cs542g Final Exam
December 5, 2006

Attempt all questions. Partial marks will be awarded for demonstrating understanding of the relevant material even if you can't fully solve the problem.

1.....

In some cases the Moving Least-Squares (MLS) approximation to scattered data points is undefined. **Describe when this can happen.**

One possible solution is to use the pseudo-inverse of the least-squares matrix involved. Another possible solution is to dynamically adjust the width of the kernel to include k sample points for some suitably large and fixed k: i.e. to get the estimate at point x, weights $W(\alpha|x - x_i|)$ are used for sample points x_i, with α chosen so that at least k weights are nonzero. **Discuss the relative merits of these two possibilities.**

2.....

The PDE $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ for time $t > 0$ and one-dimensional space x is first discretized in space with one-sided finite differences as:

$$\left(\frac{\partial u}{\partial t}\right)_i = -\frac{u_{i+1} - u_i}{\Delta x}$$

What are the eigenvalues of the Jacobian? (hint: with boundary conditions unspecified, the eigenvectors are of the form $v_j = e^{i\zeta j}$ where $i = \sqrt{-1}$ and ζ is a real constant) **What sort of time integration scheme will stably converge? Would you suggest any changes to the spatial discretization?**

3.....

The heat equation $\frac{\partial u}{\partial t} = \Delta u$ is to be solved using standard second order finite differences in space on a large 3D uniform grid. A massively parallel architecture is used, whose communication costs mean the best available solver for linear systems is Jacobi iteration. **Discuss the relative merits between an explicit time integration scheme and an implicit scheme for this problem.**

4.....

Consider the linear least squares problem $\min_x ||Ax-b||_2$ when A is large, sparse, and nearly rank-deficient. The QR decomposition is selected for solution. **How might you go about exploiting sparsity to improve the computational efficiency?** (hint: what is R in terms of A^TA?) **What is the impact of one fully dense column in A? What is the impact of one fully dense row in A?**