Notes

- Typo in blending function for homework 6 viewer
  - Thanks to Albert Wong for pointing it out
- We've almost finished surveying physics-based animation
  - If you want me to review anything, get into more detail anywhere, talk about anything more exotic, let me know!

Fire

- [Nguyen, Fedkiw, Jensen '02]
- Gaseous fuel/air mix (from a burner, or a hot piece of wood, or …) heats up
- When it reaches ignition temperature, starts to burn
  - "blue core" - see the actual flame front due to emission lines of excited hydrocarbons
- Gets really hot while burning - glows orange from blackbody radiation of smoke/soot
- Cools due to radiation, mixing
  - Left with regular smoke

Defining the flow

- Inside and outside blue core, regular incompressible flow with buoyancy
- But an interesting boundary condition at the flame front
  - Gaseous fuel and air chemically reacts to produce a different gas with a different density
  - Mass is conserved, so volume has to change
  - Gas instantly expands at the flame front
- And the flame front is moving too
  - At the speed of the flow plus the reaction speed

Interface speed

- Interface = flame front = blue core surface
- $D=V_f-S$ is the speed of the flame front
  - It moves with the fuel flow, and on top of that, moves according to reaction speed S
  - S is fixed for a given fuel mix
- We can track the flame front with a level set ϕ
- Level set moves by

$$\phi_t + D|\nabla\phi| = 0$$

$$\phi_t + u_{LS} \cdot \nabla\phi = 0$$

- Here u_{LS} is u_f-Sn

Numerical method

- For water we had to work hard to move interface accurately
- Here it's ok just to use semi-Lagrangian method (with reinitialization)
- Why?
  - We're not conserving volume of blue core - if reaction is a little too fast or slow, that's fine
  - Numerical error looks like mean curvature
  - Real physics actually says reaction speed varies with mean curvature!

Conservation of mass

- Mass per unit area entering flame front is $\rho_f(V_f-D)$ where
  - $V_f=u_f \cdot n$ is the normal component of fuel velocity
  - D is the (normal) speed of the interface
- Mass per unit area leaving flame front is $\rho_h(V_h-D)$ where
  - $V_h=u_h \cdot n$ is the normal component of hot gaseous products velocity
- Equating the two gives:

$$\rho_f\left(V_f - D\right) = \rho_h\left(V_h - D\right)$$

Velocity jump

- Plugging interface speed D into conservation of mass at the flame front gives:

$$\rho_f S = \rho_h\left(V_h - V_f + S\right)$$
$$\rho_h V_h = \rho_h V_f + \rho_f S - \rho_h S$$
$$V_h = V_f + \left(\frac{\rho_f}{\rho_h} - 1\right)S$$

Ghost velocities

- This is a "jump condition": how the normal component of velocity jumps when you go over the flame interface
- This lets us define a "ghost" velocity field that is continuous
  - When we want to get a reasonable value of u_h for semi-Lagrangian advection of hot gaseous products on the fuel side of the interface, or vice versa (and also for moving interface)
  - When we compute divergence of velocity field
- Simply take the velocity field, add/subtract $(\rho_f/\rho_h-1)Sn$

Conservation of momentum

- Momentum is also conserved at the interface
- Fuel momentum per unit area "entering" the interface is

$$\rho_f V_f \left(V_f - D\right) + p_f$$

- Hot gaseous product momentum per unit area "leaving" the interface is

$$\rho_h V_h \left(V_h - D\right) + p_h$$

- Equating the two gives

$$\rho_f V_f \left(V_f - D\right) + p_f = \rho_h V_h \left(V_h - D\right) + p_h$$

Simplifying

- Make the equation look nicer by taking conservation of mass:

$$\rho_f \left(V_f - D\right) = \rho_h \left(V_h - D\right)$$

multiplying both sides by -D:

$$\rho_f \left(-D\right)\left(V_f - D\right) = \rho_h \left(-D\right)\left(V_h - D\right)$$

and adding to previous slide's equation:

$$\rho_f \left(V_f - D\right)^2 + p_f = \rho_h \left(V_h - D\right)^2 + p_h$$

Pressure jump

- This gives us jump in pressure from one side of the interface to the other
- By adding/subtracting the jump, we can get a reasonable continuous extension of pressure from one side to the other
  - For taking the gradient of p to make the flow incompressible after advection
- Note when we solve the Poisson equation density is NOT constant, and we have to incorporate jump in p (known) just like we use it in the pressure gradient

Temperature

- We don't want to get into complex (!) chemistry of combustion
- Instead just specify a time curve for the temperature
  - Temperature known at flame front ($T_{ignition}$)
  - Temperature of a chunk of hot gaseous product rises at a given rate to T_{max} after it's created
  - Then cools due to radiation

Temperature cont'd

- For small flames (e.g. candles) can model initial temperature rise by tracking time since reaction: $Y_t+u\cdot\nabla Y=1$ and making T a function of Y
- For large flames ignore rise, just start flame at T_{max} (since transition region is very thin, close to blue core)
- Radiative cooling afterwards:

$$T_t + u \cdot \nabla T = -c_T \left(\frac{T - T_{air}}{T_{\max} - T_{air}} \right)^4$$

Smoke concentration

- Can do the same as for temperature: initially make it a function of time Y since reaction (rising from zero)
  - And ignore this regime for large flames
- Then just advect without change, like before
- Note: both temperature and smoke concentration play back into velocity equation (buoyancy force)

Note on fuel

- We assumed fuel mix is magically being injected into scene
  - Just fine for e.g. gas burners
  - Reasonable for slow-burning stuff (like thick wood)
- What about fast-burning material?
  - Can specify another reaction speed S_{fuel} for how fast solid/liquid fuel turned into flammable gas (dependent on temperature)
  - Track level set of solid/liquid fuel just like we did the blue core