Notes

- Please read O'Brien and Hodgins, "Graphical modeling and animation of brittle fracture", SIGGRAPH'99

Applying collision impulses

- For particles
  - a collision impulse can be applied directly
- For rigid bodies
  - Apply impulse to the centre-of-mass velocity and the corresponding torque to the angular momentum
  - Whole body "feels" impulse
  - Didn't discuss it, but under assumption that impulse works over an area (not just a point) can apply arbitrary torque
    - Important for rolling friction
- For deformable objects, a bit more complex

Cloth impulses

- Want to change the relative velocity of the two material points of collision in a particular way
  - Normal component zero, tangential component reduced by friction
- But those material points may be midway between actual vertices of the mesh (edge/edge or vertex/triangle)
- Need to apply impulses to the surrounding vertices that will interpolate to give the right relative velocity at the collision point

Barycentric weights

- Recall barycentric coordinates of a vertex x
  - For an edge 1-2 weights a and b such that a+b=1, $ax_1+bx_2=x$
  - For a triangle 1-2-3 weights a,b,c such that a+b+c=1, $ax_1+bx_2+cx_3=x$
- Give us linear interpolation of quantities
  - E.g. velocity at point in edge is av_1+bv_2
  - Consistent with piecewise linear mesh
- We compute these to detect if a collision occurs
- We use them to figure out the equations for relative velocity at the points in question

Distributing the impulse

- Reasonable model:
  - For edge/edge, with barycentric coordinates a,b in first edge and c,d in second edge:
    - Apply aJ and bJ to first edge's endpoints
    - Apply -cJ and -dJ to second edge
  - For vertex/triangle, with barycentric coordinates a,b,c in triangle:
    - Apply J to vertex
    - Apply -aJ, -bJ, -cJ to triangle
  - In both cases, figure out J from requirement of what happens to relative velocity of the two colliding points
  - [example]

Quasi-static approximation

- Back to 3D elasticity…
- For many deformable objects, transient velocities damp out almost instantly
  - Unimportant visually
- So assume that object instantly comes to force equilibrium: "quasi-static"
- Then just need to solve $F_{int}+F_{ext}=0$ for positions (F=ma with v=0 and a=0)
  - Note: mass irrelevant
- For linear elasticity, just a linear system to solve for positions [actually limit of B.E.]

Schur complement

- As before with modal analysis, attractive to consider just solving for boundary
- Assume internal forces are known (zero, or at least in null-space of stiffness matrix)
- Then can eliminate interior unknowns from linear system
- Left with new linear system to solve for boundary: the Schur complement
- Also called "condensation"
- [do it]

Boundary Element Method

- Can actually do the same Schur complement procedure **before** discretization, in function spaces
  - Get the Boundary Element Method (BEM), similar to FEM
- See James & Pai, "ArtDefo…", SIGGRAPH'99
  - For interactive rates, can actually do more: preinvert BEM stiffness matrix
  - Need to be smart about updating inverse when boundary conditions change…

Inelastic behaviour

- Real materials do not stretch indefinitely
- Typically, as applied force increases, permanent deformations take place
  - Begin in elastic regime
  - Then reach yield point (start of plasticity)
    - If force released after this point, returns to a different rest state
  - Then reach point of failure - material breaks (fracture)
  - [draw loading curve]

1D ideal plasticity

- Stress, strain are just scalars
- Write total strain as sum of elastic and plastic parts: $\varepsilon=\varepsilon_e+\varepsilon_p$
- Stress only depends on elastic part (so rest state includes plastic strain): $\sigma=\sigma(\varepsilon_e)$
- When $|\sigma| \geq \sigma_y$ (yield stress), change some of ε_e into ε_p so that $\sigma=\sigma_y$

1D work hardening

- For many materials, yield stress increases as plastic deformation happens: “work hardening”
  - [aside on dislocations]
- Simple model: whenever you change plastic strain, increase yield stress proportionally
  - σ_y += k $|\Delta\varepsilon_p|$
- Note work hardening is irreversible

1D Creep plasticity

- Model so far only includes plastic deformation when applied force increases
- Many materials continue to deform plastically with a constant applied load
- Called “creep”
- Have to model $d\varepsilon_p/dt$

1D fracture

- When σ increases to a fracture (failure) threshhold σ_f material breaks at that point
  - Generally very different for compression and tension
- Can model it either by:
  - Eliminating the over-stressed spring
    [note issue of volume loss]
  - Or somehow doing fracture test at nodes and splitting there instead
    [note issue of mass distribution]

For next week

- How do you determine when plasticity starts? (in 2D/3D not clear)
- How does the plastic part of strain change?
- How do you test for fracture?
- How does fracture proceed?