cs542g Final Exam
December 4, 2008

Attempt all questions. Partial marks will be awarded for demonstrating understanding of the relevant material even if you can't fully solve the problem.

1 ....

Given n irregularly placed sample points $\{x_i\}$ in 4D with measured function values $\{f_i\}$, where n is very large, how could you efficiently interpolate the values using RBFs? Address all steps involved, but you needn't derive specific formulas.

2 ....

How can you find a basis for the null-space of a given general rectangular matrix A? (i.e. the subspace of vectors x where $Ax = 0$) How would you handle the case where A is very large and sparse?

3 ....

The Generalized Symmetric Eigenproblem for a symmetric matrix A and SPD matrix B means finding vectors u and real numbers λ such that

$$Au = \lambda Bu$$

There is guaranteed to be a basis of generalized eigenvectors $u_1, \ldots, u_n$ that are B-orthogonal, i.e.

$$u_i^T B u_j = \begin{cases} 1 & : \quad i = j \\ 0 & : \quad i \neq j \end{cases}$$

How could you adapt the basic QR method we looked at to finding such a basis for this generalization? (Assume, as we did, there are no generalized eigenvalues of the same magnitude but different sign, for example.)

4 ....

A smooth convex function will always have a symmetric positive semi-definite Hessian H. It may be singular however. We dealt with this in Newton's method by adding μI to H. However, the pseudo-inverse of H lets us (in some sense) solve an equation with singular H directly. Why is this not as robust?

A similar problem can happen in Gauss-Newton, where we might hit a rank-deficient Jacobian despite the nonlinear problem being well-posed. Does the same problem with the pseudo-inverse appear here?

(hint: recall the notion of a "descent direction")

5 ....

For the 1D Poisson equation $u''(x) = f(x)$, ignoring boundary conditions, discretized on a regular grid with spacing Δx, derive the i'th row of the Galerkin stiffness matrix for piecewise linear nodal basis functions.