Notes

Shells

- Simple addition to previous bending formulation: allow for nonzero rest angles
  - i.e. rest state is curved
  - Called a "shell" model
- Instead of curvature squared, take curvature difference squared
  - Instead of θ, use $\theta-\theta_0$

Rayleigh damping

- Start with variational formulation:
  W is discrete elastic potential energy
- Suppose W is of the form $W = \frac{1}{2} k C^T A C$
  - C is a vector that is zero at undeformed state
  - A is a matrix measuring the length/area/volume of integration for each element of C
- Then elastic force is $F_{elastic} = -k \frac{\partial C}{\partial x}^T AC$
  - C says how much force, ∂C/∂X gives the direction
- Damping should be in the same direction, and proportional to ∂C/∂t:

$$F_{damping} = -d \frac{\partial C}{\partial x}^T A \frac{\partial C}{\partial t}$$

- Chain rule:
  - Linear in v, but not in x…

$$= -d \frac{\partial C}{\partial x}^T A \frac{\partial C}{\partial x} v$$

Cloth modeling

- Putting what we have so far together: cloth
- Appropriately scaled springs + bending
- Issues left to cover:
  - Time steps and stability
  - Extra spring tricks
  - Collisions

Spring timesteps

- For a fully explicit method:
  - Elastic time step limit is

$$\Delta t \sim O\left(\sqrt{\frac{mL^2}{EA}}\right) = O\left(\frac{1}{n}\right)$$

  - Damping time step limit is

$$\Delta t \sim O\left(\frac{mL^2}{DA}\right) = O\left(\frac{1}{n^2}\right)$$

  - What does this say about scalability?

Bending timesteps

- Back of the envelope from discrete energy: $\frac{\partial a}{\partial x} \sim \frac{1}{m} B \frac{|e|^2}{A} \frac{\partial^2 \theta}{\partial x^2} = O\left(\frac{L^2}{L^2 L^2 L^2}\right)$

$$\Delta t = O\left(\frac{1}{n^2}\right)$$

- Or from 1D bending problem
  - [practice variational derivatives]

Fourth order problems

- Linearize and simplify drastically, look for steady-state solution (F=0): spline equations
  - Essentially 4th derivatives are zero
  - Solutions are (bi-)cubics
- Model (nonsteady) problem: $x_{tt}=-x_{pppp}$
  - Assume solution $x(p,t) = e^{\sqrt{-1}k(p-ct)}$

    Wave of spatial frequency k, moving at speed c
  - [solve for wave parameters]
  - Dispersion relation: small waves move really fast
  - CFL limit (and stability): for fine grids, BAD
  - Thankfully, we rarely get that fine

Implicit/Explicit Methods

- Implicit bending is painful
- In graphics, usually unnecessary
  - Dominant forces on the grid resolution we use tend to be the 2nd order terms: stretching etc.
- But nice to go implicit to avoid time step restriction for stretching terms
- No problem: treat some terms (bending) explicitly, others (stretching) implicitly
  - $v_{n+1}=v_n+\Delta t/m(F_1(x_n,v_n)+F_2(x_{n+1},v_{n+1}))$
  - All bending is in F_1, half the elastic stretch in F_1, half the elastic stretch in F_2, all the damping in F_2

Hacking in strain limits

- Especially useful for cloth:
  - Biphasic nature: won't easily extend past a certain point
- Sweep through elements (e.g. springs)
  - If strain is beyond given limit, apply force to return it to closest limit
  - Also damp out strain rate to zero
- No stability limit for fairly stiff behaviour
  - But mesh-independence is an issue…
- See X. Provot, "Deformation constraints in a mass-spring model to describe rigid cloth behavior", Graphics Interface '95

Extra effects with springs

- (Brittle) fracture
  - When a spring is stretched too far, break it
  - Issue with loose ends…
- Plasticity
  - Whenever a spring is stretched too far, change the rest length part of the way
- More on this late