1 Implementation

In this assignment you will use Fourier synthesis to produce an animation of a section of ocean water (deep water waves) using the Phillips spectrum. In particular, your program should take a parameter file containing the grid size (e.g. 1024), the world-space grid size (e.g. 500m), the wind vector W, the amplitude A (used in the Phillips spectrum), how many seconds to simulate, and a frame rate. It will then produce a sequence of text files containing the height field, where the dimensions (two integers such as 1024 1024) are given first followed by the floating point values of the data.

Strongly consider using a different language which makes FFT's simpler (e.g. Octave, Matlab) for this assignment! Or you may wish to investigate the FFTW library, some versions of which are installed on the CS machines (type "use -L" to get a list of packages which have been installed).

2 Analysis

For our simple deep ocean model, which waves move the fastest?

One way to model surface tension is to change the pressure boundary condition at the surface of the water from just atmospheric pressure p_{atm}, which we relabeled to 0, to the sum of atmospheric pressure and a mean curvature term: $p_{atm} - \sigma\kappa$ where σ is the constant surface tension coefficient. For small waves, following the modeling done in class, we can approximate κ by:

$$\kappa \approx \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2}$$

Write down the new pressure boundary condition for ϕ with surface tension. Recall that it came from Bernoulli's equation evaluated at the water surface:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}|u|^2 + \frac{p}{\rho} = gh$$

with the simplifying assumption that $|u|^2 \approx 0$ and can be ignored.

Derive the dispersion relation (i.e. the relationship between ω and K) with surface tension. Remember that the solution for ϕ before applying the pressure boundary condition is

$$\phi = A\frac{\omega}{|K|}\frac{cosh\,(|K|(y+H))}{sinh\,(|K|H)}sin\,(K \cdot (x,z) - \omega t)$$

Also remember we apply the boundary condition at $y = 0$, since h is so small. (That is, replace y with 0 where you see it in the expression for ϕ.) To warm up for this, you might want to try deriving the dispersion relation without surface tension first (and make sure you get the same answer $\omega = \pm\sqrt{g|K|tanh(|K|H)}$ from class). You can use the fact that $h(x,z,t)$ will be some multiple (up to you to find) of $cos(K \cdot (x,z) - \omega t)$.

For which waves (big or small?) has c changed the most with the addition of surface tension - have they gotten faster or slower?

3 Project Ideas

You can extend this assignment in a number of ways for your final project, for example:

- Add surface tension as in the analysis, and simulate a stone dropped in a lake.
- Make the waves choppy.
- Add spray particles that fly up from the tops of very steep waves.

- Simulate the shallow-water equations for waves close to shore, and compare.
- Float objects (rigid bodies) on top of the water. (They don't need to effect the water if small enough)