathbf{x} - \mathbf{x}_j)d\mathbf{x}\right) = \sum_j m_j$$

Smoothing Attributes

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- Smoothing of attribute A,
- Given $A_1..A_n \rightarrow$ compute $A(\mathbf{x})$

$$A_s(\mathbf{x}) = \sum_j \frac{m_j}{\rho_j} A_j W(\mathbf{x} - \mathbf{x}_j)$$

- Gradient of smoothed attribute

$$\nabla A_s(\mathbf{x}) = \sum_j \frac{m_j}{\rho_j} A_j \nabla W(\mathbf{x} - \mathbf{x}_j)$$

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Equation of Motion

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$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{v}$$

- Because particles follow the fluid we have:

$$\frac{D\mathbf{v}}{Dt} = \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = \frac{d\mathbf{v}_i}{dt} = \mathbf{a}_i$$

- The acceleration a_i of particle i is, thus

$$\mathbf{a}_i = \frac{\mathbf{f}_i}{\rho_i}$$

$\mathbf{f}_i$ is body force evaluated at $\mathbf{x}_i$

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Pressure

- The pressure term yields

$$\mathbf{f}_i^{\text{pressure}} = -\nabla p(\mathbf{x}_i) = -\sum_j \frac{m_j}{\rho_j} \boxed{p_j} \nabla W(\mathbf{x}_i - \mathbf{x}_j)$$

- Symmetrize (SPH problem: actio ≠ reactio)

$$\mathbf{f}_i^{\text{pressure}} = -\sum_j \frac{m_j}{\rho_j} \boxed{\frac{p_i + p_j}{2}} \nabla W(\mathbf{x}_i - \mathbf{x}_j)$$

- Pressure $p_i = k\,\rho_i$ with k gas constant (stiffness)
- Other state laws possible [Becker07]

Remaining Forces

- External force, e.g. gravity:

$$\mathbf{f}_i^{\text{external}} = \rho_i \mathbf{g}$$

- Viscosity (simmetrized)

$$\mathbf{f}_i^{\text{viscosity}} = \mu \sum_j m \frac{\mathbf{v}_j - \mathbf{v}_i}{\rho_j} \nabla^2 W(\mathbf{x}_i - \mathbf{x}_j)$$

Remarks on SPH

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- Compressibility
  - Pressure force reacts to density variation (bouncy)
  - Predict densities, solve for incompressibility [Premoze03]
- Parameters hard to tune
- Rendering
  - Sprites for smoke, blurry surface
  - Marching cubes for liquids
- Combine particles and heightfields [O'Brien95, Thuerey07]

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Surface Tracking

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- Two main bottlenecks
  - Not the simulation!
  - Collision detection
  - Surface tracking for liquids
- Marching cubes
  - Often used to in offline simulations
  - Generates detailed geometry in non visible places, far from the camera
- Screen Space Meshes [Müller07]

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Screen Space Mesh

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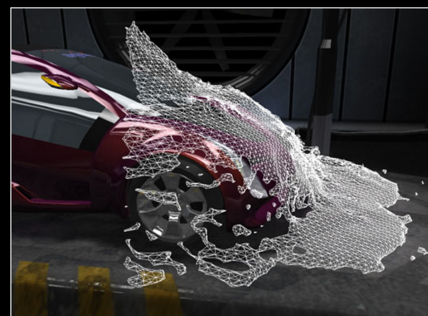

- Regular 2D mesh
- Constructed in screen space
- Modified marching squares

- Vertices projected back to world space
- Frozen mesh, side view

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SPH Demo Scenes

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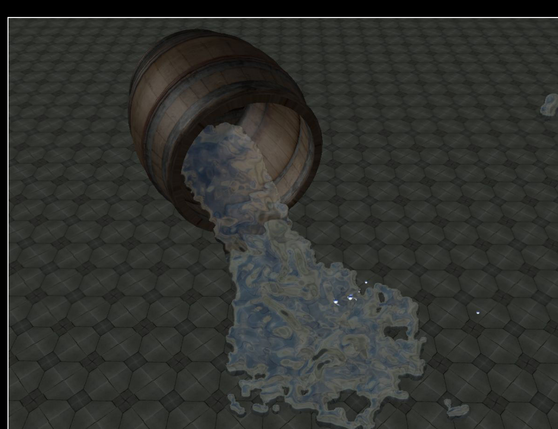

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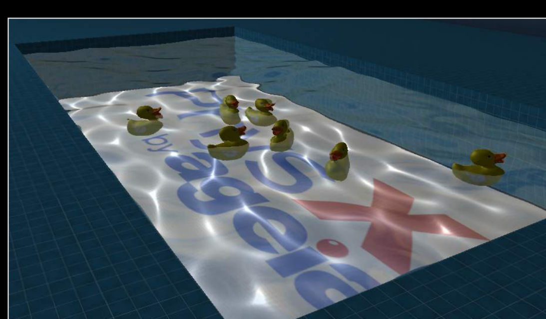
Slides & demos available soon at

www.MatthiasMueller.info

www.cs.ubc.ca/~rbridson/fluidsimulation/



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Questions?